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GEOMETRIYA 9

O'zbekiston Respublikasi Xalq ta'limi vazirligi umumiy o'rta ta'lim maktablarining 9-sinfi uchun darslik sifatida tasdiqlagan

«O'zbekiston milliy ensiklopediyasi»
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9-sinfda geometriyaning planimetriya qismini — yassi geometrik shakllarning xossalarini o‘rganish davom ettiriladi. Unda siz geometrik shakllarning o‘xshashligi, uchburchakning tomonlari va burchaklari orasidagi munosabatlar, aylana uzunligi va doira yuzi, uchburchak va aylanadagi metrik munosabatlar bilan tanishasiz.

Ushbu “Geometriya” darsligining mazmuni qat’iy aksiomatik tizim asosiga qurilgan. Unda nazariy materiallar imkon boricha sodda va ravon tilda bayon etilgan. Barcha mavzu va tushunchalarni turli-tuman hayotiy misollar orqali ochib berishga harakat qilingan. Har bir mavzudan so‘ng berilgan savollar, isbotlash, hisoblash va yasashga doir masala va misollar o‘quvchini ijodiy fikrlashga undaydi, unga o‘zlashtirilgan bilimlarni chuqurlashtirishga va mustahkamlab borishga yordam beradi. Darslik o‘zining o‘zgacha dizayni va dars materialining ko‘rgazmali qilib taqdim etilishi bilan ham ajralib turadi. Unda keltirilgan rasm va chizmalar dars materialini yaxshiroq o‘zlashtirishga xizmat qiladi.

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MUNDARIJA

SO‘ZBOSHI

Aziz o‘quvchilar!

Axborot texnologiyalari asrida yashayapmiz. Zamonaviy taraqqiyotda kechayotgan olamshumul o‘zgarishlar zamirida, albatta, fan va texnika rivoji yotibdi. Bunday sharoitda yoshlarning asosiy vazifasi buyuk ajdodlarimizga munosib avlod bo‘lib, zamon bilan hamnafas qadam tashlash, ilm-fan cho‘qqilarini qunt bilan egallashdan iboratdir. Bu borada matematikaning tutgan o‘rni beqiyosdir.

Ma’lumki, matematika siz yoshlarning kamolga yetishingizda o‘quv fani sifatida keng imkoniyatlarga ega. U tafakkuringizni rivojlantirib, aqlingizni peshlaydi, mantiqiy fikrlash, topqirlik xislatlarini shakllantiradi va turli vaziyatlarda oqilona qaror qabul qilish, tahlil qilish hamda xulosa chiqarish ko‘nikmalarini tarbiyalaydi.

Qo‘lingizdagi 9-sinf “Geometriya” darsligining asosiy vazifasi — yassi geometrik shakllarning xossalarini o‘rgatish bilan bir qatorda sizda izchil mantiqiy fikrlash qobiliyatini o‘stirib borish natijasida aqlingizni charxlashdan iborat. U o‘zlashtirilgan bilim, ko‘nikma va malakalarni kundalik turmushga tatbiq etishingizga ko‘maklashadi.

Darslikni yaratishda dunyoda to‘plangan ilg‘or tajriba namunalaridan foydalandik. Shu bilan bir qatorda yurtimizga xos sharqona va umrboqiy qadriyatlarimizga, buyuk ajdodlarimiz merosiga ham muntazam murojaat etishga harakat qildik.

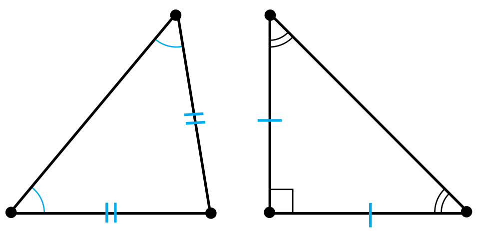
Mazkur darslikdan ta’lim olar ekansiz, sizga bu mas’uliyatli, shu bilan birga maroqli yo‘lda qunt va sabot tilab qolamiz. Geometriya asoslari bo‘yicha olgan saboqlaringiz sizni barkamollik sari yetaklab, Vatanimiz taraqqiyoti yo‘lida xizmat qilishga ko‘makchi bo‘ladi, deb ishonch bildiramiz!

- ✔ — yangi kiritilayotgan geometrik tushunchaning ta'rifi
- — teoremaning tavsifi
- — namuna tariqasida yechib ko'rsatilayotgan masala
- ? — savol, masala va topshiriqlar
- — o'quvchilar faolligini oshiruvchi mustaqil yoki guruhlarda muhokama qilinadigan topshiriqlar
- — yakka tartibda yoki guruhlarda bajariladigan amaliy ish
- — tarixiy ma'lumotlar va masalalar
- — qiziqarli masalalar va boshqotirmalar
- — Internetdan tavsiya etiladigan ma'lumotlar manzili
- **8.** — uyda yechishga tavsiya etiladigan masalalar boshqa rangda berilgan

Teorema yoki masalaning sxematik talqini

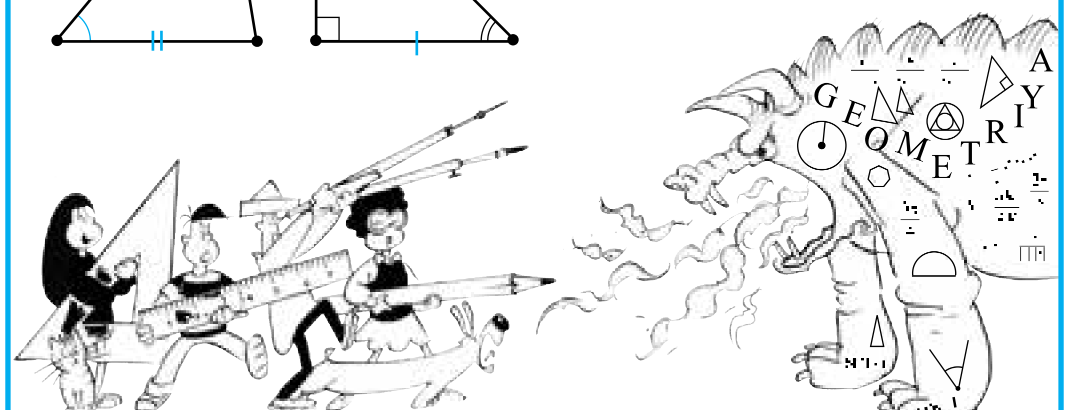
Teorema yoki masala shartida berilgan ma'lumotlar ⇒ Isbot qilinishi kerak bo'lgan xossa yoki topilishi talab etiladigan elementlar

Chizmalarda qabul qilingan alohida belgilar

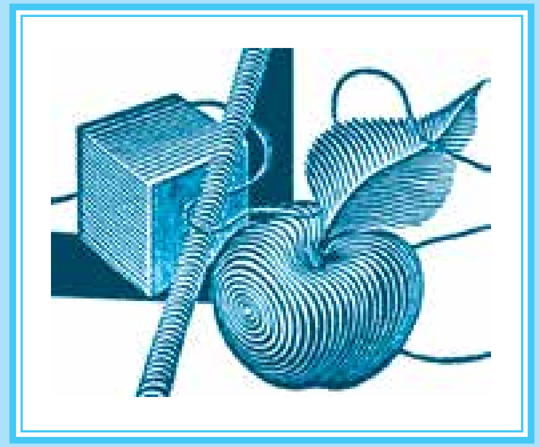


Chizmalarda teng burchaklar bir xil sondagi yoychalar bilan ajratiladi.

Chizmalarda uzunligi teng kesmalar bir xil sondagi chiziqchalar bilan ajratiladi.



Geometriyani ishg'ol qilish uchun olg'a!



TAKRORLASH

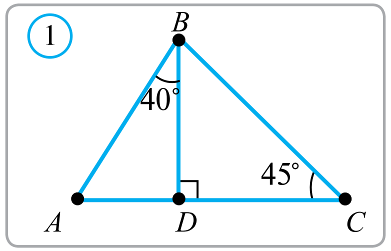
7–8-SINFLARDA O'TILGANLARNI TAKRORLASH

√ *7–8-sinflarda geometriyadan o'tilgan mavzularni takrorlab, olgan bilimlaringizni esga olasiz va erishgan ko'nikmalaringizni mustahkamlaysiz.*

√ *Bu sizga 9-sinfda geometriyani o'rganishni muvaffaqiyatli davom ettirishingizga zamin yaratadi.*

1 UCHBURCHAKLAR

Mazkur bo‘limdagi masalalar 7–8-sinflarda o‘rganilgan geometrik shakllar va ularning xossalarini yodga olish uchun berilmoqda. Masalalarni yechish uchun darslikning oxirida keltirilgan asosiy geometrik shakllarga oid ma’lumotlar hamda ularning xossalarini ifodalovchi formulalardan foydalanishingiz mumkin.



1-masala. *ABC* uchburchakning *BD* balandligi o‘tkazilgan *(1-rasm)*. Agar $\angle ABD = 40°$, $\angle BCD = 45°$ bo‘lsa, uchburchakning *A* va *B* uchidagi burchaklarini toping.

Yechilishi. 1) To‘g‘ri burchakli *ABD* uchburchakda $\angle ABD = 40°$ va uchburchak ichki burchaklarining yig‘indisi 180° ga teng bo‘lgani uchun

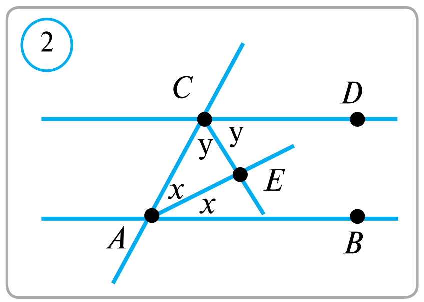
$$\angle A = 180° - (90° + 40°) = 50°.$$

2) To‘g‘ri burchakli *BCD* uchburchakda $\angle BCD = 45°$ bo‘lgani uchun

$$\angle DBC = 180° - (90° + 45°) = 45°.$$

$\angle ABC = \angle ABD + \angle DBC$ bo‘lgani uchun $\angle B = 40° + 45° = 85°$.

Javob: 50°, 85°.



2-masala. Ikki parallel to‘g‘ri chiziqni kesuvchi bilan kesganda hosil bo‘lgan ichki bir tomonli burchaklarning bissektrisalari orasidagi burchakni toping.

Yechilishi. *AC* to‘g‘ri chiziq *AB* va *CD* – parallel to‘g‘ri chiziqlarni 2-rasmda tasvirlangandek kesib o‘tgan bo‘lsin. Ichki bir tomonli *BAC* va *ACD* burchaklarning bissektrisalari *E* nuqtada kesishgan bo‘lib, $\angle EAC = x$, $\angle ECA = y$ bo‘lsin. Unda, burchak bissektrisasining ta’rifiga ko‘ra

$$\angle BAC = x + x = 2x, \qquad \angle ACD = y + y = 2y.$$

$AB \| CD$ bo‘lgani uchun ichki bir tomonli burchaklar xossasiga ko‘ra,

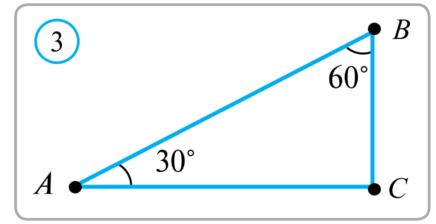
$$2x + 2y = 180°, \qquad x + y = 90°.$$

Endi, *ACE* uchburchak ichki burchaklari yig‘indisi 180° ga teng bo‘lgani uchun

$$\angle AEC = 180° - (x + y) = 180° - 90° = 90°.$$

Javob: 90°.

3-masala. *ABC* uchburchakning *AB* tomoni 6 *sm*, *A* va *B* burchaklari, mos ravishda, 30° va 60° bo‘lsa, *ABC* uchburchak yuzini toping.



Yechilishi. Uchburchakning *C* burchagini topamiz:

$$\angle C = 180° - (\angle A + \angle B) = 180° - (60° + 30°) = 90°.$$

Demak, to‘g‘ri burchakli *ABC* uchburchakning *AB* gipotenuzasi 6 *sm* va *A* burchagi 30° ekan. To‘g‘ri burchakli uchburchakda 30° li burchak qarshisidagi katet gipotenuzaning yarmiga teng bo‘lgani uchun, *BC*=3 *sm* (*3-rasm*).

Pifagor teoremasidan foydalanib *AC* katetni topamiz:

$$AC^2 = AB^2 - BC^2 = 6^2 - 3^2 = 27, \quad AC = 3\sqrt{3}\ sm.$$

Endi uchburchak yuzini topamiz:

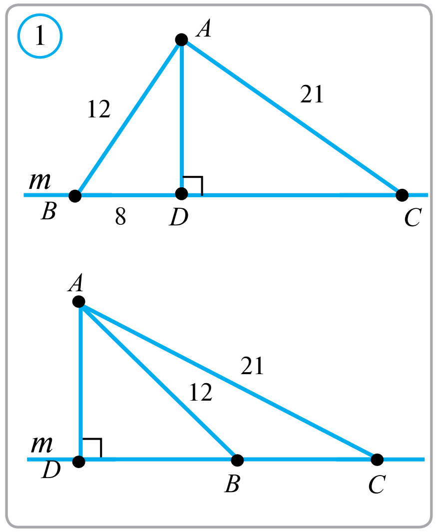
$$S_{ABC} = \frac{1}{2} AC \cdot BC = \frac{1}{2} \cdot 3\sqrt{3} \cdot 3 = \frac{9\sqrt{3}}{2}\ (sm^2).$$

Javob: $\frac{9\sqrt{3}}{2}\ sm^2$.

Savol, masala va topshiriqlar

1. *ABC* uchburchakda $\angle A = 47°$, $\angle C = 83°$ bo‘lsa, uchburchakni uchinchi ichki burchagini va tashqi burchaklarini toping.
2. Katetlari 15 *sm* va 20 *sm* bo‘lgan to‘g‘ri burchakli uchburchak gipotenuzasiga tushirilgan balandligini toping.
3. *ABC* uchburchakning *AC* tomoniga parallel to‘g‘ri chiziq *AB* va *BC* tomonlarni mos ravishda *E* va *F* nuqtalarda kesib o‘tadi. Agar $\angle BEF = 65°$ va $\angle EFC = 135°$ bo‘lsa, *ABC* uchburchak burchaklarini toping.
4. *ABC* uchburchak bissektrisalari *I* nuqtada kesishadi. Agar $\angle A = 80°$ va $\angle B = 70°$ bo‘lsa, *AIB, BIC* va *CIA* burchaklarni toping.
5. Teng yonli uchburchakning bitta tashqi burchagi 70° ga teng. Uchburchak burchaklarini toping.
6. *ABC* uchburchakning *AK* bissektrisasi o‘tkazilgan. Agar $\angle BAK = 47°$ va $\angle AKC = 103°$ bo‘lsa, uchburchak burchaklarini toping.
7*. *ABC* uchburchak balandliklari *H* nuqtada kesishadi. Agar $\angle A = 50°$, $\angle B = 60°$ bo‘lsa, *AHB, BHC* va *CHA* burchaklarni toping.
8. Uchburchakning o‘rta chiziqlari uni to‘rtta teng uchburchaklarga ajratishini isbotlang.
9*. *ABC* uchburchakda *CD* mediana davomiga bu medianaga teng *DE* kesma qo‘yilgan. *AF* mediananing davomiga *AF* medianaga teng *FH* kesma qo‘yilgan. *B, H, E* nuqtalar bitta to‘g‘ri chiziqda yotishini isbotlang.
10. *ABC* teng yonli uchburchakda (*AB*=*BC*) *AN* va *CK* bissektrisalar o‘tkazilgan.
 a) *KN* kesma *AC* tomonga parallel ekanini ko‘rsating.
 b) *AK*=*KN*=*NC* tenglik o‘rinli bo‘lishini isbotlang.

2 UCHBURCHAKLAR (davomi)



1-masala. *A* nuqtadan *m* to‘g‘ri chiziqqa uzunliklari 12 *sm* va 21 *sm* bo‘lgan ikkita og‘ma tushirilgan. Agar birinchi og‘maning *m* to‘g‘ri chiziqdagi proyeksiyasi 8 *sm* bo‘lsa, ikkinchi og‘maning proyeksiyasini toping.

Yechilishi. *m* to‘g‘ri chiziqdan tashqaridagi *A* nuqtadan shu to‘g‘ri chiziqqa *AB* va *AC* og‘malar hamda *AD* perpendikulyar tushirilgan bo‘lib, $AB=12$ *sm* va $AC=21$ *sm* bo‘lsin (*1-rasm*). Unda masala shartiga ko‘ra $BD=8$ *sm* bo‘ladi va *CD* kesma uzunligini topish kerak.

1) Pifagor teoremasidan foydalanib to‘g‘ri burchakli *ABD* uchburchakning *AD* katetini topamiz.

$AD^2=AB^2-BD^2=12^2-8^2=80$, $AD=\sqrt{80}$ *sm*.

2) To‘g‘ri burchakli *ACD* uchburchakdan Pifagor teoremasidan foydalanib *CD* kesma uzunligini topamiz.

$$CD^2=AC^2-AD^2=21^2-(\sqrt{80})^2=441-80=361,\ CD=19\ sm.$$

Javob: 19 *sm*.

2-masala. Tomonlari 13, 14 va 15 ga teng bo‘lgan uchburchak yuzini va balandliklarini toping.

Yechilishi. Geron formulasidan foydalanib, tomonlari $a=13$, $b=14$, $c=15$ bo‘lgan uchburchak yuzini topamiz:

$$p=\frac{a+b+c}{2}=\frac{13+14+15}{2}=\frac{42}{2}=21,$$

$$S=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{21\cdot(21-13)\cdot(21-14)\cdot(21-15)}=$$

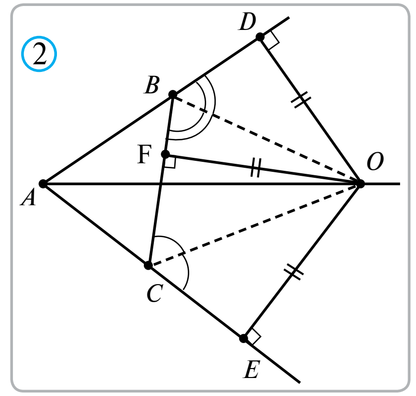
$$=\sqrt{21\cdot8\cdot7\cdot6}=\sqrt{7\cdot3\cdot8\cdot7\cdot2\cdot3}=3\cdot4\cdot7=84.$$

Endi, uchburchak yuzini hisoblash formulasi $S=\frac{1}{2}a\cdot h_a$ dan foydalanib, uchburchakning h_a balandligini topamiz:

$$h_a=\frac{2S}{a}=\frac{2\cdot84}{13}=\frac{168}{13}=12\frac{12}{13}.$$

Xuddi shuningdek h_b va h_c balandliklarni topamiz.

Javob: 84; $12\frac{12}{13}$; 12; $11\frac{1}{5}$.



3-masala. *ABC* uchburchakning *B* va *C* uchlaridagi tashqi burchaklarining bissektrisalari *O* nuqtada kesishadi. *O* nuqtaning *BAC* burchak bissektrisasida yotishini isbotlang.

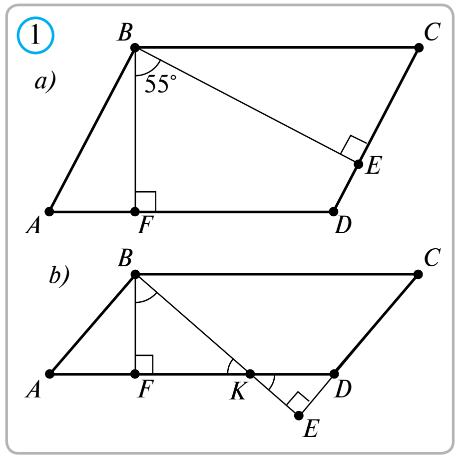
Isbot. *O* nuqtaning *AB, AC* va *BC* to‘g‘ri chiziqlardagi proyeksiyalari mos ravishda *D, E* va *F* nuqtalar bo‘lsin *(2-rasm)*. Unda, birinchidan *O* nuqta *DBC* burchakning bissektrisasida yotgani uchun $OD=OF$ bo‘ladi. Ikkinchidan, *O* nuqta *BCE* burchakning bissektrisasida yotgani uchun $OF=OE$ bo‘ladi. Shuning uchun, $OD=OF=OE$. Demak, *O* nuqta *BAC* burchak tomonlaridan teng uzoqlikda joylashgan ekan. Shuning uchun *O* nuqta *BAC* burchak bissektrisasida yotadi.

Savol, masala va topshiriqlar

1. Tomonlari 5, 6 va 7 bo‘lgan uchburchak yuzini toping.
2. Berilgan nuqtadan *a* to‘g‘ri chiziqqa uzunliklarining ayirmasi 6 ga teng bo‘lgan ikkita og‘ma tushirilgan. Og‘malarning *a* to‘g‘ri chiziqdagi proyeksiyalari 27 va 15 ga teng. Berilgan nuqtadan *a* to‘g‘ri chiziqqacha bo‘lgan masofani toping.

3*. *ABC* uchburchakning *A* va *B* uchlaridagi tashqi burchaklarining bissektrisalari *D* nuqtada kesishadi. Agar $ADB=75°$ bo‘lsa, uchburchakning *ACB* burchagini toping.

4. Asosi *AC* bo‘lgan *ABC* teng yonli uchburchakda *CD* bissektrisa o‘tkazilgan. *ADC* burchak: a) 60°; b) 75° ga teng bo‘lsa, uchburchak burchaklarini toping.
5. Bir kateti 7 *sm* ga, gipotenuzasi esa 25 *sm* ga teng bo‘lgan to‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligini toping.
6. *ABC* uchburchakning *BD* balandligi o‘tkazilgan (*D* nuqta *AC* kesmaga tegishli). Agar $BD=12$, $AD=5$ va $DC=16$ bo‘lsa, uchburchak perimetrini va yuzini toping.
7. Teng yonli uchburchakning yon tomoni 10 *sm*, asosi esa $10\sqrt{3}$ *sm*. Uchburchakning asosiga tushirilgan balandligini, yuzini va burchaklarini toping.
8. O‘tkir burchakli *ABC* uchburchakka tashqi chizilgan aylana markazi *O* nuqtada bo‘lib $\angle AOB=120°$, $\angle BOC=110°$ bo‘lsa, *ABC* uchburchak burchaklarini toping.
9. Agar *ABC* uchburchakning *CD* medianasi *AB* tomondan ikki marta kichik bo‘lsa, *ACB* burchakni toping.
10. *ABC* uchburchakning balandliklari *O* nuqtada kesishadi. Agar $\angle A=60°$, $\angle B=80°$ bo‘lsa, *AOB* burchakni toping.
11. *ABC* uchburchakning *A* va *B* uchlaridagi tashqi burchaklarining bissektrisalari *O* nuqtada kesishadi. Agar $\angle ACB=80°$ bo‘lsa, *AOB* burchakni toping.



1-masala. Agar parallelogrammning bir uchidan uning ikki tomoniga tushirilgan balandliklari orasidagi burchak 55° ga teng bo‘lsa, parallelogrammning burchaklarini toping.

Yechilishi. Parallelogrammning BF va BE balandliklari orasidagi burchak 55° bo‘lsin *(1-rasm)*. Rasmda tasvirlangan ikki hol: a) BE balandlik CD tomonga; b) BE balandlik CD tomon davomiga tushgan bo‘lishi mumkin.

a) holda $BEDF$ to‘rtburchak burchaklarining yig‘indisi 360° bo‘lgani uchun,

$$55° + 90° + \angle D + 90° = 360°.$$

Bundan, $\angle D = 125°$.

b) holda BE balandlik AD tomon bilan kesishgan nuqta K bo‘lsin. Unda,

$$\angle DKE = \angle BKF = 90° - 55° = 35°.$$

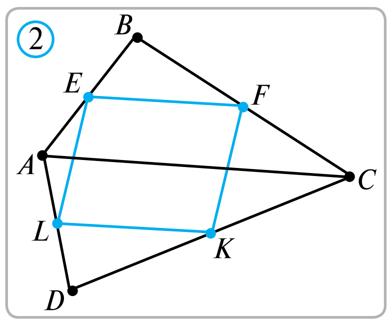
Uchburchak tashqi burchagining xossasiga ko‘ra,

$$\angle ADC = \angle DKE + \angle KED = 35° + 90° = 125°.$$

Demak, har ikkala holda ham $\angle D = 125°$. Unda,

$$\angle A = \angle C = 180° - \angle D = 55°, \quad \angle B = \angle D = 125°.$$

Javob: 55°, 125°, 55°, 125°.



2-masala. To‘rtburchak tomonlarining o‘rtalari parallelogramm uchlari bo‘lishini isbotlang.

Yechilishi. $ABCD$ to‘rtburchakning AB, BC, CD va DA tomonlari o‘rtalari mos ravishda E, F, K va L nuqtalar bo‘lsin. AC diagonalni o‘tkazamiz *(2-rasm)*. $EFKL$ — parallelogramm ekanligini ko‘rsatamiz.

EF kesma ABC uchburchakning, KL kesma esa ACD uchburchakning o‘rta chizig‘i bo‘ladi. Unda, uchburchak o‘rta chizig‘ining xossalariga ko‘ra,

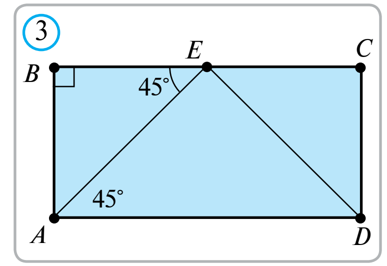
$$EF \| AC, \; KL \| AC, \qquad EF = \tfrac{1}{2}AC, \; KL = \tfrac{1}{2}AC.$$

Bundan $EF \| KL$ va $EF = LK$. Shuning uchun, parallelogramm alomatiga ko‘ra, $EFKL$ — parallelogramm.

3-masala. $ABCD$ to‘g‘ri to‘rtburchak A va D burchaklarining bissektrisalari BC tomonda kesishadi. Agar $AB = 4$ *sm* bo‘lsa, bu to‘g‘ri to‘rtburchak yuzini toping.

Yechilishi. To‘g‘ri to‘rtburchak A va D burchaklarining bissektrisalari kesishgan nuqta E bo‘lsin *(3-rasm)*. Unda, $\angle B = 90°$, $\angle BAE = 45°$ bo‘lgani uchun

$$\angle AEB = 180° - 90° - 45° = 45°.$$



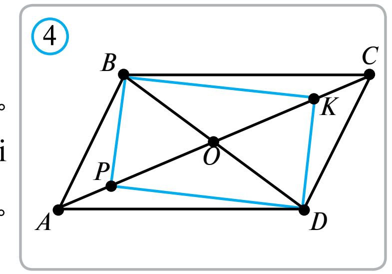
Ya’ni, ABE — teng yonli uchburchak. Unda,

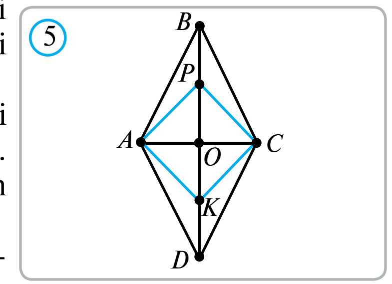
$$AB = BE = 4 \; (sm).$$

Xuddi shunga o‘xshash $EC = CD = 4$ (sm) ekanligini ko‘rsatish mumkin. Bundan $BC = BE + EC = 8\,(sm)$ va

$$S_{ABCD} = AB \cdot BC = 4 \cdot 8 = 32 \; (sm^2).$$

Javob: 32 sm^2.



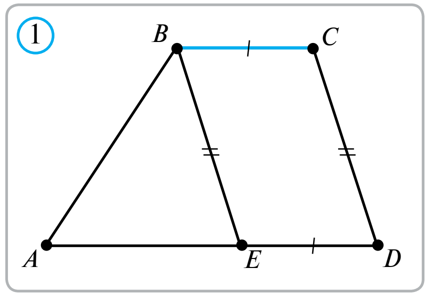


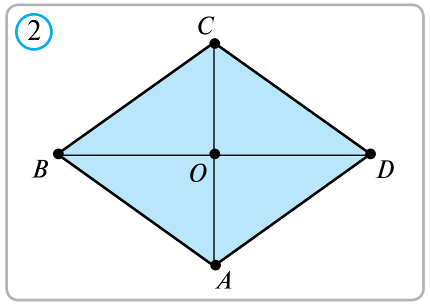
Savol, masala va topshiriqlar

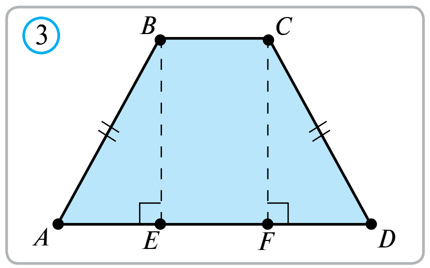
1. To‘rtburchakning uchta burchagi 47°, 83° va 120° ga tengligi ma’lum. Uning to‘rtinchi burchagini toping.
2. Parallelogrammning ikki burchagi yig‘indisi 156° ga teng. Uning burchaklarini toping.
3. To‘g‘ri to‘rtburchak diagonallari orasidagi burchak 74°. Uning bir diagonali bilan tomonlari orasidagi burchaklarni toping.
4. Teng yonli trapetsiyaning ikkita burchagi ayirmasi 40° ga teng. Uning burchaklarini toping.
5. Romb burchaklaridan biri ikkinchisidan uch marta katta. Rombning burchaklarini toping.
6. $ABCD$ to‘g‘ri to‘rtburchakning A burchagi bissektrisasi BC tomonini 2 *sm* va 6 *sm* ga teng kesmalarga ajratadi. To‘g‘ri to‘rtburchak perimetrini toping.
7. Tomonlari 3 *sm* va 6 *sm*, katta tomonlari orasidagi masofa esa 2 *sm* bo‘lgan parallelogramm yasang.
8. $ABCD$ parallelogrammning AC diagonalida P va K nuqtalar tanlangan *(4-rasm)*. Agar $OP = OB = OK$ bo‘lsa, $BKDP$ to‘g‘ri to‘rtburchak bo‘lishini isbotlang.

9*. $ABCD$ rombning BD katta diagonalida P va K nuqtalar tanlangan *(5-rasm)*. Agar $OA = OP = OK$ bo‘lsa, $APCK$ to‘rtburchak kvadrat ekanligini isbotlang.

10*. $ABCD$ parallelogrammning BD diagonalida P va K nuqtalar tanlangan. Agar $BP = KD$ bo‘lsa, $APCK$ to‘rtburchak parallelogramm ekanligini isbotlang.







1-masala. *ABCD* trapetsiyadagi *BC* kichik asosning *B* uchidan *CD* tomonga parallel to‘g‘ri chiziq o‘tkazilgan. Natijada, hosil bo‘lgan uchburchakning perimetri 24 *sm* ga teng. Agar trapetsiyaning perimetri 36 *sm* bo‘lsa, *BC* tomon uzunligini toping.

Yechilishi. Masala shartiga ko‘ra, o‘tkazilgan to‘g‘ri chiziq kesmasi *BE* bo‘lsin, *E* nuqta *AD* tomonda yotadi *(1-rasm)*. *BC* kesma trapetsiyani *ABE* uchburchak va *BCDE* parallelogrammga ajratadi. Xususan, $BC = ED$ va $CD = BE$.

Masala shartiga ko‘ra,

$P_{ABCD} = AB + BC + CD + DA = AB + BC + CD + DE + EA = AB + BE + EA + 2BC = P_{ABE} + 2BC = 24 + 2BC = 36$ (*sm*).

Bundan, $2BC = 12$ yoki $BC = 6$ *sm* ekanligini topamiz. ***Javob:*** 6 *sm*.

2-masala. Rombning diagonallaridan biri 14 *sm*, tomoni esa 25 *sm*. Romb yuzini toping.

ABCD — romb,
AC = 14 sm, AB = 25 sm. ⇒ $S_{ABCD} = ?$

Yechilishi. Romb diagonallari kesishish nuqtasi *O* bo‘lsin *(2-rasm)*. Unda, romb xossasiga ko‘ra,

$$AO = \frac{1}{2}AC = \frac{1}{2}\cdot 14 = 7\ (sm), \qquad \angle AOB = 90^\circ.$$

Pifagor teoremasidan foydalanib *OB* kesmani topamiz:

$$OB^2 = AB^2 - AO^2 = 25^2 - 7^2 = 576 \quad \text{yoki} \quad OB = 24\ sm.$$

Unda $BD = 2\cdot OB = 2\cdot 24 = 48$ (*sm*). Romb yuzini hisoblash formulasiga ko‘ra,

$$S_{ABCD} = \frac{1}{2}AC\cdot BD = \frac{1}{2}\cdot 14\cdot 48 = 7\cdot 48 = 336\ (sm^2).$$ ***Javob:*** 336 sm^2.

3-masala. Teng yonli trapetsiyaning yon tomoni 20 *sm*, asoslari esa 12 *sm* va 36 *sm*. Trapetsiya yuzini toping.

Yechilishi. *ABCD* trapetsiyada $AB = CD = 20$ *sm*, $BC = 12$ *sm*, $AD = 36$ *sm* bo‘lsin. Trapetsiyaning *BE* va *CF* balandliklarini o‘tkazamiz *(3-rasm)*.

Unda,

$$EF = BC = 12\ (sm),$$

$$AE = FD = \frac{AD - EF}{2} = \frac{36 - 12}{2} = 12\ (sm).$$

To‘g‘ri burchakli *ABE* uchburchakka Pifagor teoremasini qo‘llab, *BE* balandlikni topamiz:

$$BE^2 = AB^2 - AE^2 = 20^2 - 12^2 = 256 \quad \text{yoki} \quad BE = 16\ sm.$$

Trapetsiyaning yuzini topamiz:

$$S_{ABCD} = \frac{BC + AD}{2}\cdot BE = \frac{12 + 36}{2}\cdot 16 = 24\cdot 16 = 384\ (sm^2).$$

Javob: 384 sm^2.

Savol, masala va topshiriqlar

1. *ABCD* trapetsiyaning kichik *BC* asosi 7 *sm* ga teng. Uning *B* uchidan *CD* tomoniga parallel to‘g‘ri chiziq o‘tkazilgan. Hosil bo‘lgan uchburchak perimetri 16 *sm* ga teng. Trapetsiya perimetrini toping.
2. To‘g‘ri chiziqni kesib o‘tmaydigan kesmaning uchlari bu to‘g‘ri chiziqdan 8 *sm* va 18 *sm* uzoqlikda joylashgan. Kesma o‘rtasidan to‘g‘ri chiziqqacha bo‘lgan masofani toping.
3. Tomonlari 4 *sm* va 5 *sm*, yuzi esa 10 sm^2 bo‘lgan parallelogramm yasang.
4. Rombning diagonallaridan biri 80 *sm*, tomoni esa 81 *sm*. Romb yuzini toping.
5. Parallelogrammning 135° ga teng bo‘lgan o‘tmas burchagi uchidan tushirilgan balandligi 4 *sm* ga teng bo‘lib, u o‘zi tushgan tomonni teng ikkiga bo‘ladi.
   a) Shu tomonni toping.
   b) Parallelogrammning o‘tmas burchaklari uchlarini tutashtiruvchi diagonali bilan tomonlari orasidagi burchaklarni toping.
   d) Parallelogramm perimetri va yuzini toping.
6. Rombning o‘tmas burchagi uchidan tushirilgan balandlik romb tomonini teng ikkiga bo‘ladi. Agar rombning tomoni 6 *sm* bo‘lsa, romb yuzini toping.
7. To‘g‘ri burchakli uchburchakning gipotenuzasi 13 *sm*, katetlarining yig‘indisi esa 17 *sm*. Uchburchak yuzini toping.

8*. To‘g‘ri burchakli trapetsiyaning bir burchagi 135° ga, o‘rta chizig‘i esa 18 *sm* ga teng. Agar trapetsiya asoslari nisbati 1:8 ga teng bo‘lsa, trapetsiyaning yon tomonlarini toping.

9*. *ABCD* ($AB \| CD$) trapetsiya *O* markazli aylanaga tashqi chizilgan. $\angle AOD = 90^\circ$ ekanligini isbotlang.

MATEMATIK MASALALAR XAZINASI

Ma’lumki, keyingi paytlarda axborot kommunikatsiya texnologiyalari juda tez sur’atlar bilan rivojlanib bormoqda. Internet to‘ri borgan sari olis qishloqlarni ham qamrab olmoqda. Shu kunga kelib, Internetning World-Wide-Web — Jahon axborot tarmog‘ida shunchalik ko‘p axborot manbalari joylashtirilganki, bu xazinadan foydalanish har bir odam uchun ham qarz, ham farz hisoblanadi. Jumladan, bir-biridan qiziq shunday web-sahifalar borki, ulardan ixtiyoriy fanni, jumladan, geometriyani o‘rganish jarayonida samarali foydalanish mumkin. Quyida shu axborot manbalarining manzillarini berishni lozim topdik. Bu web-sahifalardan siz o‘zbek, rus, ingliz va boshqa tillarda matematika olamidagi eng oxirgi yangiliklar, elektron resurs markazlarida saqlanayotgan ko‘plab elektron darsliklar, matematikadan masofadan turib ta’lim olish kurslari va ularning materiallari, mazkur darslik sahifalariga kirgan va kirmagan turli-tuman nazariy materiallar, matematikadan dars berib kelayotgan tajribali o‘qituvchilarning dars ishlanmalari va metodik tavsiyalari, son-sanoqsiz masalalar, misollar va ularning yechimlari, turli davlatlarda o‘tkazilayotgan matematik ko‘rik tanlovlar va olimpiadalar to‘g‘risidagi ma’lumotlar va ularda taqdim etilgan masalalar hamda ularning yechimlari, qiziqarli matematik masalalar va ularning yechimlari bilan tanishishingiz mumkin.

http://www.eduportal.uz — Xalq ta’limi vazirligi axborot-ta’lim portali
http://www.multimedia.uz — Xalq ta’limi vazirligi qoshidagi Multimedia markazi sayti
http://www.uzedu.uz — Xalq ta’limi vazirligi sayti
http://www.edu.uz — Oliy va o‘rta maxsus ta’lim vazirligining ta’lim portali
http://www.pedagog.uz — Pedagogika ta’lim muassasalari portali
http://ziyo.edu.uz — Ta’lim muassasalari portali
http://www.matematika.uz — Matematikadan qo‘shimcha materiallar sayti
http://ziyonet.uz — Axborot-ta’lim resurslari tarmog‘i
http://cde.sakha.ru — Masofadan turib o‘qitish sayti (rus tilida)
http://www.ixl.com — Masofadan turib o‘qitish sayti portali (ingliz tilida)
http://www.iro.sakha.ru — Ta’limni rivojlantirish instituti sayti (rus tilida)
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http://int-edu.ru/soft/base_geom.html — «Живая геометрия» dasturini qo‘llab-quvvatlash sayti
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http://www.mathtype.narod.ru/ — Online-darsliklar (rus tilida)
http://www.e-pi.narod.ru/ — Hammasi e va π sonlari haqida (rus tilida)
http://mschool.kubsu.ru/ — Elektron qo‘llanmalar kutubxonasi. Sirtqi matematik olimpiadalar
http://matematika.agava.ru/ — Matematikadan 2000 dan ortiq masalalar (rus tilida)
http://mat-game.narod.ru/ — Matematik gimnastika. Matematik masalalar va boshqotirmalar
http://mathc.chat.ru/ — Matematik kaleydoskop (rus tilida)
http://mathmag.spbu.ru/ — Internetdagi matematik jurnal (rus tilida)
http://www.matematik1.narod.ru/ — Matematikadan masalalar (rus tilida)

I BOB

O‘XSHASH GEOMETRIK SHAKLLAR

Ushbu bobni o‘rganish natijasida siz quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

Bilimlar:

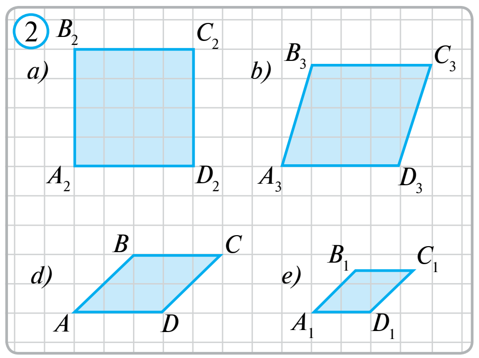
√ *o‘xshash shakllarning ta’rifini va belgilanishini bilish;*
√ *uchburchaklarning o‘xshashlik alomatlarini bilish;*
√ *gomotetiya tushunchasini bilish.*

Amaliy ko‘nikmalar:

√ *ikkita o‘xshash uchburchaklardan mos elementlarni topa olish;*
√ *uchburchaklarning o‘xshashlik alomatlarini isbotlashga va hisoblashga oid masalalarni yechishda qo‘llay olish;*
√ *gomotetiyadan foydalanib, o‘xshash ko‘pburchaklarni yasay olish.*

5 KO‘PBURCHAKLARNING O‘XSHASHLIGI

Kundalik turmushda teng shakllardan tashqari shakli (ko‘rinishi) bir xil, lekin o‘lchamlari turlicha bo‘lgan shakllarga ko‘p duch kelamiz. Tarix va geografiya fanlarida turli masshtabda ishlangan xaritalardan foydalangansiz. Sinf doskasiga ilinadigan va darsliklarda tasvirlangan respublikamizning xaritalari turli o‘lchamda, lekin ular bir xil shaklda (ko‘rinishda). Shuningdek, bitta fototasmadan turli o‘lchamdagi fotosuratlar tayyorlanadi. Bu suratlarning o‘lchamlari turlicha bo‘lsa-da, bir xil ko‘rinishda, ya’ni ular bir-biriga o‘xshaydi *(1-rasm)*.



Mashq. 2-rasmda to‘rtta romb tasvirlangan. Ulardan faqat *d*) va *e*) romblar bir xil ko‘rinishga ega. Bu romblar nimasi bilan boshqa romblardan ajralib turibdi? Keling, buni birgalikda aniqlaylik.

1. Rasmdan ko‘rinib turibdiki, $AD=3$, $A_1D_1=2$. Rombning tomonlari teng bo‘lgani uchun,

$$\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{CD}{C_1D_1} = \frac{AD}{A_1D_1} = \frac{3}{2} = 1{,}5$$

tenglikni hosil qilamiz. Bu holatda romblarning mos tomonlari proporsional deb yuritiladi.

2. $ABCD$ va $A_1B_1C_1D_1$ romblarning mos burchaklari o‘zaro teng. Haqiqatan ham, $\angle A=\angle A_1=45°$, $\angle B=\angle B_1=135°$, $\angle C=\angle C_1=45°$, $\angle D=\angle D_1=135°$.

Shunday qilib, bu romblarning bir-biriga o‘xshashligining sababi — mos tomonlarining proporsionalligi va mos burchaklarining tengligi deya olamiz. Ixtiyoriy ko‘pburchaklar o‘xshashligi tushunchasi ham shu asosda kiritiladi.

Burchaklari soni bir xil (demak, tomonlarining soni ham bir xil) bo‘lgan ko‘pburchaklar **bir xil nomli ko‘pburchaklar** deb yuritiladi.

Ikkita bir xil nomli $ABCDE$ va $A_1B_1C_1D_1E_1$ ko‘pburchaklarning burchaklari mana bu tartibda teng bo‘lsin: $\angle A=\angle A_1$, $\angle B=\angle B_1$, $\angle C=\angle C_1$, $\angle D=\angle D_1$, $\angle E=\angle E_1$.

Bunday burchaklar **mos burchaklar** deb yuritiladi. U holda, AB va A_1B_1, BC va B_1C_1, CD va C_1D_1, DE va D_1E_1, EA va E_1A_1 tomonlar **mos tomonlar** deyiladi.

Ta’rif. Bir xil nomli ko‘pburchaklardan birining burchaklari ikkinchisining burchaklariga mos ravishda teng, mos tomonlari esa proporsional bo‘lsa, bunday ko‘pburchaklar **o‘xshash ko‘pburchaklar** deb ataladi *(3-rasm)*.

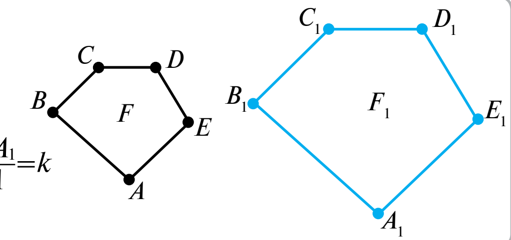
Ko‘pburchaklar o‘xshashligi $\backsim$ belgisi bilan ko‘rsatiladi.

3

Mos burchaklar teng

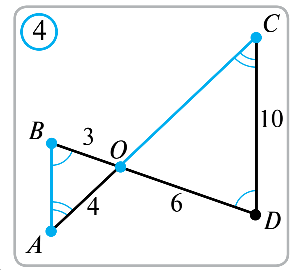
$$F \backsim F_1 \quad \begin{cases} \angle A=\angle A_1,\ \angle B=\angle B_1,\ \angle C=\angle C_1, \\ \angle D=\angle D_1,\ \angle E=\angle E_1 \\ \dfrac{A_1B_1}{AB} = \dfrac{B_1C_1}{BC} = \dfrac{C_1D_1}{CD} = \dfrac{D_1E_1}{DE} = \dfrac{E_1A_1}{EA} = k \end{cases}$$

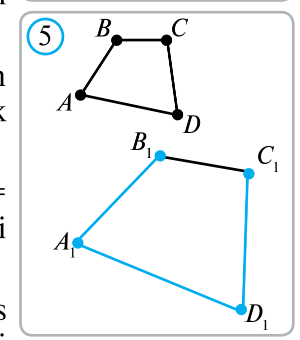
Mos tomonlar proporsional



O‘xshash ko‘pburchaklarning mos tomonlari nisbatiga teng bo‘lgan k son **o‘xshashlik koeffitsiyenti** deyiladi.

Savol, masala va topshiriqlar





1. O‘xshash ko‘pburchaklar ta’rifini ayting.
2. O‘xshashlik koeffitsiyenti nima va u qanday aniqlanadi?
3. Agar ABC va DEF uchburchaklarda $\angle A=105°$, $\angle B=35°$, $\angle E=105°$, $\angle F=40°$, $AC=4{,}4$ *sm*, $AB=5{,}2$ *sm*, $BC=7{,}6$ *sm*, $DE=15{,}6$ *sm*, $DF=22{,}8$ *sm*, $EF=13{,}2$ *sm* bo‘lsa, ular o‘xshash bo‘ladimi?
4. 2-rasmda tasvirlangan a) va b) romblar nima sababdan o‘xshash emas? b) va d) romblar-chi?
5. 4-rasmdagi ABO va CDO uchburchaklar o‘xshash bo‘lsa, AB, OC kesmalar uzunligini va o‘xshashlik koeffitsiyentini toping.
6. 5-rasmda $ABCD \backsim A_1B_1C_1D_1$. $AB=24$, $BC=18$, $CD=30$, $AD=54$, $B_1C_1=54$. A_1B_1, D_1A_1 va C_1D_1 kesmalarni toping.

7*. ABC uchburchak AB va AC tomonlarining o‘rtalari mos ravishda P va Q bo‘lsin. $\Delta ABC \backsim \Delta APQ$ ekanligini isbotlang.

6 O‘XSHASH UCHBURCHAKLAR VA ULARNING XOSSALARI

Eng sodda ko‘pburchak bo‘lmish uchburchaklar o‘xshashligini o‘rganamiz.

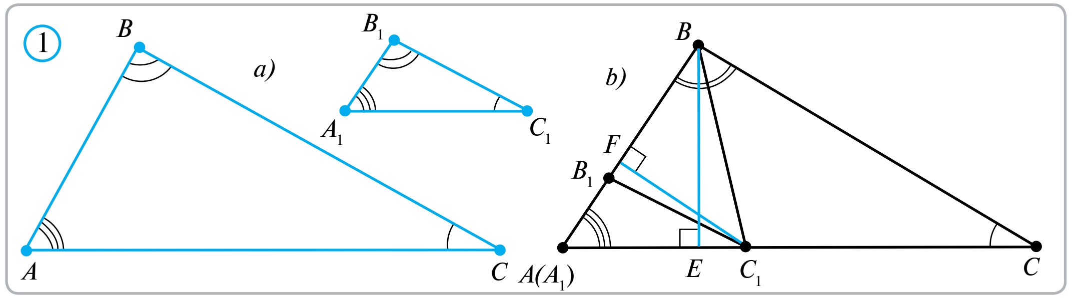
Teorema. **Ikkita o‘xshash uchburchak perimetrlarining nisbati o‘xshashlik koeffitsiyentiga teng.**

Bu teoremani mustaqil isbotlang.

Teorema. **Ikkita o‘xshash uchburchak yuzlari nisbati o‘xshashlik koeffitsiyentining kvadratiga teng.**

$\triangle ABC \backsim \triangle A_1B_1C_1$ *(1-a rasm)*, k — *o‘xshashlik koeffitsiyenti* ⇒ $S_{ABC}:S_{A_1B_1C_1}=k^2$

Isbot. Teorema shartiga ko‘ra, $\triangle ABC \backsim \triangle A_1B_1C_1$. Demak, ko‘pburchaklar o‘xshashligi ta’rifiga ko‘ra, $\angle A=\angle A_1$, $\angle B=\angle B_1$, $\angle C=\angle C_1$ va $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{CA}{C_1A_1}=k$.



$\angle A=\angle A_1$ ekanligidan foydalanib, ularni 1-b rasmdagidek ustma-ust qo‘yamiz va tegishli yasash hamda belgilashlarni amalga oshiramiz.

Quyidagi uchburchaklar yuzlarini topamiz va ularning nisbatlarini qaraymiz:

$$\left.\begin{aligned} S_{ABC}&=\frac{AC\cdot BE}{2};\\ S_{ABC_1}&=\frac{A_1C_1\cdot BE}{2};\end{aligned}\right] \Rightarrow \frac{S_{ABC}}{S_{ABC_1}}=\frac{AC}{A_1C_1} \quad (1),$$

$$\left.\begin{aligned} S_{A_1B_1C_1}&=\frac{A_1B_1\cdot C_1F}{2};\\ S_{ABC_1}&=\frac{AB\cdot C_1F}{2};\end{aligned}\right] \Rightarrow \frac{S_{A_1B_1C_1}}{S_{ABC_1}}=\frac{A_1B_1}{AB} \quad (2).$$

(1) tenglikni hadma-had (2) tenglikka bo‘lsak, teng burchakka ega bo‘lgan uchburchaklar yuzlarining nisbati uchun (3) tenglikni hosil qilamiz.

$$\frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{AB}{A_1B_1}\cdot\frac{AC}{A_1C_1} \quad (3)$$

Bu yerda shartga ko‘ra, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{CA}{C_1A_1}=k$ ekanligini hisobga olsak,

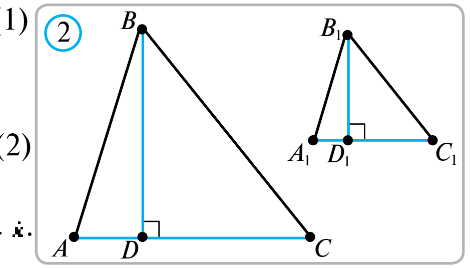
$$\frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{AB\cdot AC}{A_1B_1\cdot A_1C_1}=\frac{AB}{A_1B_1}\cdot\frac{AC}{A_1C_1}=k\cdot k=k^2$$

tenglik kelib chiqadi. ***Teorema isbotlandi.***

1-masala. O‘xshash uchburchaklarning mos tomonlari nisbati shu tomonlarga tushirilgan balandliklar nisbatiga tengligini isbotlang *(2-rasm)*.

$\triangle ABC \backsim \triangle A_1B_1C_1$, BD, B_1D_1 — *balandliklar* ⇒ $\frac{AC}{A_1C_1}=\frac{BD}{B_1D_1}$

Yechilishi. Berilgan uchburchaklarning o‘xshashlik koeffitsiyenti k bo‘lsin. Unda, $AC:A_1C_1=k$; $S_{ABC}:S_{A_1B_1C_1}=k^2$ (1) bo‘ladi. Ikkinchi tomondan,



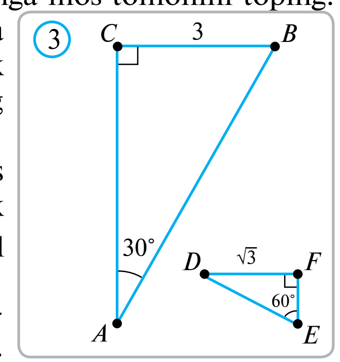
$$\frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{\frac{1}{2}AC\cdot BD}{\frac{1}{2}A_1C_1\cdot B_1D_1}=\frac{AC}{A_1C_1}\cdot\frac{BD}{B_1D_1}=k\cdot\frac{BD}{B_1D_1}. \quad (2)$$

(1) va (2) tengliklardan $k^2=k\cdot\frac{BD}{B_1D_1}$ yoki $\frac{BD}{B_1D_1}=k$.

Shunday qilib, $\frac{BD}{B_1D_1}$ nisbat ham, $\frac{AC}{A_1C_1}$ nisbat ham k ga teng, ya’ni $\frac{AC}{A_1C_1}=\frac{BD}{B_1D_1}$.

Savol, masala va topshiriqlar

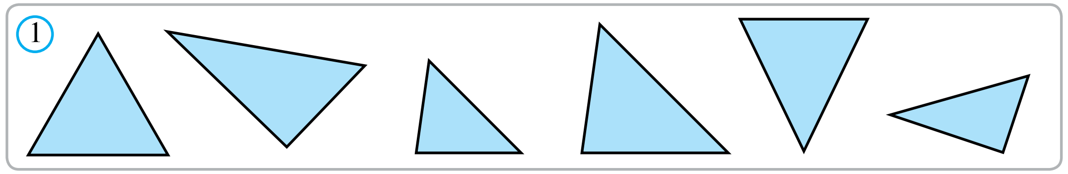
1. O‘xshash uchburchaklar yuzlari nisbati haqidagi teoremani ayting va isbotlang.
2. Ikkita o‘xshash ABC va $A_1B_1C_1$ uchburchaklar berilgan. Agar $S_{ABC}=25\ sm^2$ va $S_{A_1B_1C_1}=81\ sm^2$ bo‘lsa, o‘xshashlik koeffitsiyentini toping.
3. Ikkita o‘xshash uchburchak yuzlari 65 m^2 va 260 m^2. Birinchi uchburchakning bir tomoni 6 m bo‘lsa, ikkinchi uchburchakning unga mos tomonini toping.
4. Berilgan uchburchak tomonlari 15 *sm*, 25 *sm* va 30 *sm*. Agar perimetri 35 *sm* bo‘lgan uchburchak berilgan uchburchakka o‘xshash bo‘lsa, uning tomonlarini toping.
5. $\triangle ABC \backsim \triangle A_1B_1C_1$ va bu uchburchaklarning mos tomonlari nisbati 7:5 ga teng. Agar ABC uchburchak yuzi $A_1B_1C_1$ uchburchak yuzidan 36 m^2 ga ortiq bo‘lsa, bu uchburchaklar yuzlarini toping.
6. 3-rasmda berilganlardan foydalanib, uchburchaklarning o‘xshash yoki o‘xshash emasligini aniqlang.



7 UCHBURCHAKLAR O‘XSHASHLIGINING BIRINCHI ALOMATI

Faollashtiruvchi mashq

1-rasmda tasvirlangan uchburchaklar ichidan o‘xshashlarini aniqlang. Ularning o‘xshashligini qanday aniqladingiz?

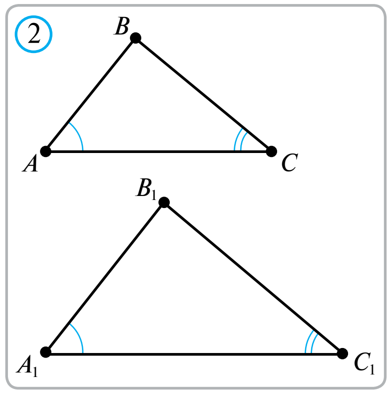


Ta’rifga ko‘ra, ikkita uchburchakning o‘xshashligini aniqlash uchun ular burchaklarining tengligini va mos tomonlarining proporsional ekanligini tekshirish lozim bo‘ladi. Uchburchaklar uchun bu ish ancha osonlashar ekan. Quyida keltiriladigan teoremalar shu xususda bo‘lib, ular “uchburchaklar o‘xshashligining alomatlari” deb nomlanadi.

Teorema. (Uchburchaklar o‘xshashligining BB alomati). **Agar bir uchburchakning ikkita burchagi ikkinchi uchburchakning ikkita burchagiga mos ravishda teng bo‘lsa, bunday uchburchaklar o‘xshash bo‘ladi** *(2-rasm)*.

$\Delta ABC, \Delta A_1B_1C_1, \angle A=\angle A_1, \angle C=\angle C_1$ ⇒ $\Delta ABC \backsim \Delta A_1B_1C_1$

Isbot. 1. Uchburchak ichki burchaklari yig‘indisi haqidagi teoremaga ko‘ra,



$$\left.\begin{aligned}\angle B&=180^\circ-(\angle A+\angle C),\\ \angle B_1&=180^\circ-(\angle A_1+\angle C_1)\end{aligned}\right\} \Rightarrow \angle B=\angle B$$

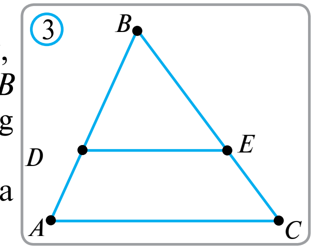
Demak, ABC va $A_1B_1C_1$ uchburchaklarning burchaklari mos ravishda teng.

2. Shartga ko‘ra, $\angle A=\angle A_1$, $\angle C=\angle C_1$. Teng burchakka ega bo‘lgan uchburchaklar yuzlarining nisbati haqidagi teoremaga ko‘ra

$$\frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{AB\cdot AC}{A_1B_1\cdot A_1C_1} \quad \text{va} \quad \frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{CA\cdot CB}{C_1A_1\cdot C_1B_1}.$$

Bu tengliklarning o‘ng qismlarini tenglab, bir xil hadlar qisqartirilsa, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}$ tenglik hosil bo‘ladi. Xuddi shu singari, $\angle A=\angle A_1$ va $\angle B=\angle B_1$ tengliklardan foydalanib, $\frac{BC}{B_1C_1}=\frac{CA}{C_1A_1}$ tenglikni olamiz. Shunday qilib, ABC va $A_1B_1C_1$ uchburchaklarning burchaklari teng va mos tomonlari proporsional, ya’ni bu uchburchaklar o‘xshash. ***Teorema isbotlandi.***

Masala. ABC uchburchakning ikki tomonini kesib o‘tuvchi va uchinchi tomoniga parallel bo‘lgan DE to‘g‘ri chiziq uchburchakdan unga o‘xshash uchburchak ajratishini isbotlang *(3-rasm)*.



Isbot. ABC va DBE uchburchaklarda $\angle B$ — umumiy, $\angle CAB = \angle EDB$ (AC va DE parallel to‘g‘ri chiziqlarni AB kesuvchi bilan kesganda hosil bo‘lgan mos burchaklar teng bo‘lgani uchun) *(3-rasm)*.

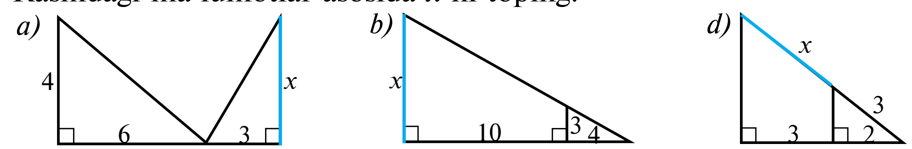
Demak, uchburchaklar o‘xshashligining BB alomatiga ko‘ra, $ABC \backsim \Delta DBE$.

Savol, masala va topshiriqlar

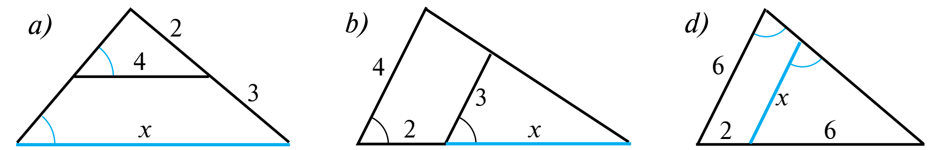
1. Uchburchaklar o‘xshashligining ta’rifi va BB alomatini o‘zaro solishtiring.

2. Uchburchaklar o‘xshashligining BB alomatini isbotlang.

3. Rasmdagi ma’lumotlar asosida x ni toping.



4. Rasmdagi ma’lumotlar asosida x ni toping.

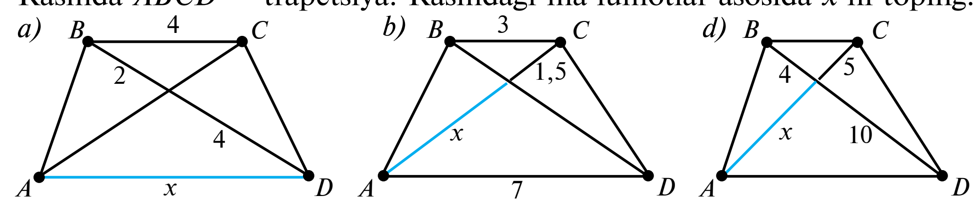


5. $ABCD$ parallelogrammning CD tomonida E nuqta olingan. AE va BC nurlar F nuqtada kesishadi.

a) Agar $DE=8$ *sm*, $EC=4$ *sm*, $BC=7$ *sm*, $AE=10$ *sm* bo‘lsa, EF va FC ni;

b) Agar $AB=8$ *sm*, $AD=5$ *sm*, $CF=2$ *sm* bo‘lsa, DE va EC ni toping.

6. Rasmda $ABCD$ — trapetsiya. Rasmdagi ma’lumotlar asosida x ni toping.



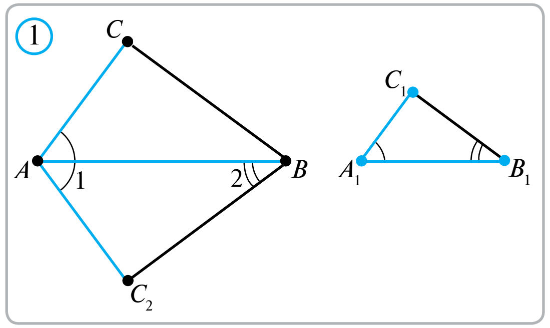
7*. Bittadan o‘tkir burchaklari teng bo‘lgan ikkita to‘g‘ri burchakli uchburchaklar o‘xshash ekanligini isbotlang.

8*. ABC uchburchakning AC tomonida D nuqta olingan. Agar $\angle ABC=\angle BDC$ bo‘lsa, ABC va BDC uchburchaklar o‘xshash ekanligini isbotlang. Shuningdek, $3AB=4BD$ va $BC=9$ *sm* bo‘lsa, AC kesmani toping.

8 UCHBURCHAKLAR O‘XSHASHLIGINING IKKINCHI ALOMATI

Teorema. (Uchburchaklar o‘xshashligining TBT alomati). **Agar bir uchburchakning ikki tomoni ikkinchi uchburchakning ikki tomoniga proporsional va bu tomonlar hosil qilgan burchaklar teng bo‘lsa, bunday uchburchaklar o‘xshash bo‘ladi** *(1-rasm)*.

$\Delta ABC,\ \Delta A_1B_1C_1,\ \angle A=\angle A_1,\ \frac{AB}{A_1B_1}=\frac{AC}{A_1C_1}$ ⇒ $\Delta ABC \sim \Delta A_1B_1C_1$



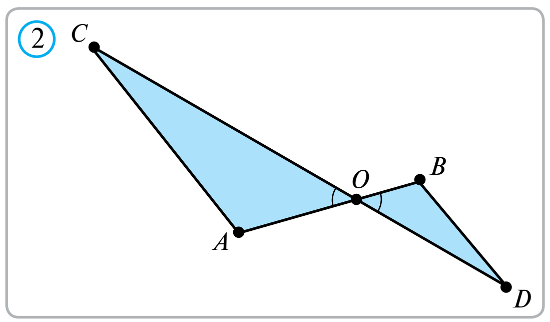
Isbot. $\angle 1=\angle A_1$, $\angle 2=\angle B_1$ bo‘ladigan qilib ABC_2 uchburchak yasaymiz *(1-rasm)*. U BB alomat bo‘yicha $A_1B_1C_1$ uchburchakka o‘xshash bo‘ladi.

$\frac{AB}{A_1B_1}=\frac{AC_2}{A_1C_1}$ ⇐ $(\Delta A_1B_1C_1 \sim \Delta ABC_2)$

$\frac{AB}{A_1B_1}=\frac{AC}{A_1C_1}$ ⇐ (shartga ko‘ra).

Bu ikki tenglikdan, $AC_2=AC$ ekanligini aniqlaymiz. Unda, uchburchaklar tengligining TBT alomatiga ko‘ra, $\Delta ABC=\Delta ABC_2$. Xususan, $\angle 2=\angle B$. Lekin yasashga ko‘ra, $\angle 2=\angle B_1$ edi. Demak, $\angle B=\angle B_1$. U holda, $\angle A=\angle A_1$ va $\angle B=\angle B_1$ bo‘lgani uchun, uchburchaklar o‘xshashligining BB alomatiga ko‘ra, $\Delta ABC \sim \Delta A_1B_1C_1$. ***Teorema isbotlandi.***

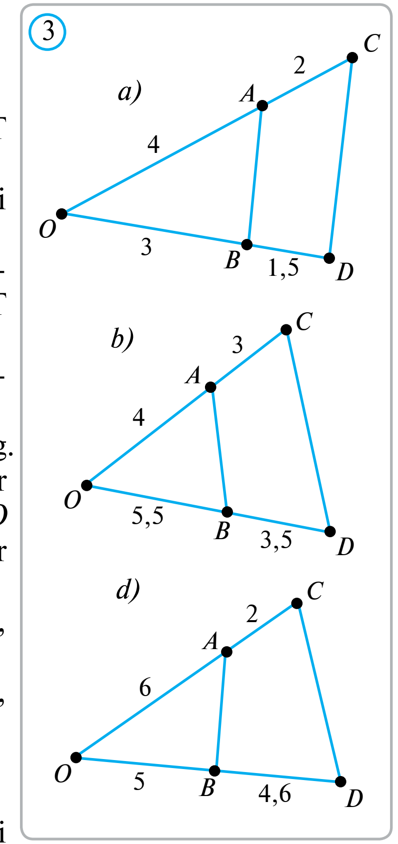
Masala. AB va CD kesmalar O nuqtada kesishadi, $AO=12$ *sm*, $BO=4$ *sm*, $CO=30$ *sm*, $DO=10$ *sm* bo‘lsa, AOC va BOD uchburchaklar yuzlari nisbatini toping.



Yechilishi: Shartga ko‘ra,

$$\left.\begin{aligned}\frac{OA}{OB}=\frac{12}{4}=3\\ \frac{OC}{OD}=\frac{30}{10}=3\end{aligned}\right\} \Rightarrow \frac{OA}{OB}=\frac{OC}{OD}=3.$$

Demak, AOC uchburchakning ikki tomoni BOD uchburchakning ikki tomoniga proporsional va bu tomonlar orasidagi mos burchaklar vertikal burchaklar bo‘lgani uchun: $\angle AOC=\angle BOD$. Shuning uchun, uchburchaklar o‘xshashligining TBT alomatiga ko‘ra, $\Delta AOC \sim \Delta BOD$ va o‘xshashlik koeffitsiyenti $k=\frac{OA}{OB}=3$. Endi o‘xshash uchburchaklar yuzlarining nisbati haqidagi teoremani qo‘llaymiz: $\frac{S_{AOC}}{S_{BOD}}=k^2=9$. ***Javob:*** 9.



Savol, masala va topshiriqlar

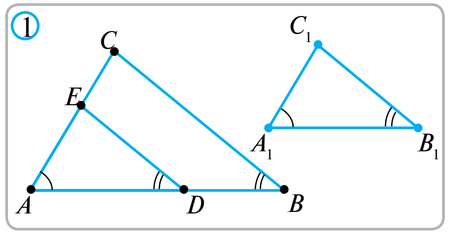
1. Uchburchaklar o‘xshashligining ta’rifi va TBT alomatini o‘zaro solishtiring.
2. Uchburchaklar o‘xshashligining TBT alomatini isbotlang.
3. Uchidagi burchaklari teng bo‘lgan teng yonli uchburchaklarning o‘xshashligini a) BB; b) TBT alomatdan foydalanib isbotlang.
4. 3-rasmda tasvirlangan OAB va OCD uchburchaklar o‘xshash bo‘ladimi? Agar o‘xshash bo‘lsa, bu uchburchaklar perimetrining nisbatini toping.
5. AC va BD nurlar O nuqtada kesishadi. Agar $AO:CO=BO:DO=3$, $AB=7$ *sm* bo‘lsa, CD kesmani hamda AOB va COD uchburchaklar yuzlari nisbatini toping.
6. ABC va $A_1B_1C_1$ uchburchaklarda $\angle A=\angle A_1$, $AB:A_1B_1=AC:A_1C_1=4:3$.
   a) Agar AB kesma A_1B_1 dan 5 *sm* ortiq bo‘lsa, AB va A_1B_1 tomonlarni toping.
   b) Agar A_1B_1 kesma AB dan 6 *sm* kam bo‘lsa, AB va A_1B_1 tomonlarini toping.
   d) Agar berilgan uchburchaklarning yuzlari yig‘indisi 400 sm^2 bo‘lsa, har qaysi uchburchakning yuzini toping.
7. Agar bir to‘g‘ri burchakli uchburchakning katetlari ikkinchi to‘g‘ri burchakli uchburchakning mos katetlariga proporsional bo‘lsa, bu uchburchaklar o‘xshash bo‘lishini isbotlang.
8. ABC uchburchakda $AB=15$ *m*, $AC=20$ *m*, $BC=32$ *m*. Uchburchakning AB tomoniga $AD=9$ *m* kesma, AC tomoniga esa $AE=12$ *m* kesma qo‘yildi. DE kesmani toping.
9. Katetlari 3 *dm* va 4 *dm* bo‘lgan to‘g‘ri burchakli uchburchak bilan bir kateti 8 *dm* va gipotenuzasi 10 *dm* bo‘lgan to‘g‘ri burchakli uchburchak o‘xshash bo‘lishini isbotlang.

10*. AB kesma va l to‘g‘ri chiziq O nuqtada kesishadi. l to‘g‘ri chiziqqa AA_1 va BB_1 perpendikularlar tushirilgan. Agar $AA_1=2$ *sm*, $OA_1=4$ *sm* va $OB_1=3$ *sm* bo‘lsa, BB_1, OA va AB kesmalarni toping.

9 UCHBURCHAKLAR O‘XSHASHLIGINING UCHINCHI ALOMATI

Teorema. (Uchburchaklar o‘xshashligining TTT alomati). **Agar bir uchburchakning uchta tomoni ikkinchi uchburchakning uchta tomoniga mos ravishda proporsional bo‘lsa, bunday uchburchaklar o‘xshash bo‘ladi.**

$\Delta ABC, \Delta A_1B_1C_1, \frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{CA}{C_1A_1}$ *(1-rasm)* ⇒ $\Delta ABC \sim \Delta A_1B_1C_1$



Isbot. *ABC* uchburchakning *AB* tomonida $AD = A_1B_1$ bo‘ladigan qilib *D* nuqtani belgilaymiz. *D* nuqtadan *BC* tomonga parallel qilib o‘tkazilgan to‘g‘ri chiziq *AC* tomonni *E* nuqtada kessin. Unda uchburchaklar o‘xshashligining BB alomatiga ko‘ra, ΔADE va ΔABC o‘xshash bo‘ladi. U holda teorema shartiga va ta’rifiga ko‘ra:

$$\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1} \quad \text{va} \quad \frac{AB}{AD} \quad \frac{BC}{DE}.$$

Ammo yasashga ko‘ra, $A_1B_1=AD$. Unda yuqoridagi tengliklardan $B_1C_1=DE$ tenglik hosil bo‘ladi. Shunday qilib, uchburchaklar tengligining TBT alomatiga ko‘ra, ΔADE va $\Delta A_1B_1C_1$ teng va $\Delta ADE \sim \Delta ABC$. Demak, $\Delta ABC \sim \Delta A_1B_1C_1$. ***Teorema isbotlandi.***

Masala. Agar ikkita teng yonli uchburchakdan birining asosi va yon tomoni ikkinchisining asosi va yon tomoniga proporsional bo‘lsa, bu uchburchaklarning o‘xshash ekanligini isbotlang.

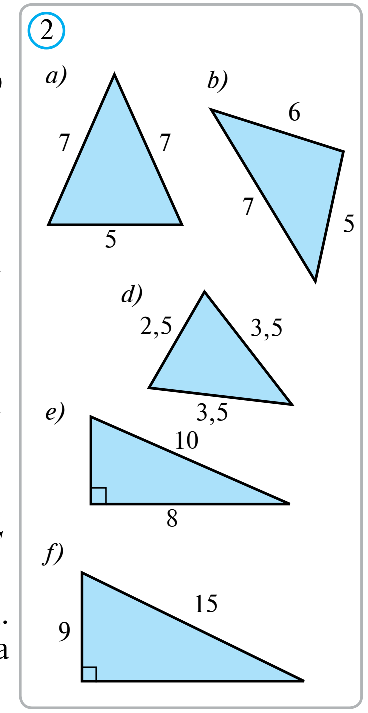
$\Delta ABC, \; AB=BC, \; \Delta A_1B_1C_1, \; A_1B_1=B_1C_1, \; \frac{AB}{A_1B_1}=\frac{AC}{A_1C_1}$ ⇒ $\Delta ABC \sim \Delta A_1B_1C_1$

Isbot. Berilgan $AB=BC$, $A_1B_1=B_1C_1$ tengliklar va $\frac{AB}{A_1B_1}=\frac{AC}{A_1C_1}$ nisbatdan $\frac{AB}{A_1B_1}=\frac{AC}{A_1C_1}=\frac{BC}{B_1C_1}$ tengliklarni hosil qilamiz. Demak, uchburchaklar o‘xshashligining TTT alomatiga ko‘ra, $\Delta ABC \sim \Delta A_1B_1C_1$.

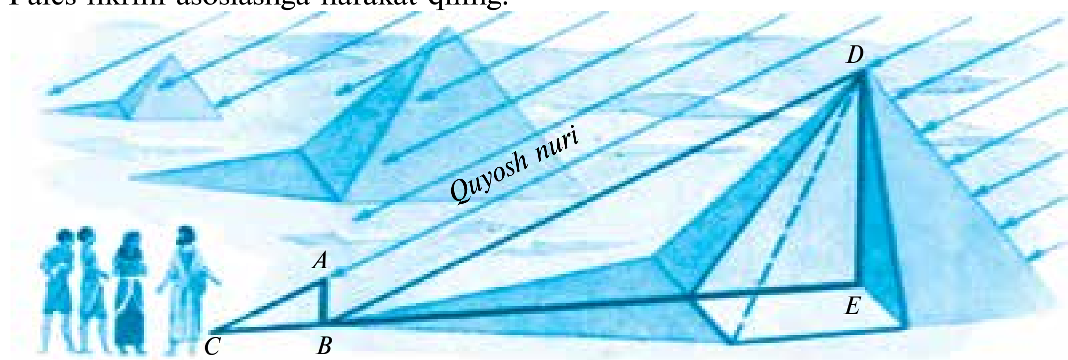
Savol, masala va topshiriqlar

1. Uchburchaklar o‘xshashligining TTT alomatini ayting va isbotini bayon qiling.
2. $AC=14$ *sm*, $AB=11$ *sm*, $BC=13$ *sm*, $A_1C_1=28$ *sm*, $A_1B_1=22$ *sm*, $B_1C_1=26$ *sm* ekanligi ma’lum. *ABC* va $A_1B_1C_1$ uchburchaklar o‘xshash bo‘ladimi?
3. 2-rasmdagi o‘xshash uchburchaklar juftliklarini ko‘rsating.

4. *ABCD* trapetsiyaning *AB* va *CD* yon tomonlari davom ettirilsa, *E* nuqtada kesishadi. Agar $AB=5$ *sm*, $BC=10$ *sm*, $CD=6$ *sm*, $AD=15$ *sm* bo‘lsa, *AED* uchburchak yuzini toping.
5. Trapetsiyaning asoslari 6 *sm* va 9 *sm*, balandligi 10 *sm*. Trapetsiyaning diagonallari kesishgan nuqtadan asoslarigacha bo‘lgan masofalarni toping.
6. Istalgan ikkita teng tomonli uchburchak o‘xshash bo‘lishini isbotlang.
7. Asosi 12 *sm*, balandligi 8 *sm* bo‘lgan teng yonli uchburchak ichiga kvadrat shunday ichki chizilganki, kvadratning ikkita uchi uchburchak asosida, qolgan ikki uchi esa yon tomonlarda yotadi. Kvadrat tomonini toping.
8*. O‘tkir burchakli *ABC* uchburchakning AA_1 va BB_1 balandliklari o‘tkazilgan. $\Delta ABC \sim \Delta A_1B_1C$ ekanligini isbotlang.
9. Ikkita o‘xshash uchburchak yuzlari 6 va 24 ga teng. Ulardan birining perimetri ikkinchisinikidan 6 ga ortiq. Katta uchburchakning perimetrini toping.



Tarixiy lavhalar. Bu voqea miloddan avvalgi VI asrda bo‘lgan. Bu vaqtda yunonlar geometriya bilan deyarli shug‘ullanishmas edi. Yunon faylasufi Fales misr fani bilan tanishish uchun tashrif buyurgan. Misrliklar unga qiyin masala beradi: ulkan piramidalardan birining balandligini qanday hisoblash mumkin? Fales bu masalaning sodda va jozibali yechimini topdi. U tayoqchani yerga qoqdi va shunday dedi: “Qachonki shu tayoqcha soyasining uzunligi tayoqchaning uzunligi bilan teng bo‘lsa, piramida soyasining uzunligi piramida balandligi bilan teng bo‘ladi”. Fales fikrini asoslashga harakat qiling!



10 TO‘G‘RI BURCHAKLI UCHBURCHAKLARNING O‘XSHASHLIK ALOMATLARI

Ma’lumki, to‘g‘ri burchakli uchburchaklarning bittadan burchaklari to‘g‘ri burchakdan iborat bo‘ladi. Shuning uchun bunday uchburchaklarning o‘xshashlik alomatlari ancha soddalashadi.

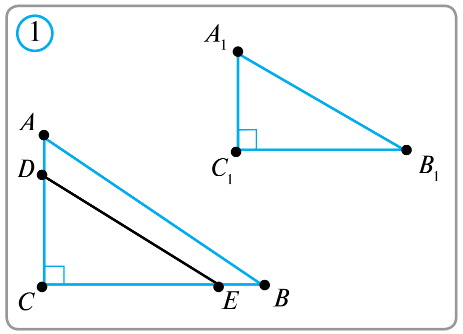
1-teorema. **To‘g‘ri burchakli uchburchaklarning bittadan o‘tkir burchagi mos ravishda teng bo‘lsa, ular o‘xshash bo‘ladi.**

2-teorema. **To‘g‘ri burchakli uchburchaklarning katetlari mos ravishda proporsional bo‘lsa, ular o‘xshash bo‘ladi.**

3-teorema. **To‘g‘ri burchakli uchburchaklardan birining gipotenuzasi va kateti ikkinchisining gipotenuzasi va katetiga mos ravishda proporsional bo‘lsa, ular o‘xshash bo‘ladi.**

Bu alomatlardan dastlabki ikkitasining to‘g‘riligi o‘z-o‘zidan ravshan. Keling, uchinchi alomatni isbotlaylik.

$\Delta ABC, \Delta A_1B_1C_1$, $\angle C=90°, \angle C_1=90°$, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}$ ⇒ $\Delta ABC \backsim \Delta A_1B_1C_1$



Isbot. ABC uchburchakning BC tomoniga $CE=C_1B_1$ bo‘ladigan qilib CE kesmani qo‘yamiz va $DE \parallel AB$ ni o‘tkazamiz *(1-rasm)*. Unda uchburchaklar o‘xshashligining BB alomatiga ko‘ra, ΔDEC va ΔABC o‘xshash bo‘ladi. O‘xshash uchburchaklar mos tomonlarining proporsionalligidan:

$$\frac{AB}{DE}=\frac{CB}{CE}.$$

Yasashga ko‘ra, $CE=C_1B_1$. Demak,

$$\frac{AB}{DE}=\frac{CB}{C_1B_1} \qquad (1)$$

tenglik o‘rinli. Boshqa tomondan, teorema shartiga ko‘ra, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}$ (2)

(1) va (2) tengliklardan $DE=A_1B_1$ ekanligini aniqlaymiz.

$A_1B_1C_1$ va DEC uchburchaklarni qaraymiz: 1. $CE=C_1B_1$ (yasashga ko‘ra); 2. $DE=A_1B_1$ (isbotlangan tenglik).

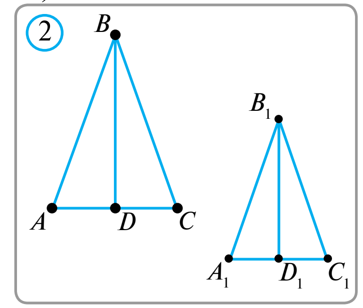
To‘g‘ri burchakli uchburchaklarning bittadan kateti hamda gipotenuzasi bo‘yicha tenglik alomatiga ko‘ra, $\Delta A_1B_1C_1 = \Delta DEC$.

Ikkinchi tomondan esa $\Delta ABC \backsim \Delta DEC$. U holda, $\Delta ABC \backsim \Delta A_1B_1C_1$ bo‘ladi.

Teorema isbotlandi.

Masala. Agar ikkita teng yonli uchburchakdan birining yon tomoni va balandligi ikkinchisining yon tomoni va balandligiga proporsional bo‘lsa, bu uchburchaklarning o‘xshash ekanligini isbotlang *(2-rasm)*.

Isbot. To‘g‘ri burchakli ABD va $A_1B_1D_1$ uchburchaklarni qaraymiz. Shartga ko‘ra, ularning bittadan kateti va gipotenuzasi o‘zaro proporsional. Demak, 3-teoremaga asosan $\Delta ABD \backsim \Delta A_1B_1D_1$. Unda $\angle DBA \backsim \angle D_1B_1A_1$.

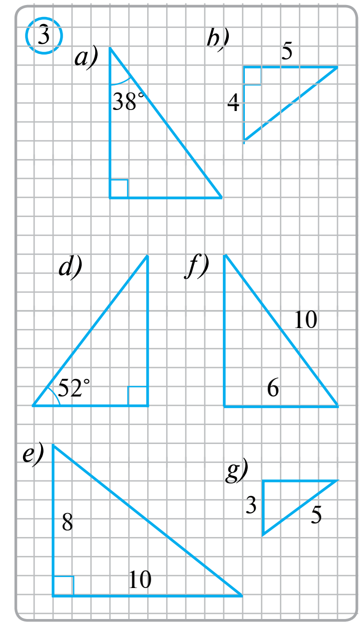


Teng yonli uchburchak asosiga tushirilgan balandlikning bissektrisa ham bo‘lishini hisobga olsak, $\angle B=2\angle DBA=2\angle D_1B_1A_1=\angle B_1$ bo‘ladi.

Natijada, ABC va $A_1B_1C_1$ uchburchaklarda

$\angle B=\angle B_1$ va $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}$ tengliklarga ega bo‘lamiz.

Demak, uchburchaklar o‘xshashligining TBT alomatiga ko‘ra, $\Delta ABC \backsim \Delta A_1B_1C_1$. So‘ralgan tasdiq isbotlandi.



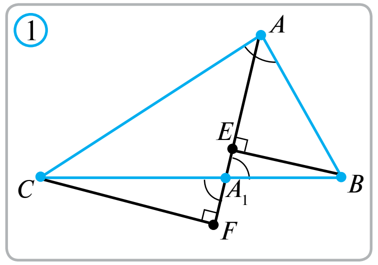
Savol, masala va topshiriqlar

1. To‘g‘ri burchakli uchburchaklarning o‘xshashlik alomatlarini ayting va isbotlang.
2. 3-rasmdan o‘xshash uchburchaklarni toping.
3. Katetlari 3 *m* va 4 *m* bo‘lgan to‘g‘ri burchakli uchburchakka o‘xshash uchburchakning bir kateti 27 *m* bo‘lsa, ikkinchi kateti necha *m* bo‘ladi?
4. Yuzlari 21 m^2 va 84 m^2 bo‘lgan ikkita to‘g‘ri burchakli uchburchaklar o‘xshash. Agar birinchi uchburchakning bir kateti 6 *m* bo‘lsa, ikkinchi uchburchak katetlarini toping.
5. Bir aylanaga ikkita o‘xshash to‘g‘ri burchakli uchburchak ichki chizilgan. Bu uchburchaklarning tengligini isbotlang.

6*. Katetlari 10 *sm* va 12 *sm* bo‘lgan to‘g‘ri burchakli uchburchakka bitta burchagi umumiy bo‘lgan kvadrat ichki chizilgan. Agar kvadratning bitta uchi gipotenuzada ekanligi ma’lum bo‘lsa, kvadratning tomonini toping.

7*. ABC uchburchak berilgan. Unga $ADEF$ romb shunday ichki chizilganki, D, E va F nuqtalar mos ravishda uchburchakning AB, BC va CA tomonlarida yotadi. Agar $AB=c$, $AC=b$ bo‘lsa, romb tomonini toping.

11 O‘XSHASHLIK ALOMATLARINING ISBOTLASHGA DOIR MASALALARGA TATBIQLARI



1-masala. Uchburchak bissektrisasi o‘zi tushgan tomonni qolgan ikki tomonga proporsional kesmalarga ajratishini isbotlang.

$\Delta ABC, AA_1$ — *bissektrisa (1-rasm)* ⇒ $\dfrac{AB}{A_1B} = \dfrac{AC}{A_1C}$

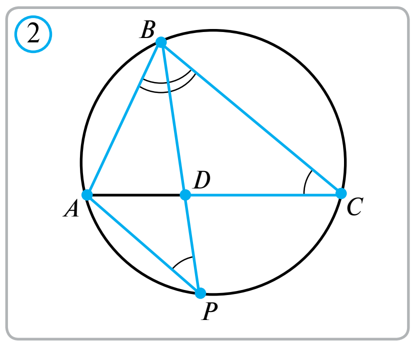
Yechilishi. AA_1 to‘g‘ri chiziqqa BE va CF perpendikularlar tushiramiz. Unda $\angle CAF = \angle BAE$ bo‘lgani uchun, to‘g‘ri burchakli CAF va BAE uchburchaklar o‘xshash bo‘ladi. O‘xshash uchburchaklarning mos tomonlari proporsionalligidan

$$\Delta CAF \sim \Delta BAE \Rightarrow \frac{AC}{AB} = \frac{CF}{BE}. \quad (1)$$

Shunga o‘xshash

$$\Delta CA_1F \sim \Delta BA_1E \Rightarrow \frac{CA_1}{BA_1} = \frac{CF}{BF}. \quad (2)$$

(1) va (2) tengliklarni solishtirsak, $\frac{AC}{AB} = \frac{CA_1}{BA_1}$ yoki $\frac{AB}{A_1B} = \frac{AC}{A_1C}$ bo‘ladi. Bu A_1B va A_1C kesmalar AB va AC kesmalarga proporsional ekanligini anglatadi.



2-masala. ABC uchburchakning BD bissektrisasi uchburchakka tashqi chizilgan aylanani B va P nuqtalarda kesadi. $\Delta ABP \sim \Delta BDC$ ekanligini isbotlang *(2-rasm)*.

Yechilishi. ΔABP va $\angle BDC$ da:

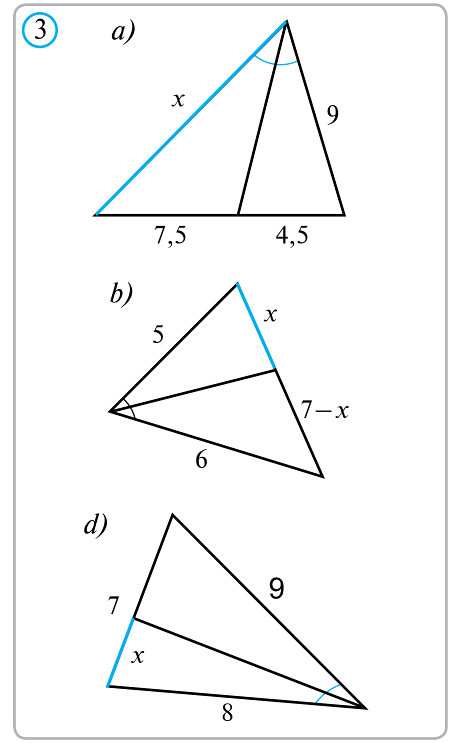
1. $\angle DBC = \angle ABP \Leftarrow$ shartga ko‘ra;
2. $\angle DCB = \angle APB \Leftarrow$ chunki ular bitta yoyga tiralgan.

Demak, uchburchaklar o‘xshashligining BB alomatiga ko‘ra, $\Delta ABP \sim \Delta BDC$.

Savol, masala va topshiriqlar

1. Uchburchak bissektrisasi o‘zi tushgan tomonda ajratgan kesmalari va uchburchakning qolgan tomonlari orasidagi proporsionallikni yozib ko‘rsating.
2. To‘g‘ri burchakli ABC uchburchakning C to‘g‘ri burchagidan CD balandlik o‘tkazilgan. $\angle ACD = \angle CBD$ bo‘lishini isbotlang. Hosil bo‘lgan shaklda nechta

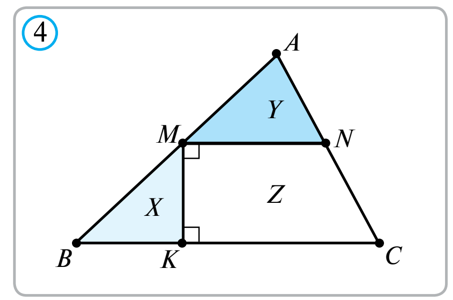
o‘zaro o‘xshash uchburchaklarni ko‘rsata olasiz?
3. 3-rasmdagi ma’lumotlar asosida x ni toping.
4. ABC uchburchaklarning AD bissektrisasi o‘tkazilgan. Agar $CD = 4,5$ *m*; $BD = 13,5$ *m* va ABC uchburchak perimetri 42 *m* bo‘lsa, uning AB va AC tomonlarini toping.
5. ABC uchburchak medianalari N nuqtada kesishadi. Agar ABC uchburchak yuzi 87 dm^2 bo‘lsa, ANB uchburchak yuzi nimaga teng?
6. ABC uchburchak medianalari kesishgan N nuqtadan AB va BC tomonlargacha bo‘lgan masofalar mos ravishda 3 *dm* va 4 *dm*. Agar $AB = 8$ *dm* bo‘lsa, BC tomonni hisoblang.
7*. Trapetsiyaning asosiga parallel to‘g‘ri chiziq yon tomonlaridan birini $m:n$ nisbatda bo‘lishi ma’lum. Bu to‘g‘ri chiziq uning ikkinchi yon tomonini qanday nisbatda bo‘ladi?



Qiziqarli masalalar

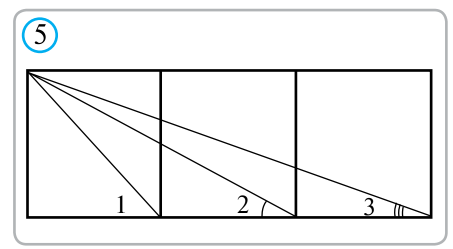
Geometriya va ingliz tili. Quyida ingliz tilida berilgan geometrik masalani yechib ko‘ring-chi! Bu bilan ham ingliz tilidan, ham geometriyadan nimaga qodirligingizni sinab olasiz.

1) *Dissection Puzzle:* Let M be the mid-point of the side AB of a given triangle ABC. The triangle has been dissected into parts X, Y, Z along the lines MN and MK passing through M such that MN is parallel while MK is perpendikular to the base BC (*picture 4*). Show how the three pieces can be fitted together to make a rectangle, respectively two different parallelograms.



2) Look at the picture 5 and proof

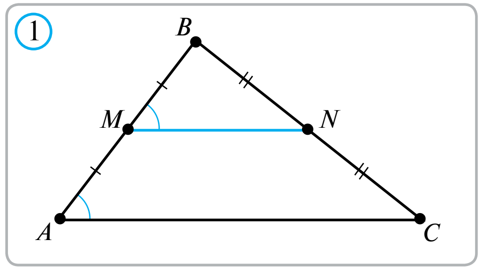
$$\angle 1 + \angle 2 + \angle 3 = 90°.$$



12 MASALALAR YECHISH

1-masala. Uchburchaklarning o‘xshashligidan foydalanib, uchburchak o‘rta chizig‘i uchburchakning bir tomoniga parallel va shu tomonning yarmiga teng ekanligini isbotlang.

$\Delta ABC, MN$ — o‘rta chiziq (1-rasm): $MA=MB, NC=NB$ ⇨ $MN \parallel AC,\ MN=\frac{1}{2}AC$



Yechilishi. ΔABC va ΔMBN uchun:

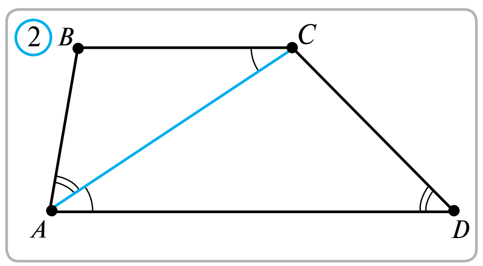
$\angle B$ — umumiy, $\frac{BM}{AB}=\frac{BN}{BC}=\frac{1}{2}$.

Shuning uchun, uchburchaklar o‘xshashligining TBT alomatiga ko‘ra, bu ikki uchburchak o‘xshash. Endi mushohadani mana bunday davom ettiramiz:

$$\Delta MBN \backsim \Delta ABC \Rightarrow \begin{cases} \angle BMN=\angle A, \\ \frac{MN}{AC}=\frac{1}{2} \end{cases} \Rightarrow \begin{cases} MN \parallel AC, \\ MN=\frac{1}{2}AC. \end{cases}$$

2-masala. Agar asoslari BC va AD bo‘lgan $ABCD$ trapetsiyaning AC diagonali uni ikkita o‘xshash uchburchakka ajratsa, $AC^2=BC\cdot AD$ bo‘lishini isbotlang.

$ABCD$ — trapetsiya, $BC \parallel AD$, $\Delta ABC \backsim \Delta DCA$ (2-rasm) ⇨ $AC^2=BC\cdot AD$



Yechilishi. **1-qadam.** ABC va ACD uchburchaklarning burchaklarini taqqoslaymiz. $\angle ACB=\angle CAD$, chunki bu burchaklar — ichki almashinuvchi burchaklar. $\angle B\neq\angle D$, chunki $ABCD$ — trapetsiya(aks holda,

$$\angle D+\angle A=\angle B+\angle A=180^\circ,$$

ya’ni $AB \parallel CD$ bo‘lib, $ABCD$ trapetsiya bo‘lmay qolar edi). U holda, $\angle D=\angle BAC$ va $\angle ACD=\angle B$.

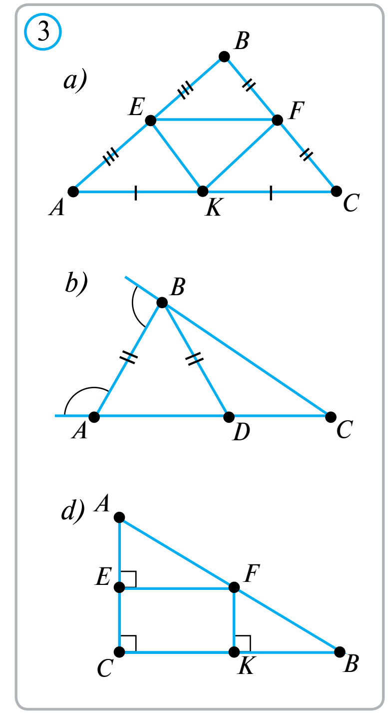
2-qadam. Endi ABC va DCA o‘xshash uchburchaklarning mos tomonlari nisbatini yozamiz: $\frac{AC}{BC}=\frac{AD}{AC}$, bundan $AC^2=BC\cdot AD$.

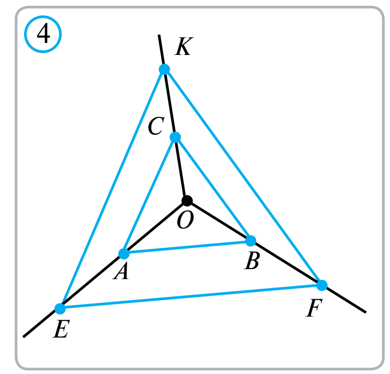
Savol, masala va topshiriqlar

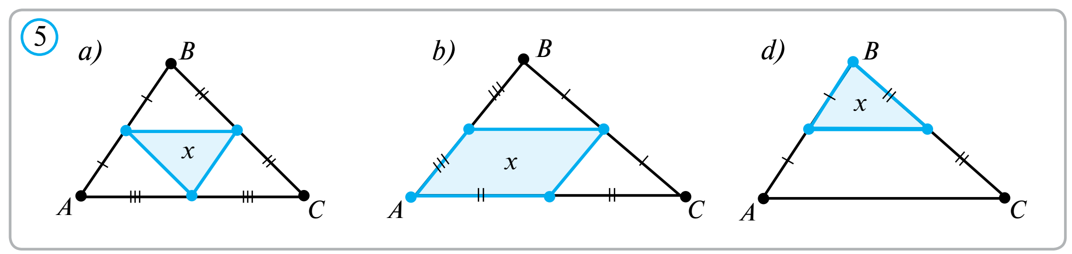
1. a) Bo‘yi 170 *sm* bo‘lgan odam soyasining uzunligi 1 *m* bo‘lsa, balandligi 5,4 *m* bo‘lgan simyog‘och soyasining uzunligini toping.
   b) Ikkita teng yonli uchburchakning uchidagi burchaklari teng. Birinchi uchburchakning yon tomoni 17 *sm*, asosi 10 *sm* ga, ikkinchi uchburchakning asosi 8 *sm* ga teng. Ikkinchi uchburchakning yon tomonini toping.
2. 3-rasmdagi har bir chizmadan o‘xshash uchburchaklarni ko‘rsating.
3. ABC uchburchakning AP medianasi BC tomonga parallel va uchlari AB va AC tomonlarda yotgan istalgan kesmani teng ikkiga bo‘lishini isbotlang.
4. Uchburchakning uchlari uning o‘rta chizig‘ini o‘z ichiga olgan to‘g‘ri chiziqdan teng masofada yotishini isbotlang.
5. Aylanaga ichki chizilgan $ABCD$ to‘rtburchak diagonallari O nuqtada kesishadi. $\Delta AOB \backsim \Delta COD$ ekanligini isbotlang.
6. ABC uchburchak ichki sohasida O nuqta va OA, OB, OC nurlarda mos ravishda E, F, K nuqtalar olingan *(4-rasm)*. Agar $AB \parallel EF$ va $BC \parallel FK$ bo‘lsa, ABC va EFK uchburchaklar o‘xshash ekanligini isbotlang.

7*. Trapetsiyaning diagonallari kesishish nuqtasidan o‘tuvchi to‘g‘ri chiziq trapetsiya asoslaridan birini $m:n$ nisbatda bo‘ladi. Bu to‘g‘ri chiziq ikkinchi asosni qanday nisbatda bo‘ladi?

8. Agar ABC uchburchakning yuzi S ga teng bo‘lsa, 5-rasmda x bilan belgilangan soha yuzini toping.







13 BILIMINGIZNI SINAB KO‘RING

I. Testlar

1. Quyidagi ta’riflardan qaysi biri to‘g‘ri?

A) Ikkita uchburchakning burchaklari mos ravishda teng bo‘lsa, ular o‘xshash deyiladi;
B) Ikkita uchburchakning tomonlari mos ravishda teng bo‘lsa, ular o‘xshash deyiladi;
D) Ikkita uchburchakning mos tomonlari proporsional va mos burchaklari teng bo‘lsa, ular o‘xshash deyiladi;
E) Ikkita uchburchakning mos tomonlari va mos burchaklari teng bo‘lsa, ular o‘xshash deyiladi.

2. Ikkita o‘xshash uchburchak yuzlarining nisbati nimaga teng?

A) O‘xshashlik koeffitsiyentiga;
B) Ularning mos tomonlari nisbatiga;
D) Ularning perimetrlari nisbatiga;
E) O‘xshashlik koeffitsiyentining kvadratiga.

3. Quyidagi tasdiqlardan qaysi biri to‘g‘ri?

A) Uchburchaklardan birining ikkita burchagi ikkinchisining ikkita burchagiga teng bo‘lsa, ular o‘xshash bo‘ladi;
B) Uchburchaklardan birining ikkita tomoni ikkinchisining ikki tomoniga teng bo‘lsa, ular o‘xshash bo‘ladi;
D) Ikkita uchburchakning bittadan burchaklari teng va ikkitadan tomonlari proporsional bo‘lsa, ular o‘xshash bo‘ladi;
E) Ikkita uchburchakning bittadan burchaklari teng va bittadan tomonlari proporsional bo‘lsa, ular o‘xshash bo‘ladi.

4. To‘g‘risini toping. Agar ikkita uchburchak o‘xshash bo‘lsa, ularning

A) Balandliklari teng bo‘ladi; B) Tomonlari proporsional bo‘ladi;
D) Tomonlari teng bo‘ladi; E) Yuzlari teng bo‘ladi.

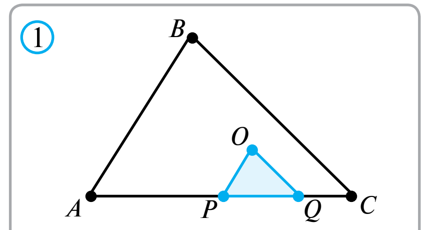
5. O‘xshash uchburchaklarning perimetrlari nisbati nimaga teng?

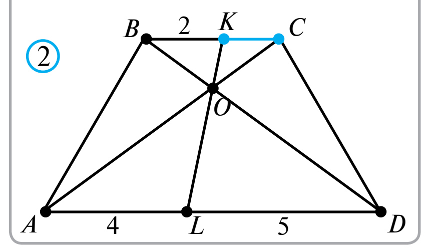
A) Mos tomonlar nisbatining kvadratiga; B) O‘xshashlik koeffitsiyentiga;
D) O‘xshashlik koeffitsiyentining kvadratiga; E) Yuzlari nisbatiga.

II. Masalalar

1. ABC uchburchakning AB va AC tomonlari o‘rtalari mos ravishda E va F nuqtalar bo‘lsin. Agar AEF uchburchak yuzi 3 sm^2 bo‘lsa, ABC uchburchak yuzini toping.
2. ABC uchburchakning AC tomoniga parallel to‘g‘ri chiziq AB va BC tomonlarni mos ravishda N va P nuqtalarda kesadi. Agar $AN=4$, $NB=3$, $BP=3{,}6$ bo‘lsa, BC tomonni toping.

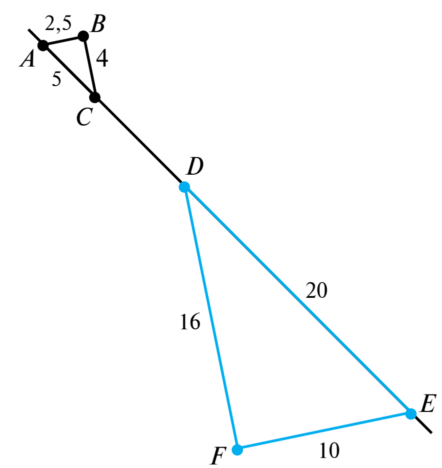
3. O‘tkir burchakli ABC uchburchakning AB tomonida K nuqta olingan. Agar $AK=3$, $BK=2$ va uchburchakning BD balandligi 4 ga teng bo‘lsa, K nuqtadan AC kesmagacha bo‘lgan masofani toping.
4. $ABCD$ parallelogramning BC tomoni o‘rtasidagi K nuqtadan o‘tkazilgan DK nur bilan AB nur F nuqtada kesishadi. Agar $AD=4$, $DK=5$ va $DC=5$ bo‘lsa, AFD uchburchak perimetrini hisoblang.
5. ABC uchburchak ichki sohasida olingan O nuqtadan AB va BC tomonlarga parallel to‘g‘ri chiziqlar o‘tkazilgan. Bu to‘g‘ri chiziqlar AC tomonni mos ravishda P va Q nuqtalarda kesadi. Agar $PQ=2$, $AC=7$ va ABC uchburchak yuzi 98 ga teng bo‘lsa, POQ uchburchak yuzini aniqlang (*1-rasm*).
6. $ABCD$ trapetsiyaning BC va AD asoslarida mos ravishda K va L nuqtalar olingan. KL kesma trapetsiyaning diagonallari kesishgan nuqtadan o‘tadi. Agar $AL=4$, $LD=5$ va $BK=2$ bo‘lsa, KC kesmani toping (*2-rasm*).



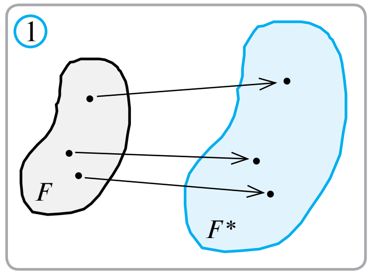


III. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi)

1. $ABCD$ trapetsiyaning AC diagonali uni o‘xshash ΔABC va ΔACD uchburchaklarga ajratadi. Bunda $BC=4$ m, $AD=9$ m bo‘lsa, AC diagonal uzunligini hisoblang.
2. Ikkita o‘xshash uchburchakning yuzi 50 dm^2 va 32 dm^2, ularning perimetrlari yig‘indisi 117 dm bo‘lsa, har bir uchburchakning perimetrini toping.
3. Rasmda tasvirlangan uchburchaklarning o‘xshashligini isbotlang. BC va DF to‘g‘ri chiziqlarning o‘zaro joylashishi to‘g‘risida nima deya olasiz?
4. *(Qo‘shimcha)*. O‘tkir burchakli ABC uchburchakning BD va AE balandliklari o‘tkazilgan. $DC \cdot AC = EC \cdot BC$ bo‘lishini isbotlang.



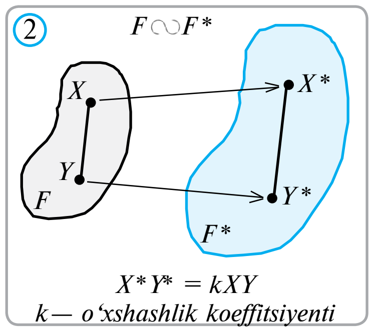
14 GEOMETRIK SHAKLLARNING O‘XSHASHLIGI



Oldingi darslarda ko‘pburchaklarning o‘xshashligi tushunchasi bilan tanishdik. Bu tushunchani faqat ko‘pburchaklar uchun emas, balki istalgan geometrik shakllar uchun ham kiritish mumkin.

Agar F va F^* shakllar berilgan bo‘lib, F shaklning har bir nuqtasiga F^* shaklning biror nuqtasi mos qo‘yilgan bo‘lsa va bunda F^* shaklning har bir nuqtasiga F shaklning faqat bitta nuqtasi mos kelsa, *(1-rasm)* ***F* shakl *F** shaklga almashtirilgan** deyiladi.

Ta’rif. Agar F shaklni F^* shaklga almashtirishda nuqtalar orasidagi masofalar bir xil son marta o‘zgarsa, bunday almashtirishga **o‘xshashlik almashtirishi** deyiladi *(2-rasm)*.

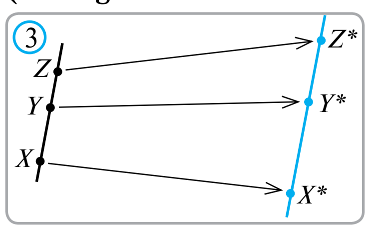


Bu ta’rifni quyidagicha talqin qilish mumkin: Aytaylik, biror almashtirish natijasida F shaklning ixtiyoriy X, Y nuqtalariga F^* shaklning X^*, Y^* nuqtalari mos qo‘yilgan bo‘lsin. Agar $X^*Y^*=k\cdot XY$, $k>0$ bo‘lsa, bunday almashtirishga **o‘xshashlik almashtirishi** deyiladi. Bunda k — barcha X va Y nuqtalar uchun bir xil son bo‘lib, u o‘xshashlik koeffitsiyenti deb yuritiladi.

Agar F va F^* shakllar berilgan bo‘lib, bu shakllardan birining ikkinchisiga o‘tkazadigan o‘xshashlik almashtirishi mavjud bo‘lsa, F va F^* shakllar o‘zaro **o‘xshash** deyiladi. Shakllarning o‘xshashligi $F \backsim F^*$ kabi yoziladi. Agar o‘xshashlik koeffitsiyenti k ni ham ko‘rsatish lozim bo‘lsa, $F \overset{k}{\backsim} F^*$ tarzda ham belgilanadi.

Agar o‘xshashlik almashtirishida X nuqtaga X^* nuqta mos qo‘yilgan bo‘lsa, **X nuqta X* nuqtaga almashdi** yoki **o‘tdi** deyiladi.

Teorema. **O‘xshashlik almashtirishi a) to‘g‘ri chiziqni to‘g‘ri chiziqqa; b) nurni nurga; d) burchakni (uning kattaligini saqlagan holda) burchakka; e) kesmani (uzunligi bu kesmadan *k* marta uzun bo‘lgan) kesmaga o‘tkazadi.**



Isbot. a) O‘xshashlik koeffitsiyenti k bo‘lgan almashtirishda bir to‘g‘ri chiziqda yotgan turli X, Y va Z nuqtalar mos ravishda X^*, Y^* va Z^* nuqtalarga almashsin *(3-rasm)*.

X, Y, Z nuqtalardan biri, aytaylik, Y qolgan ikkitasining orasida yotsin. U holda $XZ=XY+YZ$.

O‘xshashlik almashtirishi ta’rifiga ko‘ra:

$$X^*Z^*=k\cdot XZ=k\cdot(XY+YZ)=k\cdot XY+k\cdot YZ=X^*Y^*+Y^*Z^*.$$

Bu tenglikdan X^*, Y^* va Z^* nuqtalarning bir to‘g‘ri chiziqda yotishi kelib chiqadi.

Teoremaning isbotini faqat a) tasdiq uchun keltirdik. Qolgan tasdiqlarda isbotlashni sizga mashq tariqasida qoldiramiz.

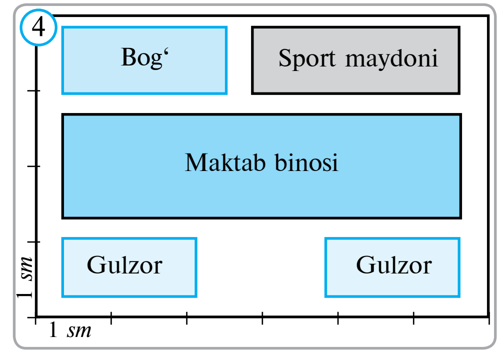
Savol, masala va topshiriqlar

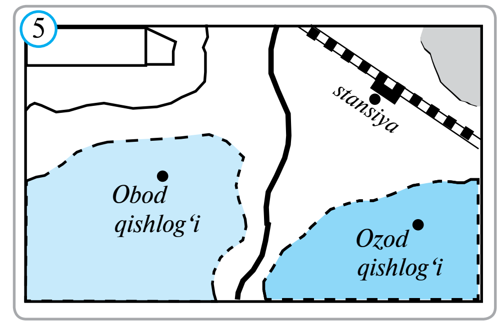
1. O‘xshashlik almashtirishi nima?
2. Qanday shakllar o‘xshash deyiladi?
3. Eni 3 *sm*, bo‘yi 4 *sm* bo‘lgan to‘g‘ri to‘rtburchakka o‘xshash, o‘xshashlik koeffitsiyenti 2 ga teng bo‘lgan to‘rtburchak yasang.
4. 4-rasmda maktab hovlisining tarxi 1:1000 masshtabda tasvirlangan. O‘lchash ishlarini bajarib,
   a) hovlining; b) maktab binosining;
   d) gulzorlarning; e) sport maydonining;
   f) bog‘ning haqiqiy o‘lchamlarini toping.
5. Agar xarita 1:50000 masshtabda tasvirlangan bo‘lsa *(5-rasm)*, Obod va Ozod qishloqlari markazlari orasidagi masofani toping.
6. O‘xshashlik almashtirishida nurlar orasidagi burchak saqlanishini isbotlang.

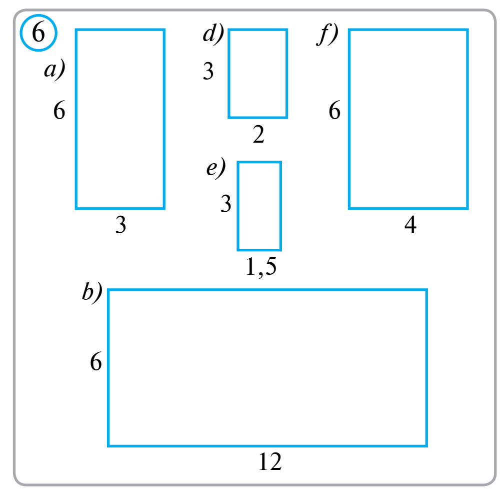
7*. O‘xshashlik almashtirishida a) parallelogramm parallelogrammga; b) kvadrat kvadratga; d) to‘g‘ri to‘rtburchak to‘g‘ri to‘rtburchakka; e) trapetsiya trapetsiyaga almashishini isbotlang.

8*. ABC uchburchak o‘xshashlik almashtirishida $A^*B^*C^*$ uchburchakka almashadi. Agar o‘xshashlik koeffitsiyenti 0,6 ga va ABC uchburchak perimetri 12 *sm* ga teng bo‘lsa, $A^*B^*C^*$ uchburchak perimetrini toping.

9. 6-rasmdan o‘xshash to‘g‘ri to‘rtburchaklar juftliklarini toping va o‘xshashlik koeffitsiyentlarini aniqlang.







15 O‘XSHASH KO‘PBURCHAKLARNING XOSSALARI

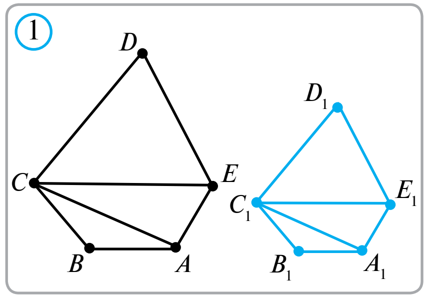
1-teorema. **O‘xshash ko‘pburchaklar perimetrlarining nisbati o‘xshashlik koeffitsiyentiga teng.**

Isbot. Haqiqatan ham, $A_1A_2...A_n$ va $B_1B_2...B_n$ ko‘pburchaklar o‘xshash va o‘xshashlik koeffitsiyenti k bo‘lsa, $B_1B_2=k\cdot A_1A_2$, $B_2B_3=k\cdot A_2A_3$, ... , $B_nB_1=k\cdot A_nA_1$ bo‘ladi. Bundan

$$P=B_1B_2+B_2B_3+...+B_nB_1=k\cdot A_1A_2+k\cdot A_2A_3+...+k\cdot A_nA_1=k\cdot(A_1A_2+A_2A_3+...+A_nA_1)=k\cdot P_1$$

tenglikni hosil qilamiz. ***Teorema isbotlandi.***

2-teorema. **O‘xshash ko‘pburchaklarni bir xil sondagi o‘xshash uchburchaklarga ajratish mumkin.**



Isbot. Aytaylik, $ABCDE$ va $A_1B_1C_1D_1E_1$ ko‘pburchaklar o‘xshash bo‘lib, o‘xshashlik koeffitsiyenti k bo‘lsin.

O‘zaro mos C va C_1 uchlardan CA, CE va C_1A_1, C_1E_1 diagonallarni o‘tkazamiz. Natijada, ko‘pburchaklar bir xil sondagi uchburchaklarga ajraldi. Hosil bo‘lgan uch juft mos uchburchaklarning o‘xshashligini ko‘rsatamiz.

1. $\Delta ABC \backsim \Delta A_1B_1C_1$. Chunki, bu uchburchaklarda, shartga ko‘ra, $\angle B=\angle B_1$, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=k$. Uchburchaklar o‘xshashligining TBT alomatiga ko‘ra, $\Delta ABC \backsim \Delta A_1B_1C_1$.

2. $\Delta CDE \backsim \Delta C_1D_1E_1$. Bu o‘xshashlik 1-banddagi kabi isbotlanadi.

3. $\Delta ACE \backsim \Delta A_1C_1E_1$. Haqiqatan, $\angle CAE$ va $\angle C_1A_1E_1$ burchaklarni qaraymiz:

$$\angle CAE=\angle BAE-\angle CAB,\ \angle C_1A_1E_1=\angle B_1A_1E_1-\angle C_1A_1B_1.$$

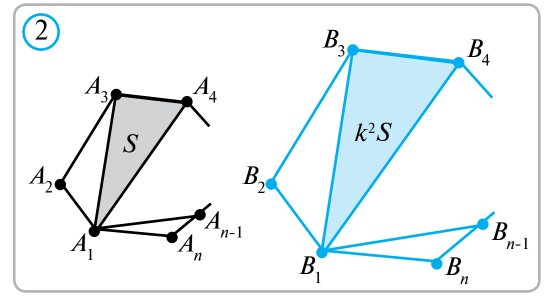
Bu yerda, $\angle BAE=\angle B_1A_1E_1$ (berilgan o‘xshash beshburchaklarning mos burchaklari). $\angle CAB=\angle C_1A_1B_1$ (o‘xshash ABC va $A_1B_1C_1$ uchburchaklarning mos burchaklari).

Demak, $\angle CAE=\angle C_1A_1E_1$.

AC va AE hamda A_1C_1 va A_1E_1 tomonlarni qaraymiz: $AC=kA_1C_1$, chunki ular o‘zaro o‘xshash ABC va $A_1B_1C_1$ uchburchaklarning mos tomonlari, $AE=kA_1E_1$, chunki ular ham berilgan o‘xshash beshburchaklarning mos tomonlari. Demak, uchburchaklar o‘xshashligining TBT alomatiga ko‘ra, $\Delta ACE \backsim \Delta A_1C_1E_1$. Ixtiyoriy o‘xshash ko‘pburchaklar uchun ham shu kabi mushohadalar yaroqli bo‘lishi ravshan.

Teorema isbotlandi.

3-teorema. **O‘xshash ko‘pburchaklar yuzlarining nisbati o‘xshashlik koeffitsiyentining kvadratiga teng.**



Isbot. Aytaylik, $A_1A_2...A_n$ va $B_1B_2...B_n$ ko‘pburchaklar o‘xshash va k — o‘xshashlik koeffitsiyenti bo‘lsin. U holda $A_1A_2A_3$, $A_1A_3A_4$, ..., $A_1A_{n-1}A_n$ uchburchaklar mos ravishda, $B_1B_2B_3$, $B_1B_3B_4$, ..., $B_1B_{n-1}B_n$ uchburchaklarga o‘xshash bo‘lib, o‘xshash uchburchaklar yuzlarining nisbati k^2 ga teng bo‘ladi (*2-rasm*):

$S_{A_1A_2A_3}=k^2S_{B_1B_2B_3}$, $S_{A_1A_3A_4}=k^2S_{B_1B_3B_4}$, ... , $S_{A_1A_{n-1}A_n}=k^2S_{B_1B_{n-1}B_n}$. Bu tengliklarning mos qismlarini qo‘shsak, $S_{A_1A_2...A_n}=k^2\,S_{B_1B_2...B_n}$ bo‘ladi. ***Teorema isbotlandi.***

Masala. Perimetrlari 18 *sm* va 24 *sm* bo‘lgan ikkita o‘xshash ko‘pburchak yuzlarining nisbatini toping.

Yechilishi. 1) O‘xshash ko‘pburchaklar perimetrlarining nisbati o‘xshashlik koeffitsiyentiga teng ekanligidan foydalanib, $k=24:18=4:3$ ekanligini topamiz.

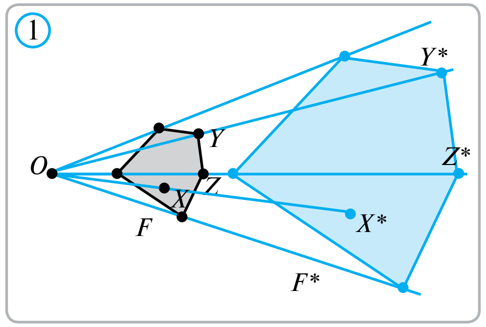
2) O‘xshash ko‘pburchaklar yuzlarining nisbati o‘xshashlik koeffitsiyentining kvadratiga teng bo‘lgani uchun izlangan nisbat $k^2=\frac{16}{9}$ ga teng. ***Javob:*** $\frac{16}{9}$.

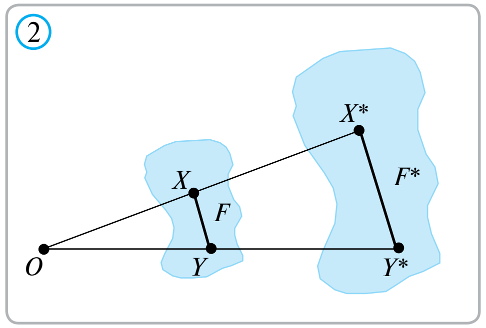
Savol, masala va topshiriqlar

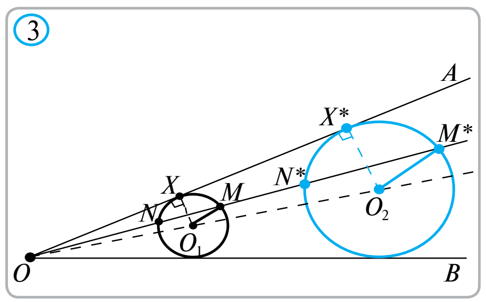
1. O‘xshash ko‘pburchaklar perimetrlarining nisbati nimaga teng?
2. O‘xshash ko‘pburchaklar yuzlarining nisbati haqidagi teoremani sharhlang.
3. Uchburchak bilan to‘rtburchak o‘xshash bo‘lishi mumkinmi?
4. Yuzlari 6 m^2 va 24 m^2 bo‘lgan ikkita to‘rtburchak o‘xshash. O‘xshashlik koeffitsiyentini toping.
5. Ikkita ko‘pburchakning perimetrlari 18 *sm* va 36 *sm* ga, yuzlarining yig‘indisi esa 30 sm^2 ga teng. Ko‘pburchaklar yuzlarini toping.
6. Perimetri 84 *sm* bo‘lgan uchburchakning bir tomoniga parallel qilib o‘tkazilgan to‘g‘ri chiziq undan perimetri 42 *sm* ga va yuzi 26 sm^2 ga teng uchburchak ajratdi. Berilgan uchburchak yuzini toping.
7. *O* nuqtaga nisbatan simmetrik shakllar o‘xshash bo‘ladimi? O‘qqa nisbatan simmetrik shakllar-chi? Ularning o‘xshashlik koeffitsiyenti nimaga teng?
8. To‘rtburchak shaklidagi paxta maydoni xaritada yuzi 12 sm^2 bo‘lgan to‘rtburchak bilan tasvirlanadi. Agar xarita masshtabi 1:1000 bo‘lsa, maydonning haqiqiy yuzini hisoblang.

9*. Yuzlari 8 sm^2 va 32 sm^2 bo‘lgan ikkita o‘xshash uchburchak perimetrlarining yig‘indisi 48 *sm* ga teng. Uchburchaklarning perimetrlarini toping.

16 GOMOTETIYA VA O‘XSHASHLIK







Eng sodda o‘xshash almashtirishlardan biri gomotetiyadir. Aytaylik, F — shakl, O — nuqta va k — musbat son berilgan bo‘lsin. F shaklning istalgan X nuqtasi orqali OX nur o‘tkazamiz va bu nurda uzunligi $k\cdot OX$ bo‘lgan OX^* kesmani qo‘yamiz *(1-rasm)*. Shu usul bilan F shaklning har bir X nuqtasiga X^* nuqtani mos qo‘yadigan almashtirish **gomotetiya** deyiladi. Bunda, O nuqta gomotetiya markazi, k soni gomotetiya koeffitsiyenti, F va gomotetiya natijasida F shakl almashadigan F^* shakllar esa **gomotetik shakllar** deyiladi.

Teorema. **Gomotetiya o‘xshashlik almashtirishi bo‘ladi.**

Isbot. Ixtiyoriy O markazli, k koeffitsiyentli gomotetiyada F shaklning X va Y nuqtalari X^* va Y^* nuqtalarga o‘tsin *(2-rasm)*. U holda, gomotetiya ta’rifiga ko‘ra, XOY va X^*OY^* uchburchaklarda $\angle O$ — umumiy va $\frac{OX^*}{OX}=\frac{OY^*}{OY}=k$ bo‘ladi. Demak, XOY va X^*OY^* uchburchaklar ikki tomoni va ular orasidagi burchagi bo‘yicha o‘xshash. Shuning uchun $\frac{X^*Y^*}{XY}=\frac{OX^*}{OX}=k$, xususan, $X^*Y^*=k\cdot XY$. ***Teorema isbotlandi.***

Masala. AOB burchak tomonlariga urinuvchi ixtiyoriy ikki aylana gomotetik bo‘lishini va O nuqta bu gomotetiya uchun markaz ekanligini isbotlang.

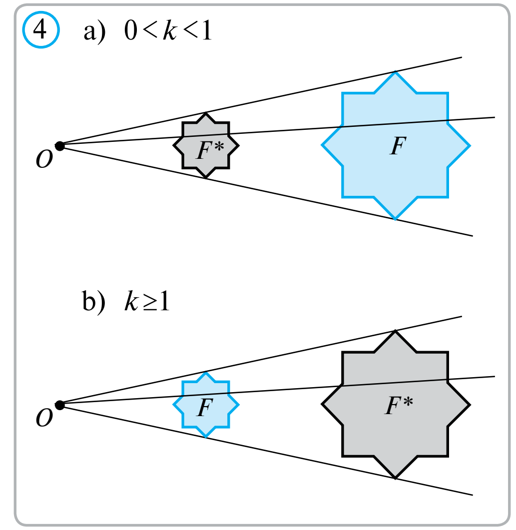
Yechilishi. Markazlari O_1 va O_2 bo‘lgan aylanalar AOB burchak tomonlariga urinsin *(3-rasm)*. Bu aylanalarning gomotetik ekanligini isbotlaymiz.

Aylanalar OA nurga mos ravishda X va X^* nuqtalarda uringan bo‘lsin *(3-rasm)*. U holda, $\Delta OXO_1 \sim \Delta OX^*O_2$ (chunki $\angle XOO_1=\angle X^*OO_2$ va $\angle OXO_1=\angle OX^*O_2=90°$).

Bundan, $\frac{OX^*}{OX}=\frac{OO_2}{OO_1}$.

O‘ng tomondagi nisbatni k bilan belgilaymiz va koeffitsiyenti $k=\frac{OO_2}{OO_1}$, markazi O bo‘lgan gomotetiyani qaraymiz. Aytaylik, bu gomotetiyada O_1 markazli aylananing istalgan M nuqtasi M^* nuqtaga almashgan bo‘lsin. U holda $O_2M^*=k\cdot O_1M$ yoki $O_2M^*=\frac{O_2X^*}{O_1X}\cdot O_1M$.

Bundan, $O_1X=O_1M$ bo‘lgani uchun $O_2M^*=O_2X^*$ tenglikni hosil qilamiz. Bu M^* nuqta markazi O_2 nuqtada, radiusi O_2X^* ga teng bo‘lgan aylanada yotishini bildiradi. Demak, qaralayotgan aylanalar o‘zaro gomotetik ekan.



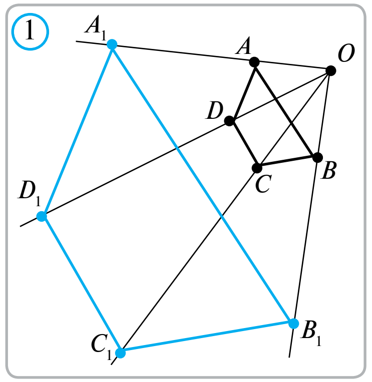
Faollashtiruvchi mashq

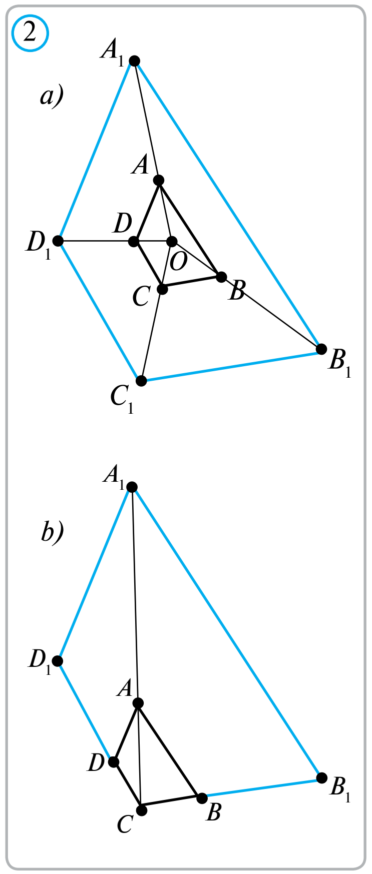
4-rasmda gomotetiya koeffitsiyenti a) $0<k<1$; b) $k\geq 1$ bo‘lgan gomotetik shakllar tasvirlangan. Gomotetiya koeffitsiyentining qiymatiga qarab gomotetik shakllarning “siqilishi” yoki “cho‘zilishi” haqida qanday xulosa chiqarish mumkin?

Savol, masala va topshiriqlar

1. Gomotetiya nima? Gomotetiya markazi, koeffitsiyenti-chi?
2. Gomotetiya o‘xshashlik almashtirishi ekanligini izohlang.
3. Uchburchak chizing. Uchburchak a) ichki sohasida; b) tashqi sohasida O nuqta belgilang va koeffitsiyenti 2 ga teng bo‘lgan O markazli gomotetiyani qarab, berilgan uchburchakka gomotetik uchburchak yasang.
4. Perimetrlari 18 *sm* va 27 *sm* bo‘lgan ikkita romb o‘zaro gomotetik. Bu romblar tomonlari va yuzlarining nisbatlarini toping.
5. Gomotetiyada X nuqta X^* nuqtaga, Y nuqta Y^* nuqtaga o‘tadi. Agar X, X^*, Y, Y^* nuqtalar bir to‘g‘ri chiziqda yotmasa, shu gomotetiya markazini toping.
6. Koeffitsiyenti 2 ga teng bo‘lgan gomotetiyada X nuqta X^* nuqtaga o‘tishi ma’lum. Shu gomotetiya markazini yasang.
7. Aylanaga gomotetik shakl aylana bo‘lishini isbotlang.
8. Aylana chizing. Markazi aylana markazida va koeffitsiyenti a) $\frac{1}{2}$; b) 2; d) 3; e) $\frac{1}{3}$ ga teng bo‘lgan gomotetiyada chizilgan aylanaga gomotetik bo‘lgan shakllarni quring.
9. Burchak va uning ichki sohasida A nuqta berilgan. Burchak tomonlariga urinib, A nuqtadan o‘tuvchi aylana yasang.

17 O‘XSHASH KO‘PBURCHAKLARNI YASASH





Shu paytgacha teoremalarni isbotlashda va masalalarni yechishda turli o‘xshash uchburchaklarni yasab keldik. O‘xshash ko‘pburchaklar qanday yasaladi? Quyida shu bilan tanishasiz.

Masala. Berilgan $ABCD$ to‘rtburchakka o‘xshash, o‘xshashlik koeffitsiyenti 3 ga teng bo‘lgan $A_1B_1C_1D_1$ to‘rtburchak yasang *(1-rasm)*.

Yasash. Tekislikda ixtiyoriy O nuqtani olamiz. Undan va to‘rtburchakning uchlaridan o‘tuvchi OA, OB, OC va OD nurlarni o‘tkazamiz. Bu nurlarda O nuqtadan $OA_1 = 3OA$, $OB_1 = 3OB$, $OC_1 = 3OC$ va $OD_1 = 3OD$ kesmalarni qo‘yamiz. Hosil bo‘lgan $A_1B_1C_1D_1$ to‘rtburchak izlangan to‘rtburchakdir.

Asoslash. $ABCD \sim A_1B_1C_1D_1$ ekanligini isbotlaymiz.

1. ***Mos tomonlarning proporsionalligi.***

a) $\Delta AOD \sim \Delta A_1OD_1 \Rightarrow \frac{A_1D_1}{AD} = \frac{O_1D_1}{OD} = \frac{OA_1}{OA} = 3;$ (1)

b) $\Delta DOC \sim \Delta D_1OC_1 \Rightarrow \frac{OD_1}{OD} = \frac{D_1C_1}{DC} = \frac{OC_1}{OC} = 3.$ (2)

(1) va (2) tenglikdan $\frac{A_1D_1}{AD} = \frac{D_1C_1}{DC}$ ekanligini hosil qilamiz.

To‘rtburchaklarning boshqa mos tomonlari proporsionalligini xuddi shunga o‘xshash isbotlash mumkin.

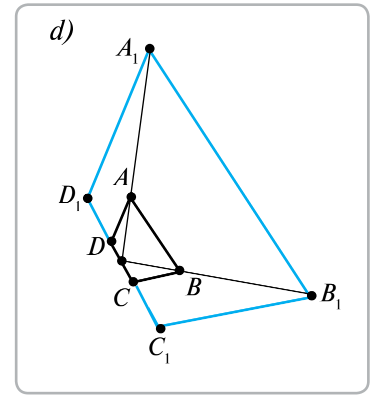
2. ***Mos burchaklarning tengligi.***

O‘xshash uchburchaklarning mos burchaklari teng bo‘lgani uchun, $\angle A_1D_1O = \angle ADO$, $\angle C_1D_1O = \angle CDO$. U holda,

$$\angle A_1D_1C_1 = \angle A_1D_1O + \angle C_1D_1O =$$
$$= \angle ADO + \angle CDO = \angle ADC,$$

ya’ni to‘rtburchaklarning mos $A_1D_1C_1$ va ADC burchaklari teng. Xuddi shunga o‘xshash to‘rtburchaklarning boshqa mos burchaklari tengligi isbotlanadi.

Demak, $ABCD$ va $A_1B_1C_1D_1$ to‘rtburchaklar o‘xshash. Tomonlari ixtiyoriy sonda bo‘lgan ko‘pburchakka o‘xshash ko‘pburchak ham xuddi shu kabi yasaladi.



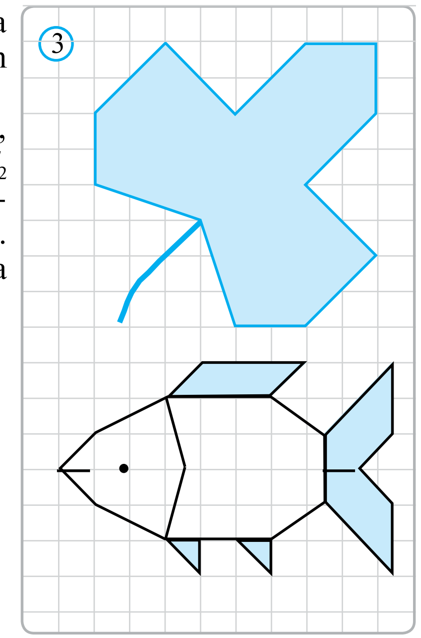
Gomotetiya markazini bu masalada to‘rtburchak tashqi sohasidan tanladik. Umuman olganda, gomotetiya markazini to‘rtburchakning ichki sohasida *(2-a rasm)*, biror uchida *(2-b rasm)* yoki biror tomonida *(2-d rasm)* yotadigan qilib tanlashimiz ham mumkin edi. Gomotetiya markazini qayerda olmaylik, berilgan $ABCD$ to‘rtburchakka o‘xshash va o‘xshashlik koeffitsiyenti 3 ga teng bo‘lgan to‘rtburchaklar o‘zaro teng bo‘ladi.

? *Savol, masala va topshiriqlar*

1. Berilgan ko‘pburchakka o‘xshash ko‘pburchakni yasash ketma-ketligini ayting.

2. Daftaringizga biror $ABCDE$ beshburchak chizing. Gomotetiya yordamida bu beshburchakka o‘xshash, o‘xshashlik koeffitsiyenti 0,5 ga teng bo‘lgan beshburchak yasang. Gomotetiya markazi a) C nuqtada; b) beshburchak ichida; d) AB tomonda bo‘lgan hollarni alohida ko‘ring.

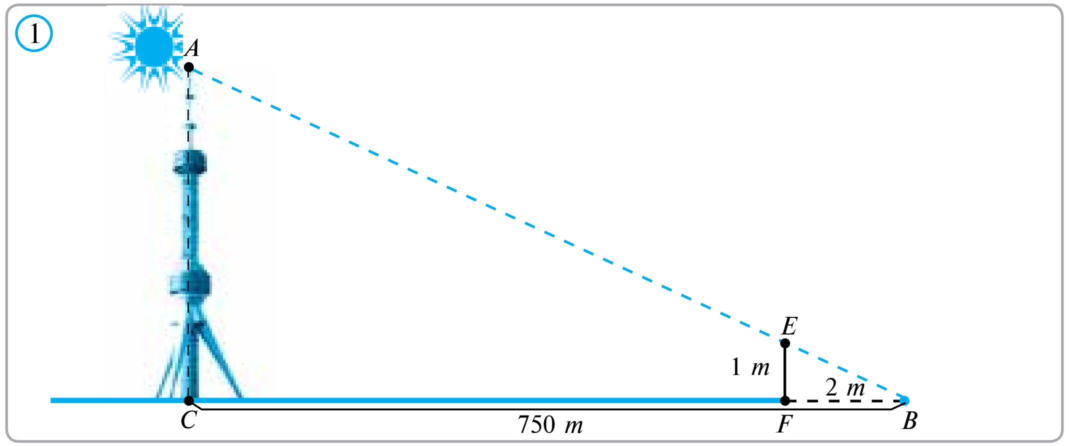
3. Kataklarni inobatga olgan holda, 3-rasmda berilgan shakllarni daftaringizga chizing: a) yaproqqa o‘xshashlik koeffitsiyenti 3 ga teng bo‘lgan yaproq; b) baliqchaga o‘xshashlik koeffitsiyenti 0,8 ga teng bo‘lgan baliqchani gomotetiya yordamida chizing.



4. F_1 ko‘pburchak F_2 ko‘pburchakka o‘xshash, k — o‘xshashlik koeffitsiyenti. P_1, P_2, S_1, S_2 harflar bilan mos ravishda bu ko‘pburchaklarning perimetrlari va yuzlari belgilangan. Quyidagi jadvalni daftaringizga ko‘chiring va uni to‘ldiring.

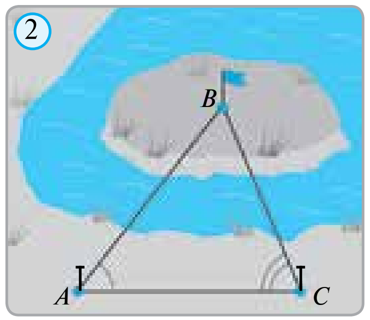
	P_1	P_2	S_1	S_2	k
a)	84		100	25	
b)	14	28		48	
d)		150	200	100	
e)		30	24		3

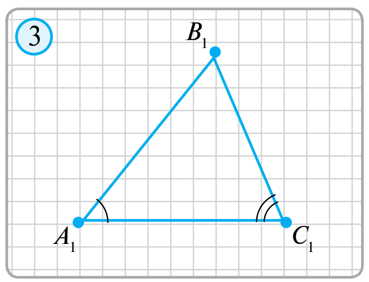
18 AMALIY MASHG‘ULOT



1. Balandlikni aniqlash.

Yerda turib, Toshkent teleminorasining balandligini topaylik. Minoraning uchi — A nuqtaning soyasi B nuqta bo‘lsin. EF tayoqni vertikal tarzda shunday qoqamizki (*1-rasm*), tayoqning E uchi soyasi ham B nuqtada bo‘lsin. Minoraning asosini C bilan belgilaymiz. Hosil bo‘lgan, to‘g‘ri burchakli ABC va EBF uchburchaklar o‘xshash bo‘ladi. Shuning uchun,





$$\frac{AC}{EF}=\frac{BC}{BF} \text{ yoki } AC=\frac{BC\cdot EF}{BF}.$$

BC, BF masofalarni va EF tayoq uzunligini o‘lchab, hosil bo‘lgan formuladan teleminora balandligi — AC kesma uzunligini topamiz. Masalan, agar $EF=1$ *m*, $BC=750$ *m*, $FB=2$ *m* ekani ma’lum bo‘lsa, u holda $AC=375$ *m* bo‘ladi.

2. Borib bo‘lmaydigan joygacha bo‘lgan masofani o‘lchash.

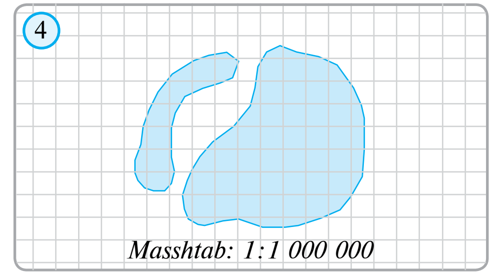
Aytaylik, A nuqtadan borish mumkin bo‘lmagan B nuqtagacha bo‘lgan masofani aniqlash lozim bo‘lsin *(2-rasm)*. A nuqtadan borib bo‘ladigan shunday C nuqtani belgilaymizki, undan qaraganda A va B nuqtalar ko‘rinib tursin hamda AC masofani o‘lchab bo‘lsin.

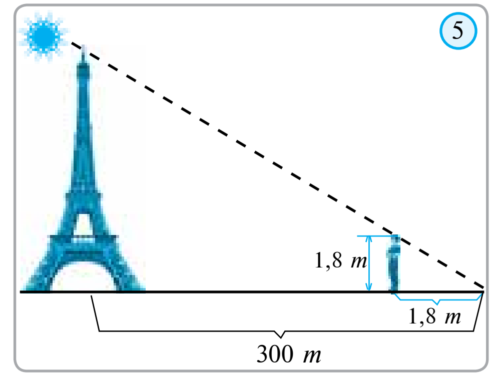
Asboblar yordamida BAC va ACB burchaklarni o‘lchaymiz. Aytaylik, $\angle BAC=\alpha$ va $\angle ACB=\beta$

bo‘lsin. Qog‘ozga $\angle A_1=\alpha$, $\angle C_1=\beta$ bo‘lgan $A_1B_1C_1$ uchburchak yasaymiz. Unda ABC va $A_1B_1C_1$ uchburchaklar ikki burchagi bo‘yicha o‘xshash bo‘ladi (*2- va 3-rasmlar*). Bundan,

$$\frac{AB}{A_1B_1}=\frac{AC}{A_1C_1} \text{ yoki } AB=\frac{AC\cdot A_1B_1}{A_1C_1}.$$

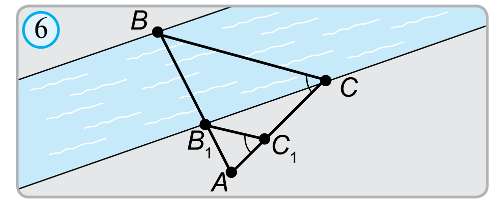
AC masofa va A_1B_1, A_1C_1 kesmalarni o‘lchab, natijada hosil bo‘lgan formula yordamida AB kesma hisoblanadi. Hisoblash ishlarini osonlashtirish maqsadida $AC:A_1C_1$ nisbatni 100:1, 1000:1 kabi nisbatda olish mumkin. Masalan, $AC=130$ *m*, $\angle A=73°$, $\angle C=58°$ bo‘lsa, qog‘ozda $A_1B_1C_1$ uchburchakni $\angle A_1=73°$, $\angle C_1=58°$, $A_1C_1=130$ *mm* qilib chizamiz. A_1B_1 kesmani o‘lchab, uning 153 *mm* ekanligini topamiz. Unda, izlangan masofa 153 *m* bo‘ladi.

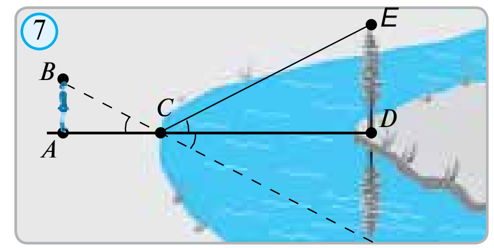




3. Orol dengizi haqida amaliy ish.

4-rasmda suv havzasining kosmik kemadan olingan oldingi surati tasvirlangan. U asosida tegishli o‘lchash va hisoblash ishlarini bajarib, suv havzasi yuzining taqribiy qiymatini toping.



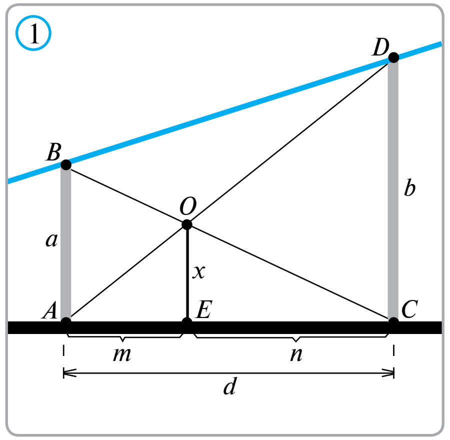


Savol, masala va topshiriqlar

1. Agar bo‘yi 1,7 *m* bo‘lgan odam soyasining uzunligi 2,5 *m* bo‘lsa, soyasining uzunligi 10,2 *m* bo‘lgan daraxt balandligi qancha bo‘ladi?
2. 5-rasmda tasvirlangan minora balandligini aniqlang.
3. 6-rasmdagi ikkita o‘xshash AB_1C_1 va ABC uchburchaklar yordamida daryoning kengligini (enini) aniqlash zarur. Agar $AC=100$ *m*, $AC_1=32$ *m* va $AB_1=34$ *m* bo‘lsa, daryoning eni (BB_1) ni toping.
4. Anhor qirg‘og‘idagi DE daraxtning suvdagi aksi A nuqtadagi odamga ko‘rinayapti. Agar $AB=165$ *sm*, $AC=120$ *sm*, $CD=4,8$ *m* bo‘lsa, daraxt balandligini toping *(7-rasm)*.
5. Hovlida biror daraxtni tanlang va uning balandligini aniqlang. Bu ishni qanday bajarganingiz haqida hisobot tayyorlang.

19 MASALALAR YECHISH

1-masala. Uzunliklari mos ravishda a va b bo‘lgan AB va CD ustunlar tik qilib o‘rnatilgan. Ularning mustahkamligini oshirish maqsadida A va D, B va C uchlari O nuqtada kesishuvchi po‘lat simlar bilan mahkamlangan *(1-rasm)*. Rasmda berilgan ma’lumotlar asosida a) $\frac{m}{m+n} = \frac{x}{b}$ va $\frac{n}{m+n} = \frac{x}{a}$ tengliklarni isbotlang; b) $\frac{x}{a} + \frac{x}{b} = 1$ tenglikning to‘g‘ri ekanligini ko‘rsating va uni sharhlang.



Yechilishi. a) Masala shartiga ko‘ra:

1. $AOE \backsim ADC$. Shuning uchun,

$$\frac{AE}{AC} = \frac{OE}{DC}, \text{ ya'ni } \frac{m}{m+n} = \frac{x}{b}. \quad (1)$$

2. $\Delta EOC \backsim \Delta ABC$. Shuning uchun,

$$\frac{EC}{AC} = \frac{OE}{AB}, \text{ ya'ni } \frac{n}{m+n} = \frac{x}{a}. \quad (2)$$

b) (1) va (2) tengliklarni hadma-had qo‘shsak,

$$\frac{m}{m+n} + \frac{n}{m+n} = \frac{x}{b} + \frac{x}{a} \text{ yoki } \frac{x}{a} + \frac{x}{b} = 1$$

tenglikni hosil qilamiz. Demak, ustunlar qanday o‘rnatilmasin, po‘lat simlar kesishgan O nuqta yerdan bir xil balandlikda bo‘lar ekan.

2-masala. $ABCD$ trapetsiyaning AB va CD yon tomonlarida M va N nuqtalar olingan. Bunda MN kesma trapetsiya asoslariga parallel va trapetsiya diagonallari kesishgan O nuqtadan o‘tadi. Agar $BC = a$, $AD = b$ bo‘lsa, a) MO; b) ON; d) MN kesmalarni toping (*2-rasm*).

Yechilishi. 1) AOD va BOC uchburchaklar BB alomatga ko‘ra o‘xshash, chunki $\angle BOC = \angle AOD$, $\angle OBC = \angle ADO$. Bundan,

$$\frac{OC}{OA} = \frac{BC}{AD} \text{ yoki } \frac{OC}{OA} = \frac{a}{b}. \quad (1)$$

2) ABC va AOM uchburchaklar ham BB alomatga ko‘ra o‘xshash, chunki $\angle AMO = \angle ABC$, $\angle ACB = \angle AOM$. Bundan,

$$\frac{AC}{OA} = \frac{BC}{MO} \text{ yoki } \frac{OA + OC}{OA} = \frac{a}{MO} \Rightarrow 1 + \frac{OC}{OA} = \frac{a}{MO}, \ \frac{OC}{OA} = \frac{a}{MO} - 1. \quad (2)$$

3) (1) va (2) tengliklarning o‘ng qismlarini tenglashtirib,

$$\frac{a}{MO} - 1 = \frac{a}{b}$$

tenglikni va undan

$$MO = \frac{ab}{a+b} \quad (3)$$

ekanligini topamiz.

4) Yuqoridagidek yo‘l tutib

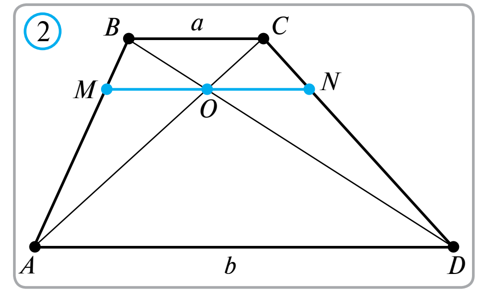
$$ON = \frac{ab}{a-b} \quad (4)$$

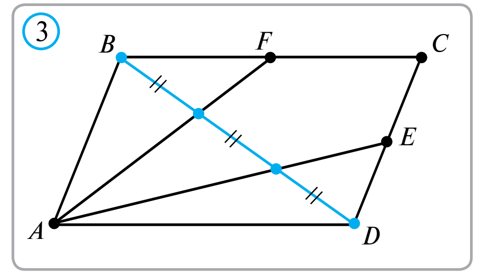
tenglikni, keyin esa (3) va (4) tengliklarning mos qismlarini qo‘shib

$$MN = \frac{2ab}{a+b}$$

tenglikni hosil qilamiz.

Javob: a) $\frac{ab}{a-b}$; b) $\frac{ab}{a-b}$; d) $\frac{2ab}{a+b}$.





Eslatma. *Bu masala yechimidan $MO = ON$ ekanligi kelib chiqadi.*

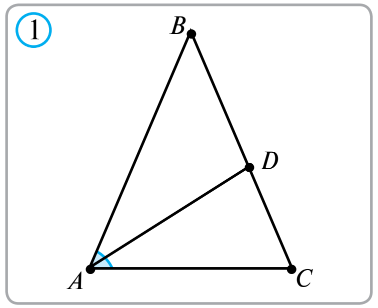
Savol, masala va topshiriqlar

1. ABC uchburchakning AB va BC yon tomonlarida D va E nuqtalar olingan. Agar $AC \| DE$, $AC = 6$, $DB = 3$ va $DE = 2$ bo‘lsa, AB tomonni toping.
2. Ikkita o‘xshash ko‘pburchakning yuzlari 8 dm^2 va 72 dm^2 ga teng, ulardan birining perimetri ikkinchisinikidan 26 dm ga kam. Katta ko‘pburchakning perimetrini toping.
3. Perimetri 1 m bo‘lgan $A_1B_1C_1$ uchburchak $A_2B_2C_2$ uchburchakning tomonlari o‘rtalarini, $A_2B_2C_2$ uchburchak $A_3B_3C_3$ uchburchak tomonlari o‘rtalarini, $A_3B_3C_3$ uchburchak esa $A_4B_4C_4$ uchburchak tomonlari o‘rtalarini tutashtirishdan hosil qilingan bo‘lsa, $A_4B_4C_4$ uchburchakning perimetri qancha bo‘ladi?
4. Ikkita o‘xshash uchburchakning perimetrlari 18 dm va 36 dm ga, yuzlarining yig‘indisi 30 dm^2 ga teng. Katta uchburchakning yuzini toping.
5. Romb tomonlarining o‘rtalari to‘g‘ri to‘rtburchak uchlari bo‘lishini isbotlang.
6. ABC uchburchak yasang. Bu uchburchakka o‘xshash va yuzi ABC uchburchak yuzidan 9 marta kichik bo‘lgan $A_1B_1C_1$ uchburchakni yasang.

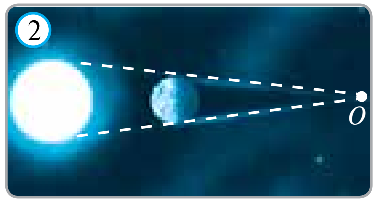
7*. E va F nuqtalar mos ravishda $ABCD$ parallelogramning CD va BC tomonlari o‘rtalari. AF va AE to‘g‘ri chiziqlar BD diagonalni teng uch qismga bo‘lishini isbotlang (*3-rasm*).

20 MASALALAR YECHISH

1. Teng yonli uchburchakning asosidagi burchak bissektrisasi bu uchburchakdan o‘ziga o‘xshash uchburchak ajratadi. Uchburchak burchaklarini aniqlang (*1-rasm*, $AB=BC$, $\triangle ABC \backsim \triangle CAD$).

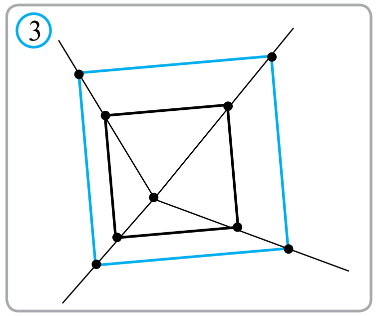


2. Aylana yasang va unda O nuqta belgilang. Markazi O nuqtada va koeffitsiyenti 2 ga teng bo‘lgan gomotetiyada berilgan aylanaga gomotetik bo‘lgan aylana yasang.
3. Ikkita o‘xshash ko‘pburchak perimetrlarining nisbati 2:3 kabi. Katta ko‘pburchakning yuzi 27 bo‘lsa, kichik ko‘pburchakning yuzini toping.
4. 2-rasmda Quyoshning to‘la tutilgan holati tasvirlangan. Agar Quyosh radiusi 686784 *km*, Oy radiusi 1760 *km* va Yerdan Oygacha bo‘lgan masofa 384400 *km* bo‘lsa, Yerdan Quyoshgacha bo‘lgan masofani toping.

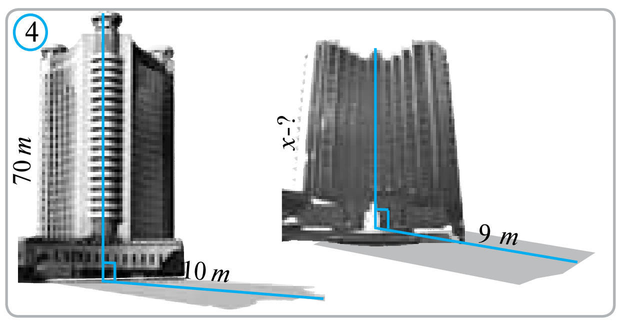


5. a) Bitta aylanaga ikkita o‘xshash ko‘pburchak ichki chizilgan. Bu ko‘pburchaklar teng bo‘ladimi?
   b) Bitta aylanaga ikkita o‘xshash ko‘pburchak tashqi chizilgan. Bu ko‘pburchaklar teng bo‘ladimi?

6*. Bir kvadratning tomonlari ikkinchi kvadrat tomonlariga parallel. Agar kvadratlar bir-biriga teng bo‘lmasa, ular gomotetik bo‘lishini isbotlang (*3-rasm*).



7. ABC uchburchakning AB va BC tomonlari to‘rtta teng kesmalarga bo‘lindi va bo‘linish nuqtalari AC tomonga parallel kesmalar bilan tutashtirildi. Agar $AC=24$ *sm* bo‘lsa, hosil bo‘lgan kesmalar uzunliklarini toping.
8. Agar rasmlar ayni bir paytda suratga olingan bo‘lsa, berilgan ma’lumotlar asosida ikkinchi binoning balandligini toping (*4-rasm*).



21 I BOBGA DOIR QO‘SHIMCHA MASALALAR VA MA’LUMOTLAR

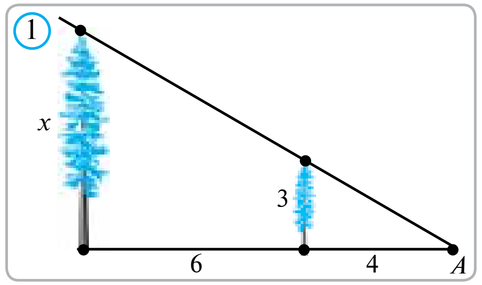
I. Testlar

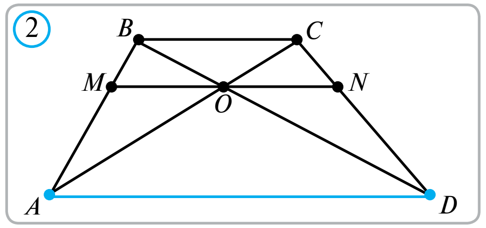
1. **Ikkita o‘xshash uchburchak uchun noto‘g‘ri tasdiqni toping:**
   A. Yuzlari nisbati o‘xshashlik koeffitsiyentiga teng;
   B. Mos medianalari nisbati o‘xshashlik koeffitsiyentiga teng;
   D. Mos bissektrisalari nisbati o‘xshashlik koeffitsiyentiga teng;
   E. Mos balandliklari nisbati o‘xshashlik koeffitsiyentiga teng.
2. **Ikkita gomotetik ko‘pburchak uchun to‘g‘ri tasdiqni toping:**
   A. Ular teng; B. Ular o‘xshash;
   D. Ular tengdosh; E. To‘g‘ri javob yo‘q.
3. **Uchburchak medianalari uchun noto‘g‘ri tasdiqni ko‘rsating:**
   A. Bir nuqtada kesishadi; B. Kesishish nuqtasida 2:1 nisbatda bo‘linadi;
   D. Bir-biriga teng; E. Har biri uchburchakni ikkita tengdosh qismga ajratadi.
4. **Uchburchak bissektrisalari uchun noto‘g‘ri tasdiqni ko‘rsating:**
   A. Bir nuqtada kesishadi; B. Kesishish nuqtasida 2:1 nisbatda bo‘linadi;
   D. O‘zi tushgan tomonni qolgan ikki tomonga proporsional kesmalarga ajratadi;
   E. O‘zi chiqqan uchdagi burchakni teng ikkiga bo‘ladi.
5. **Ikkita o‘xshash ko‘pburchak uchun noto‘g‘ri tasdiqni toping:**
   A. Ularning tomonlari soni teng; B. Ularning burchaklari soni teng;
   D. Mos tomonlari proporsional; E. Yuzlarining nisbati o‘xshashlik koeffitsiyentiga teng.

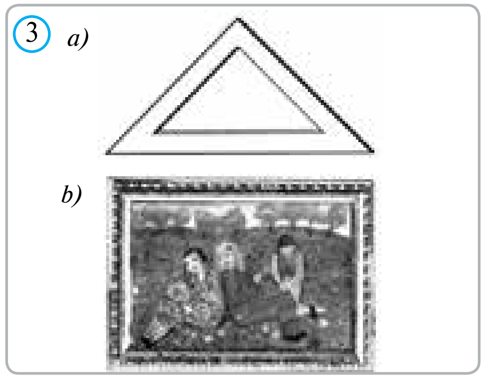
II. Masalalar

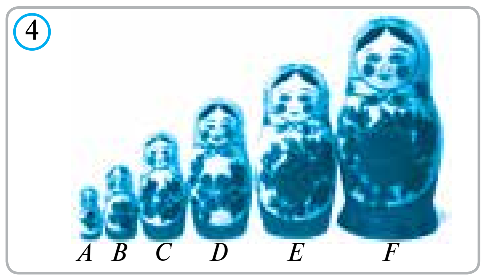
1. Asoslari 6 *m* va 12 *m* bo‘lgan trapetsiya diagonallari kesishgan nuqtadan asoslarga parallel to‘g‘ri chiziq o‘tkazilgan. To‘g‘ri chiziqning trapetsiya ichidagi qismi uzunligini toping.
2. ABC uchburchakda $BC=BA=10$, $AC=8$. Agar AA_1 va CC_1 uchburchak bissektrisalari bo‘lsa, A_1C_1 kesmani toping.
3. A nuqtadan borib bo‘lmaydigan B nuqtagacha bo‘lgan masofani aniqlash uchun tekis joyda C nuqta tanlandi. Keyin AC masofa, BAC va ACB burchaklar o‘lchandi va ABC uchburchakka o‘xshash $A_1B_1C_1$ uchburchak yasaldi. Agar $AC = 42$ *m*, $A_1C_1 = 6{,}3$ *sm*, $A_1B_1 = 7{,}2$ *sm* bo‘lsa, AB masofani toping.
4. Koeffitsiyenti $k=3$ bo‘lgan gomotetiyada F ko‘pburchak F_1 ko‘pburchakka almashadi. Agar F_1 ko‘pburchakning perimetri 12 *sm* va yuzi 4,5 sm^2 bo‘lsa, F ko‘pburchakning perimetri va yuzini toping.
5. Bo‘yi 180 *sm* bo‘lgan odam soyasining uzunligi 2,4 *m* bo‘lgan paytda balandligi 4 *m* bo‘lgan simyog‘och soyasining uzunligi necha metr bo‘ladi?

6. Xaritada Toshkent va Urganch shaharlari orasidagi masofa 8,67 *sm*. Agar xarita masshtabi 1:10 000 000 bo‘lsa, Toshkent va Urganch shaharlari orasidagi masofani toping.









III. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi)

1. 1-rasmda berilgan ma’lumotlar asosida daraxt balandligini toping.

2. *ABC* uchburchakning tomonlari $AB = 5$ *sm*, $AC = 6$ *sm*, $BC = 7$ *sm*. Bu uchburchakning *AC* tomoniga parallel to‘g‘ri chiziq *AB* tomonini *P* nuqtada, *BC* tomonini esa *K* nuqtada kesadi. Agar $PK = 2$ *sm* bo‘lsa, *PBK* uchburchak perimetrini toping.

3. 2-rasmda $AD||BC||MN$. Agar $BC = 6$ *sm*, $AD = 10$ *sm* bo‘lsa, *MN* kesmani toping.

4. *(Qo‘shimcha)*. Romb tomonlarining o‘rtalari to‘g‘ri to‘rtburchakning uchlari bo‘lishini isbotlang.

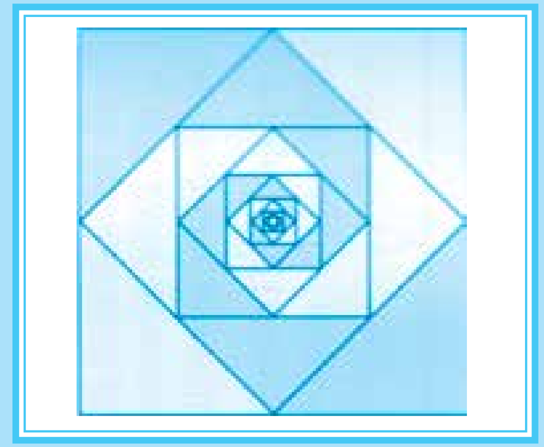
Qiziqarli masalalar

1. 4 marta kattalashtirib ko‘rsatilgan ko‘zgu-lupa bilan qaralganda 2° li burchak kattaligi qanchaga o‘zgaradi?

2. a) Uchburchakli chizg‘ich rasmida tasvirlangan ichki va tashqi uchburchaklar o‘xshashmi *(3-a rasm)*?
b) 3-b rasmdagi romning ichki va tashqi qirralarini tasvirlovchi to‘rtburchaklar o‘xshashmi?

3. Quyidagi rus tilida berilgan masalani yechib ko‘ring. Bu bilan ham rus tilidan, ham geometriyadan nimaga qodirligingizni bilib olasiz.

На 4-рисунке изображена русская игрушка “матрёшка”. Выполнив соответствующие измерения, найти коэффициент подобия игрушек:
a) *A* и *B*; b) *A* и *D*; d) *C* и *F*; e) *B* и *E*.



II BOB

UCHBURCHAK TOMONLARI VA BURCHAKLARI ORASIDAGI MUNOSABATLAR

Ushbu bobni o‘rganish natijasida siz quyidagi bilim, ko‘nikma va malakaga ega bo‘lasiz:

Bilimlar:

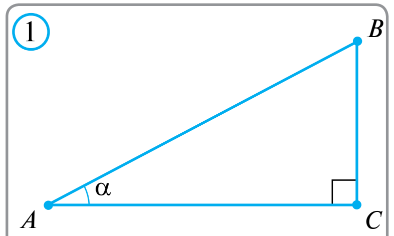
√ *ixtiyoriy burchakning sinusi,kosinusi,tangensi va kotangensi ta’riflarini bilish;*
√ *burchakning radian o‘lchovini bilish;*
√ *asosiy trigonometrik ayniyatlarni bilish;*
√ *uchburchakning yuzini burchak sinusi yordamida hisoblash formulasini bilish;*
√ *sinuslar va kosinuslar teoremasini bilish.*

Amaliy ko‘nikmalar:

√ *ba’zi burchaklarning sinusi,kosinusi,tangensi va kotangensini hisoblay olish;*
√ *asosiy trigonometrik ayniyatlarni misollar yechishda qo‘llay olish;*
√ *uchburchak yuzini uning ikki tomoni va ular orasidagi burchagi bo‘yicha hisoblay olish;*
√ *sinuslar,kosinuslar teoremasidan foydalanib hisoblashga va isbotlashga doir masalalarni yechish.*

22 O‘TKIR BURCHAKNING SINUSI, KOSINUSI, TANGENSI VA KOTANGENSI

1



$\sin\alpha = \frac{BC}{AB}$; $\cos\alpha = \frac{AC}{AB}$;

$\text{tg}\,\alpha = \frac{BC}{AC}$; $\text{ctg}\,\alpha = \frac{AC}{BC}$.

To‘g‘ri burchakli ABC uchburchakda $\angle C=90°$ bo‘lsa, AB tomon gipotenuza, BC tomon — A burchak qarshisidagi katet, AC tomon esa A burchakka yopishgan katet deyiladi *(1-rasm)*.

To‘g‘ri burchakli uchburchak o‘tkir burchagining **sinusi** deb, shu burchak qarshisidagi katetning gipotenuzaga nisbatiga aytiladi.

To‘g‘ri burchakli uchburchak o‘tkir burchagining **kosinusi** deb, shu burchakka yopishgan katetning gipotenuzaga nisbatiga aytiladi.

To‘g‘ri burchakli uchburchak o‘tkir burchagining **tangensi** deb, shu burchak qarshisidagi katetning yopishgan katetga nisbatiga aytiladi.

To‘g‘ri burchakli uchburchak o‘tkir burchagining **kotangensi** deb, shu burchakka yopishgan katetning qarshisidagi katetga nisbatiga aytiladi.

α burchakning sinusi, kosinusi, tangensi va kotangensi mos ravishda **sin**α**, cos**α**, tg**α va **ctg**α shaklida belgilanadi (o‘qilishi: ***«sinus alfa», «kosinus alfa», «tangens alfa», «kotangens alfa»***).

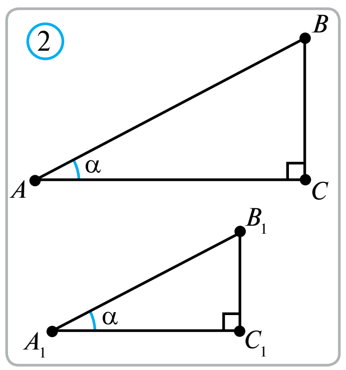
Yuqoridagi ta’riflardan quyidagi formulalar kelib chiqadi:

1. $\frac{\sin A}{\cos A} = \frac{BC}{AB} \cdot \frac{AB}{AC} = \frac{BC}{AC}$; $\text{tg}A = \frac{BC}{AC}$. $\Rightarrow$ $\text{tg}A = \frac{\sin A}{\cos A}$.

2. $\frac{\cos A}{\sin A} = \frac{AC}{AB} \cdot \frac{AB}{BC} = \frac{AC}{BC}$; $\text{ctg}A = \frac{AC}{BC}$. $\Rightarrow$ $\text{ctg}A = \frac{\cos A}{\sin A}$.

3. $\text{tg}\,A \cdot \text{ctg}\,A = \frac{BC}{AC} \cdot \frac{AC}{BC} = 1 \Rightarrow$ $\text{tg}\,A \cdot \text{ctg}\,A = 1$.

2



Teorema. **Bir to‘g‘ri burchakli uchburchakning o‘tkir burchagi ikkinchi to‘g‘ri burchakli uchburchakning o‘tkir burchagiga teng bo‘lsa, bu o‘tkir burchaklarning sinuslari (kosinusi, tangensi va kotangensi) ham teng bo‘ladi.**

Isbot. To‘g‘ri burchakli ABC va $A_1B_1C_1$ uchburchaklarda $(\angle C=\angle C_1=90°)$ $\angle A=\angle A_1$ bo‘lsin *(2-rasm)*. U holda, ABC va $A_1B_1C_1$ uchburchaklar BB alomatga ko‘ra o‘xshash bo‘ladi. Shuning uchun, $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \frac{AC}{A_1C_1}$. Bu tengliklardan $\frac{BC}{AB} = \frac{B_1C_1}{A_1C_1}$ yoki $\sin A=\sin A_1$ ekanligini topamiz.

Bu o‘tkir buchaklarning kosinusi, tangensi va kotangenslari ham teng bo‘lishi yuqoridagiga o‘xshash isbotlanadi. ***Teorema isbotlandi.***

Masala. ABC uchburchakda $\angle C=90°$, $AC=8$ *sm*, $BC=15$ *sm* bo‘lsa, uning B burchagi sinusi, kosinusi, tangensi va kotangensini toping.

Yechilishi. Pifagor teoremasidan foydalanib, uchburchakning gipotenuzasini topamiz:

$AB^2=AC^2+BC^2=8^2+15^2=289$, $AB=17$ (*sm*).

Uchburchakning B burchagi qarshisidagi katet AC, B burchagiga yopishgan katet esa BC (*1-rasm*). Unda, ta’riflarga ko‘ra,

$\sin B = \frac{AC}{AB} = \frac{8}{17}$; $\cos B = \frac{BC}{AB} = \frac{15}{17}$;

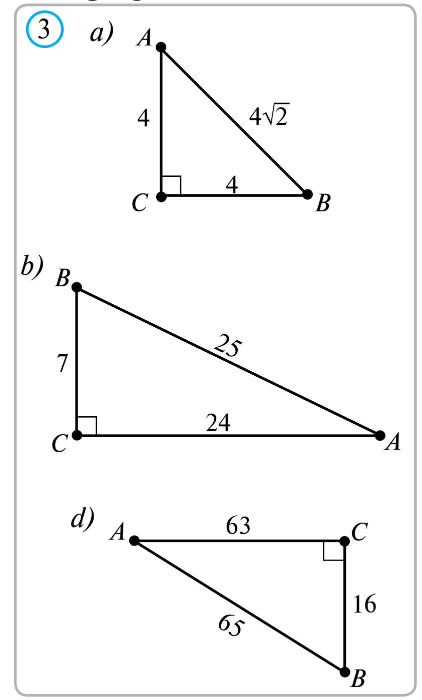
$\text{tg}B = \frac{AC}{BC} = \frac{8}{15}$; $\text{ctg}B = \frac{BC}{AC} = \frac{15}{8}$.

Yoki $\text{tg}B = \frac{\sin B}{\cos B} = \frac{8}{17} \cdot \frac{17}{15} = \frac{8}{15}$;

$\text{ctg}B = \frac{\cos B}{\sin B} = \frac{15}{17} \cdot \frac{17}{8} = \frac{15}{8}$.

Javob: $\frac{8}{17}$; $\frac{15}{17}$; $\frac{8}{15}$; $\frac{15}{8}$.

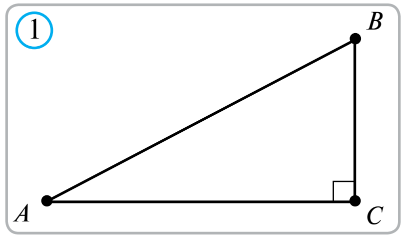
3



Savol, masala va topshiriqlar

1. To‘g‘ri burchakli uchburchakning o‘tkir burchagi sinusi, kosinusi, tangensi va kotangensi deb nimaga aytiladi?
2. O‘tkir burchakning sinusi, kosinusi, tangensi va kotangensi nimaga bog‘liq, nimaga bog‘liq emas?
3. 3-rasmdagi ma’lumotlar asosida $\sin A$, $\cos A$, $\sin B$, $\cos B$ ni toping.
4. To‘g‘ri burchakli ABC uchburchakning AB gipotenuzasi 13 *sm* ga, AC kateti esa 12 *sm* ga teng. Uchburchakning A burchagi sinusi, kosinusi, tangensi va kotangensini toping.
5. Agar to‘g‘ri burchakli ABC $(\angle C=90°)$ uchburchakda a) $AB=25$, $BC=7$; b) $AC=5$, $BC=12$; d) $AB=41$, $AC=40$; e) $AC=24$, $AB=25$ bo‘lsa, A va B burchaklarning sinusi, kosinusi, tangensi va kotangenslarini toping.
6. Agar ABC uchburchakda $\angle C=90°$, $\cos A=\frac{60}{61}$ va $AC=3$ *sm* bo‘lsa, uchburchakning qolgan tomonlarini toping.
7. Agar ABC uchburchakda $\angle C=90°$, $\sin A=\frac{8}{17}$ va $BC=16$ *sm* bo‘lsa, uchburchakning qolgan tomonlarini toping.

23 MASALALAR YECHISH



Masalalar yechishda juda asqotadigan yana bir muhim tenglikning to‘g‘riligini ko‘rsataylik: to‘g‘ri burchakli ABC uchburchakda (*1-rasm*) Pifagor teoremasiga ko‘ra: $AB^2=BC^2+AC^2$. U holda,

$$\sin^2 A+\cos^2 A=\frac{BC^2}{AB^2}+\frac{AC^2}{AB^2}=\frac{BC^2+AC^2}{AB^2}=\frac{AB^2}{AB^2}=1.$$

$$\sin^2 A+\cos^2 A=1$$

tenglik trigonometriyaning asosiy ayniyati deb ataladi (“trigonometriya” so‘zi yunoncha “uchburchaklarni o‘lchayman” degan ma’noni anglatadi).

1-masala. Agar $\cos\alpha=\frac{1}{2}$ bo‘lsa, $\sin\alpha$, $\text{tg}\,\alpha$ va $\text{ctg}\,\alpha$ ni toping.

Yechilishi. Asosiy trigonometrik ayniyatga ko‘ra:

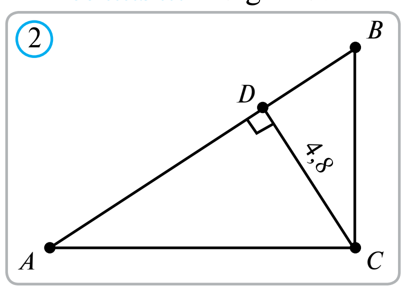
$$\sin^2\alpha=1-\cos^2\alpha \;\Rightarrow\; \sin\alpha=\sqrt{1-\cos^2\alpha}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}.$$

Unda,

$$\text{tg}\,\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\sqrt{3}}{2}\cdot\frac{2}{1}=\sqrt{3},\qquad \text{ctg}\,\alpha=\frac{1}{\text{tg}\,\alpha}=\frac{1}{\sqrt{3}}\;.$$

2-masala. ABC uchburchakda $\angle C=90°$ va $\sin A=0,6$. Agar uchburchakning CD balandligi 4,8 *sm* bo‘lsa, uning AC katetini va bu katetning gipotenuzadagi proyeksiyasini toping.

Yechilishi. To‘g‘ri burchakli ADC uchburchakni qaraymiz *(2-rasm)*. Unda,



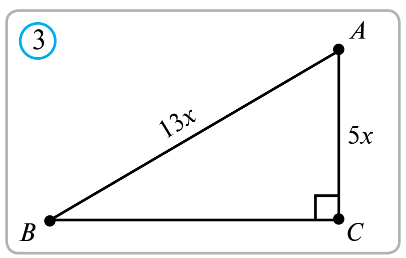
sinusning ta’rifiga ko‘ra,

$\sin A=\frac{DC}{AC}$. Bundan, $AC=\frac{DC}{\sin A}=\frac{4,8}{0,6}=8$ (*sm*).

Pifagor teoremasidan foydalanib AC katetning gipotenuzadagi proyeksiyasi AD ni topamiz:

$$AD=\sqrt{AC^2-CD^2}=\sqrt{8^2-4,8^2}=6,4\ (sm).$$

Javob: 8 *sm*; 6,4 *sm*.



3-masala. Agar ABC uchburchakda $\angle C=90°$ va $\cos A=\frac{5}{13}$ bo‘lsa, uchburchak tomonlari qanday nisbatda bo‘ladi (*3-rasm*).

Yechilishi. Burchak kosinusining ta’rifiga ko‘ra,

$$\cos A=\frac{AC}{AB}.\qquad \text{Demak,}\quad \frac{AC}{AB}=\frac{5}{13}.$$

Agar $AC=5x$ desak, unda

$$AB=\frac{13\cdot AC}{5}=13x.$$

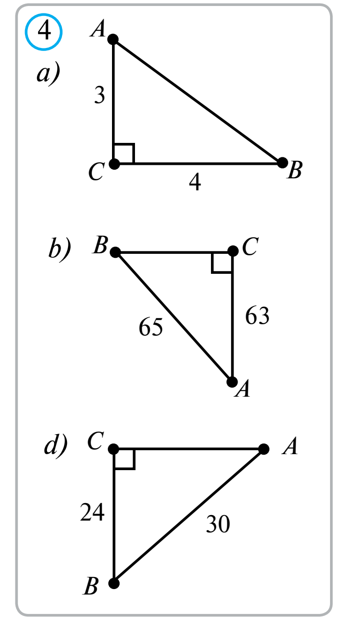
Pifagor teoremasiga ko‘ra,

$$BC=\sqrt{AC^2-BC^2}=\sqrt{169x^2-25x^2}=12x.$$

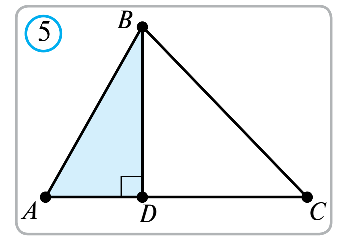
Shunday qilib, $AC:BC:AB=5:12:13$.

Javob: 5 : 12 : 13 kabi.

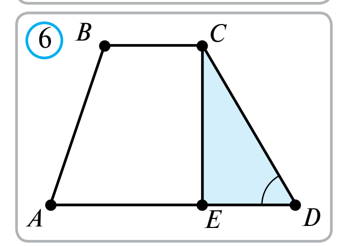
Savol, masala va topshiriqlar



1. 4-rasmdagi ma’lumotlar asosida quyidagilarni: a) $\sin A$, $\cos A$, $\text{tg}\,A$, $\text{ctg}\,A$; b) $\sin B$, $\cos B$, $\text{tg}\,B$, $\text{ctg}\,B$ ni toping.
2. Agar $\sin\alpha=0,5$ bo‘lsa, $\cos\alpha$, $\text{tg}\,\alpha$ va $\text{ctg}\,\alpha$ ni toping.
3. Agar $\cos\alpha=0,6$ bo‘lsa, $\sin\alpha$, $\text{tg}\,\alpha$ va $\text{ctg}\,\alpha$ ni toping.
4. To‘g‘ri burchakli ABC ($\angle C=90°$) uchburchakda $BC=17$ *sm* va $\sin B=\frac{15}{17}$ bo‘lsa: a) uchburchakning CD balandligini; b) BC katetning gipotenuzadagi proyeksiyasini; d) gipotenuzani; e) ikkinchi katetni toping.
5. Agar ABC uchburchakda $\angle C=90°$, $\sin A=\frac{1}{3}$ va $BC=15$ *sm* bo‘lsa, uchburchakning gipotenuzasiga tushirilgan balandligini toping.



6*. Agar a) $\sin\alpha=\frac{2}{3}$; b) $\cos\alpha=\alpha$; d) $\text{tg}\,\alpha=\frac{3}{4}$; e) $\text{ctg}\,\alpha=\frac{4}{5}$ bo‘lsa, α burchakni yasang.



7. ABC uchburchakda $AC=12$ *sm*, $AB=10$ *sm*, $\sin A=0,7$ bo‘lsa, uchburchak yuzini toping *(5-rasm)*.
8. ABC uchburchakda BD — balandlik, $AC=7$ *sm*, $AD=2$ *sm* va $\text{tg}\,A=3$ bo‘lsa, uchburchak yuzini toping *(5-rasm)*.
9. $ABCD$ ($BC\|AD$) trapetsiyada $\sin D=0,5$; $CD=8$, $BC=6$, $AD=10$ bo‘lsa, trapetsiya yuzini toping *(6-rasm)*.
10. $ABCD$ rombda $\sin A=0,8$ va $AB=15$ *sm* bo‘lsa, romb yuzini toping.

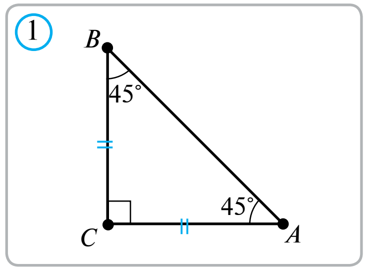
11*. Teng yonli uchburchakning asosiga tushirilgan balandligi 5 *sm*, asosi esa $10\sqrt{3}$ *sm* bo‘lsa, uchburchakning a) burchaklarini; b) yon tomonini; d) yuzini toping.

12. To‘g‘ri burchakli ABC uchburchakda $\sin A=\frac{3}{7}$ va $\sin B=\frac{4}{7}$ bo‘lishi mumkinmi?

24 BA'ZI BURCHAKLARNING SINUSI, KOSINUSI, TANGENSI VA KOTANGENSINI HISOBLASH

1. 45 gradusli burchakning sinusi, kosinusi, tangensi va kotangensini hisoblash.

Teng yonli to'g'ri burchakli ABC uchburchakni qaraymiz *(1-rasm)*. Bu uchburchakda $AC=BC$, $\angle A=\angle B=45°$ bo'lsin. Unda Pifagor teoremasiga ko'ra, $AB^2=AC^2+BC^2=2AC^2$ yoki $AB=AC\sqrt{2}$.



Bundan $AC = BC = \frac{AB}{\sqrt{2}} = \frac{AB\sqrt{2}}{2}$ ni hosil qilamiz.

Shunday qilib,

$$\sin 45° = \sin A = \frac{BC}{AB} = \frac{\sqrt{2}}{2}; \quad \cos 45° = \cos A = \frac{AC}{AB} = \frac{\sqrt{2}}{2};$$

$$\text{tg}45° = \text{tg}A = \frac{BC}{AC} = 1; \quad \text{ctg}45° = \text{ctg}A = \frac{AC}{BC} = 1.$$

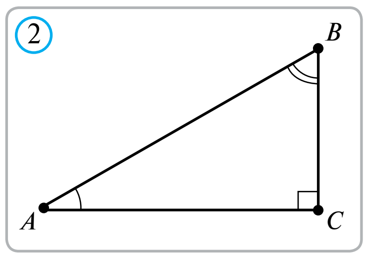
1-masala. To'g'ri burchakli ABC ($\angle C=90°$) uchburchakda $\angle A=45°$ va $BC=6$ *sm*. Uchburchakning qolgan tomonlarini toping (*1-rasm*).

Yechilishi. $\frac{AC}{BC} = \text{ctg}45°$ yoki $\frac{AC}{BC} = 1$. $AC=BC=6$ (*sm*);

$\frac{BC}{AB} = \sin 45°$ yoki $\frac{BC}{AB} = \frac{\sqrt{2}}{2}$. $AB=BC\sqrt{2}=6\sqrt{2}$ (*sm*).

Javob: 6 *sm*; $6\sqrt{2}$ *sm*.

2. 30° va 60° burchaklarning sinusi, kosinusi, tangensi va kotangensini hisoblash.



Burchaklari $\angle A=30°$, $\angle B=60°$ va $\angle C=90°$ bo'lgan ABC uchburchakni qaraymiz *(2-rasm)*. 30 gradusli burchak qarshisida yotgan katet gipotenuzaning yarmiga teng bo'lgani uchun $BC = \frac{1}{2}AB$ yoki $\frac{BC}{AB} = \frac{1}{2}$. Bundan

$$\sin 30° = \sin A = \frac{BC}{AB} = \frac{1}{2}; \quad \cos 60° = \cos B = \frac{BC}{AB} = \frac{1}{2}$$

tengliklarni topamiz. Asosiy trigonometrik ayniyatga ko'ra:

$$\cos 30° = \sqrt{1-\sin^2 30°} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}, \quad \sin 60° = \sqrt{1-\cos^2 60°} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$$

Topilganlarga ko'ra, $\text{tg}30° = \frac{\sin 30°}{\cos 30°} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$, $\text{tg}60° = \frac{\sin 60°}{\cos 60°} = \sqrt{3}$;

$\text{ctg}30° = \frac{\cos 30°}{\sin 30°} = \sqrt{3}$; $\quad \text{ctg}60° = \frac{\cos 60°}{\sin 60°} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

3. α ning 30°, 45°, 60° ga teng qiymatlarida topilgan $\sin\alpha$, $\cos\alpha$, $\text{tg}\alpha$ va $\text{ctg}\alpha$ uchun qiymatlarni jadval ko'rinishida jamlaymiz:

α	30°	45°	60°
$\sin\alpha$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\alpha$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\text{tg}\alpha$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\text{ctg}\alpha$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

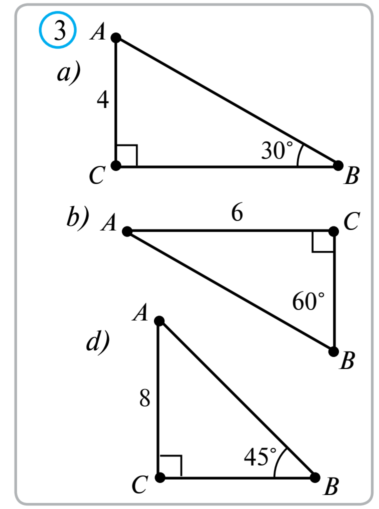
2-masala. To'g'ri burchakli uchburchakning gipotenuzasi 10 *sm* va burchaklaridan biri 60°. Uning qolgan tomonlarini toping.

Yechilishi. 2-rasmdan foydalanamiz. Unda

$$BC = AB\sin A = 10\cdot\sin 30° = 10\cdot\frac{1}{2} = 5\ (sm),$$

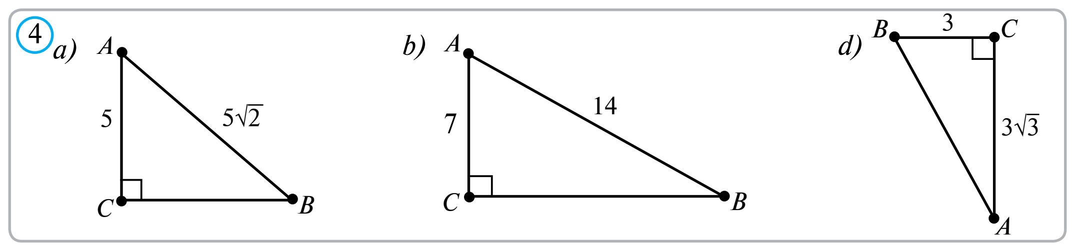
$$AC = AB\cos A = 10\cdot\cos 30° = 10\cdot\frac{\sqrt{3}}{2} = 5\sqrt{3}\ (sm).$$

Javob: 5 *sm*; $5\sqrt{3}$ *sm*.



Savol, masala va topshiriqlar

1. α burchak 30°,45°, 60° ga teng bo'lsa, $\sin\alpha$, $\cos\alpha$, $\text{tg}\alpha$ va $\text{ctg}\alpha$ qiymati nimaga teng? Javobingizni asoslang.
2. 3-rasmdagi uchburchaklar perimetrlarini toping.
3. 4-rasmdagi uchburchaklar burchaklarini toping.
4. α ning 30°, 45°, 60° ga teng qiymatlarida $\sin\alpha$, $\cos\alpha$, $\text{tg}\alpha$ va $\text{ctg}\alpha$ uchun qiymatlar jadvalini yod oling.
5. To'g'ri burchakli uchburchakning bir o'tkir burchagi 30°, unga yopishgan katet 6 *dm*. Uning qolgan tomonlarini toping.
6. Teng yonli uchburchakning asosi 10 *sm* ga, bir burchagi esa 120° ga teng. Uning yuzini toping.
7. ABC uchburchakda $\angle C=90°$, $AB=25$ *sm*, $\sin A=\frac{7}{25}$ .Uchburchakning qolgan tomonlarini va $\cos A$, $\text{tg}A$ hamda $\text{ctg}A$ ni toping.
8. Diagonallari $5\sqrt{3}$ *sm* va 5 *sm* bo'lgan rombning burchaklarini toping.



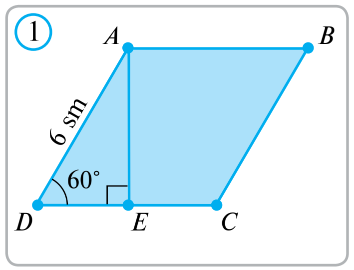
25 MASALALAR YECHISH

Faollashtiruvchi mashq

Jadvalning bo‘sh kataklarini to‘ldiring.

α	sinα	cosα	tgα	ctgα
	$\frac{\sqrt{3}}{2}$			
			$\frac{\sqrt{3}}{3}$	
		$\frac{\sqrt{2}}{2}$		

1-masala. Agar $ABCD$ rombda $\angle A=120^\circ$ va $AB=6$ *sm* bo‘lsa, rombning balandligi va yuzini toping (*1-rasm*).



Yechilishi. 1) Rombning bir tomoniga yopishgan burchaklari yig‘indisi 180° ga teng bo‘lgani uchun $\angle D=180^\circ-\angle A=60^\circ$. Rombning AE balandligini o‘tkazib *(1-rasm)*, to‘g‘ri burchakli AED uchburchak hosil qilamiz. Unda,

$$\frac{AE}{AD}=\sin D=\sin 60^\circ=\frac{\sqrt{3}}{2} \text{ yoki } AE=\frac{\sqrt{3}}{2}AD=3\sqrt{3}\ (sm).$$

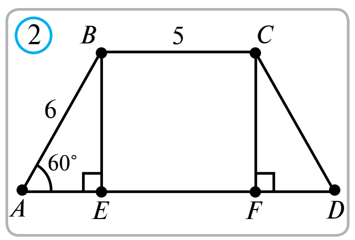
2) Endi rombning yuzini topamiz:

$$S_{ABCD}=DC\cdot AE=6\cdot 3\sqrt{3}=18\sqrt{3}\ (sm^2).$$

Javob: $h=3\sqrt{3}$ *sm*; $S_{ABCD}=18\sqrt{3}$ sm^2.

2-masala. $ABCD$ teng yonli trapetsiyaning BC kichik asosi 5 *sm*. Agar $\angle A=60^\circ$, $AB=6$ *sm* bo‘lsa, trapetsiyaning yuzini toping.

Yechilishi. Trapetsiyaning BE va CF balandliklarini o‘tkazamiz *(2-rasm)*. Unda, to‘g‘ri burchakli ABE uchburchakdan



$$AE=AB\cos 60^\circ=6\cdot\frac{1}{2}=3\ (sm),$$

$$BE=AB\sin 60^\circ=6\cdot\frac{\sqrt{3}}{2}=3\sqrt{3}\ (sm).$$

Bundan tashqari $AE=FD$, $EF=BC$ bo‘lgani uchun,

$$AD=AE+EF+FD=3+5+3=11\ (sm).$$

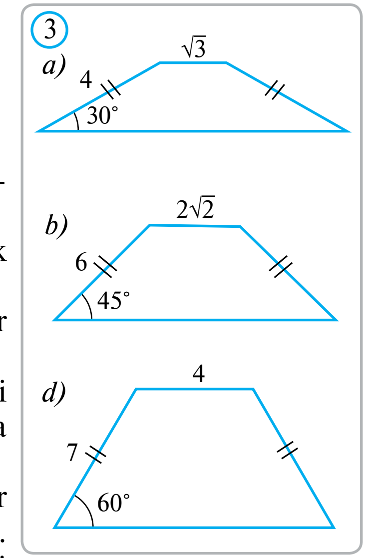
Trapetsiyaning yuzini topish formulasiga ko‘ra,

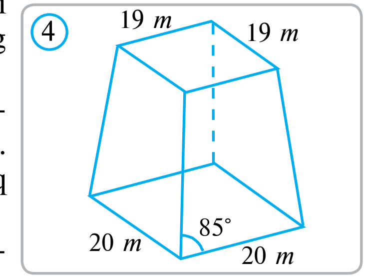
$$S_{ABCD}=\frac{BC+AD}{2}\cdot BE=\frac{5+11}{2}\cdot 3\sqrt{3}=24\sqrt{3}\ (sm^2).$$

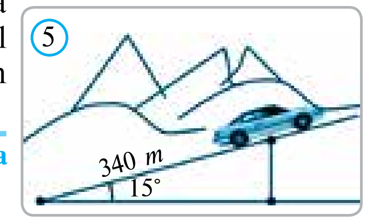
Javob: $24\sqrt{3}$ sm^2.

Savol, masala va topshiriqlar

1. Teng yonli to‘g‘ri burchakli uchburchakning gipotenuzasi 12 *sm*. Uning yuzini hisoblang.
2. Balandligi $4\sqrt{3}$ *sm* bo‘lgan teng tomonli uchburchak perimetrini toping.
3. 3-rasmda berilganlarga ko‘ra teng yonli trapetsiyalar yuzini toping.
4. To‘g‘ri burchakli trapetsiyaning o‘tkir burchagi 30° ga, balandligi 4 *sm* ga va kichik asosi 6 *sm* ga teng. Trapetsiyaning perimetri va yuzini toping.
5. Aylana vatari 120 gradusli yoyni tortib turadi. Agar aylana radiusi 10 *sm* bo‘lsa, vatar uzunligini toping.
6*. Teng yonli uchburchakning uchidagi burchagi a) 120°; b) 90°; d) 60°. Uchburchak balandligining asosiga nisbatini hisoblang.
7*. 4-rasmda tasvirlangan paxta xirmonining yon yoqlari teng yonli trapetsiya, usti esa kvadrat shaklida. Rasmda berilganlardan foydalanib, xirmonni to‘liq yopish uchun qancha mato zarurligini aniqlang.
8. Yengil mashina dovonning yuqoriga ko‘tarilish qismida 340 *m* yo‘l bosdi. Agar yo‘lning gorizontga nisbatan ko‘tarilish burchagi 15° bo‘lsa, yengil mashina necha metr balandlikka ko‘tarilgan (*5-rasm*)?







Maxsus kalkulatorda burchakning sinusi, kosinusi va tangensini hisoblash

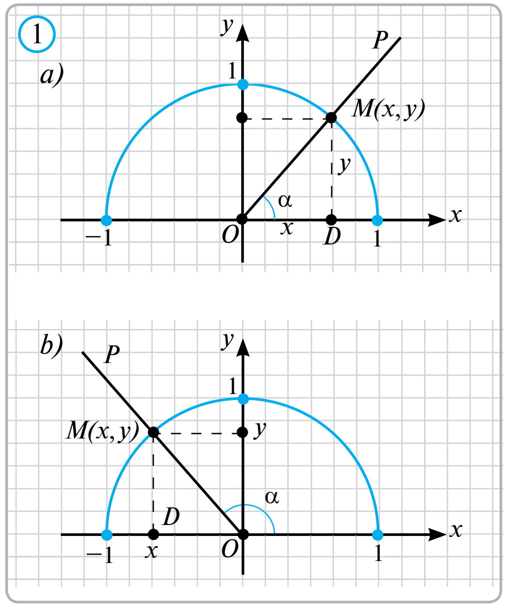
sin va cos tugmachalari bor maxsus kalkulatorda trigonometrik funksiyalarning qiymatlari quyidagicha hisoblanadi:

Burchak graduslarda berilgan bo‘lsin, masalan, sin30°:

1. Kalkulator yoqilib, DEG (gradus) tugmachasi bosiladi.
2. So‘ng tugmachalar C 3 0 Sin tartibda bosiladi va tegishli javob: 0,5 olinadi. sin30° = 0,5.

Agar maxsus kalkulator bo‘lmasa, darslik oxiridagi ilovada keltirilgan trigonometrik funksiyalarning qiymatlari jadvalidan foydalanishingiz mumkin.

26 0° DAN 180° GACHA BO‘LGAN BURCHAKNING SINUSI, KOSINUSI, TANGENSI VA KOTANGENSI



To‘g‘ri burchakli *Oxy* koordinatalar sistemasining I va II choraklarida joylashgan, radiusi birlik kesmaga teng, markazi koordinatalar boshida bo‘lgan yarim aylana yasaymiz (*1-rasm*). Aylanani *M* ($x;y$) nuqtada kesuvchi *OP* nurni o‘tkazamiz. Bu nurning *Ox* nur bilan hosil qilgan burchagini α bilan belgilaymiz. *OP* nurning *Ox* nur bilan ustma-ust tushgan holdagi burchakni 0° li burchak sifatida qabul qilamiz.

Ma’lumki, α o‘tkir burchak bo‘lganda (*1-a rasm*), bu burchakning sinusi, kosinusi, tangensi va kotangensi to‘g‘ri burchakli *ODM* uchburchakdan

$$\sin\alpha=\frac{DM}{MO};\quad \cos\alpha=\frac{OD}{MO},$$
$$\mathrm{tg}\alpha=\frac{DM}{OD},\quad \mathrm{ctg}\alpha=\frac{OD}{DM}$$

tengliklar yordamida aniqlanadi. Agar $MO=1$, $DM=y$, $OD=x$ ekanligini hisobga olsak,

$$\sin\alpha=y,\quad \cos\alpha=x,\quad \mathrm{tg}\alpha=\frac{y}{x},\quad \mathrm{ctg}\alpha=\frac{x}{y} \qquad (1)$$

tengliklarga ega bo‘lamiz.

Umumiy holda, 0° dan 180° gacha bo‘lgan burchakning sinusi, kosinusi, tangensi va kotangensini ham (1) formula orqali aniqlaymiz:

Istalgan α ($0°\le\alpha\le180°$) burchakning **sinusi** deb *M* nuqtaning ordinatasi — *y* ga aytiladi. Istalgan α ($0°\le\alpha\le180°$) burchakning **kosinusi** deb *M* nuqtaning absissasi — *x* ga aytiladi. Istalgan α ($0°\le\alpha\le180°$, $\alpha\ne90°$) burchakning **tangensi** deb *M* nuqta ordinatasining absissasiga nisbatiga aytiladi. Istalgan α ($0°<\alpha<180°$) burchakning **kotangensi** deb *M* nuqta absissasining ordinatasiga nisbatiga aytiladi.

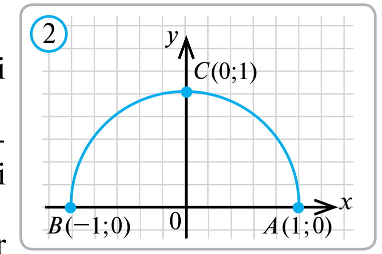
OMD uchburchakda $OD^2+DM^2=MO^2$ yoki $x^2+y^2=1$. $\sin\alpha=y$ va $\cos\alpha=x$ ekanligini hisobga olsak, istalgan α ($0°\le\alpha\le180°$) burchak uchun

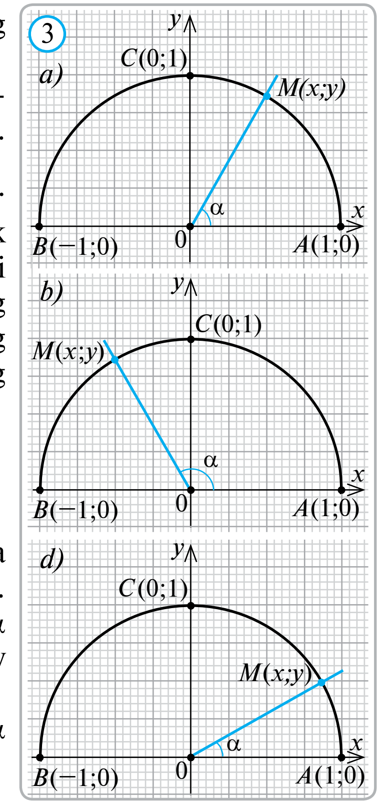
$$\sin^2\alpha+\cos^2\alpha=1 \qquad (2)$$

tenglikni hosil qilamiz. Bu tenglik **asosiy trigonometrik ayniyat** deb atalib, u oldingi darslarda o‘tkir burchaklar uchun isbotlangan edi.

Amaliy topshiriq

1. Birlik kesmani 5 *sm* ga teng deb olib, to‘g‘ri burchakli koordinatalar sistemasini chizing.
2. Koordinatalar sistemasining I va II choragida joylashgan, markazi koordinatalar boshida va radiusi birlik kesmaga teng yarim aylana chizing.
3. Yarim aylanani *M* nuqtada kesadigan va *Ox* nur bilan a) $\alpha=67°$; b) $\alpha=118°$; d) $\alpha=150°$ ga teng burchak tashkil qiladigan *OM* nur yasang.
4. O‘lchashlar yordamida *M* nuqtaning koordinatalarini hamda sinα, cosα, tgα va ctgα qiymatlarini toping.





Masala. 0°, 90° va 180° li burchaklarning sinusini toping.

Yechilishi. 0° li burchak *OA*, 90° li burchak *OC*, 180° li burchak *OB* nur yordamida aniqlanadi (*2-rasm*). Ta’rifga ko‘ra, sin0° — *A* (1;0) nuqtaning ordinatasi sifatida 0 ga, sin90° — *C* (0;1) nuqtaning ordinatasi sifatida 1 ga, sin180° esa *B* (−1;0) nuqtaning ordinatasi sifatida 0 ga teng bo‘ladi.

Javob: $\sin0°=0$, $\sin90°=1$, $\sin180°=0$.

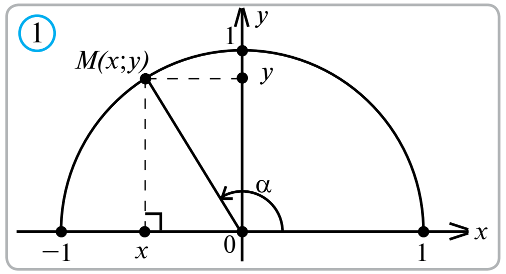
Savol, masala va topshiriqlar

1. 0° dan 180° gacha bo‘lgan burchakning sinusi va kosinusi deganda nima tushunilishini aytib bering.
2. α burchakning tangensi va kotangensi nima? α burchakning tangensi va kotangensi α ning qanday qiymatlarida aniqlanmagan?
3. Agar $90°<\alpha<180°$ bo‘lsa, sinα, cosα, tgα va ctgα qiymatlarining ishorasini aniqlang.
4. Agar $0°\le\alpha\le180°$ bo‘lsa, $0\le\sin\alpha\le1$ va $-1\le\cos\alpha\le1$ tengsizliklar o‘rinli bo‘lishini tushuntiring.
5. 3-rasmdagi α burchakni o‘lchang va uning sinusi, kosinusi, tangensi va kotangensini tegishli o‘lchashlar yordamida aniqlang.

6*. 1-a rasmda tasvirlangan yarim aylanani chizing. *Ox* nur bilan 45° va 135° li burchak hosil qiluvchi nurlarni yasang. Chizilgan rasmdan foydalanib, sin45° ni sin135° bilan va cos45° ni cos135° bilan o‘zaro solishtiring.

7*. Balandligi 3 *sm* va o‘tkir burchagi 30° bo‘lgan rombning perimetri va yuzini hisoblang.

27 ASOSIY TRIGONOMETRIK AYNIYATLAR



Faollashtiruvchi mashq

1-rasmdan foydalanib nuqtalar o‘rnini to‘ldiring:

$\sin\alpha = \ldots$; $\cos\alpha = \ldots$;

$\mathrm{tg}\,\alpha = \frac{\ldots}{\ldots}$; $\mathrm{ctg}\,\alpha = \frac{\ldots}{\ldots}$.

Ta’riflarga ko‘ra, har bir o‘tkir burchakka bu burchak sinusining (kosinusi, tangensi va kotangensining) bitta qiymati mos qo‘yilayapti. Bu mosliklar o‘tkir burchakning trigonometrik funksiyalari: sinus, kosinus, tangens va kotangens funksiyalarini aniqlaydi. Bu funksiyalar ko‘pincha uchburchaklarni yechishda qo‘llanishi sababli, ular trigonometrik funksiyalar deb ataladi.

“Trigonometriya” so‘zi — yunoncha “uchburchaklarni yechish” degan ma’noni anglatadi.

Endi α $(0° \le \alpha \le 180°)$ burchakning sinusi, kosinusi, tangensi va kotangensi orasidagi munosabatlarni aniqlaylik.

1. **Trigonometriyaning asosiy ayniyati** deb ataluvchi, α ning $0° \le \alpha \le 180°$ qiymatlari uchun o‘rinli bo‘lgan ushbu

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad (1)$$

formula bilan oldingi darslarda tanishgan edik.

2. Ta’rifga ko‘ra, $\mathrm{tg}\,\alpha = \frac{y}{x}$, $\mathrm{ctg}\,\alpha = \frac{x}{y}$, $x = \cos\alpha$, $y = \sin\alpha$ bo‘lgani uchun,

$$\mathrm{tg}\,\alpha = \frac{\sin\alpha}{\cos\alpha}\ (\alpha \ne 90°), \quad \mathrm{ctg}\,\alpha = \frac{\cos\alpha}{\sin\alpha}\ (\alpha \ne 0,\ \alpha \ne 180°),$$

$$\mathrm{tg}\,\alpha \cdot \mathrm{ctg}\,\alpha = 1\ (\alpha \ne 0,\ \alpha \ne 90°,\ \alpha \ne 180°) \qquad (2)$$

ayniyatlar o‘rinlidir.

3. (1) tenglikning har ikki qismini oldin $\cos^2\alpha$ ga, keyin esa $\sin^2\alpha$ ga bo‘lib,

$$1 + \mathrm{tg}^2\alpha = \frac{1}{\cos^2\alpha}\ (\alpha \ne 90°), \qquad 1 + \mathrm{ctg}^2\alpha = \frac{1}{\sin^2\alpha},\ (\alpha \ne 0,\ \alpha \ne 180°) \qquad (3)$$

ayniyatlarni hosil qilamiz.

Masala. Agar $\sin\alpha = 0{,}6$ va $90° \ne \alpha \ne 180°$ bo‘lsa, $\cos\alpha$, $\mathrm{tg}\,\alpha$ va $\mathrm{ctg}\,\alpha$ qiymatini toping.

Yechilishi. Asosiy trigonometrik ayniyatdan foydalanib $\cos\alpha$ ni hisoblaymiz:

$$\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-0{,}6^2} = -\sqrt{1-0{,}36} = -\sqrt{0{,}64} = -0{,}8.$$

$90° \le \alpha \le 180°$, ya’ni α II chorakda bo‘lganda, $\cos\alpha \le 0$. Shu bois ildiz “–” ishora bilan olindi.

(2) formulalarga asosan,

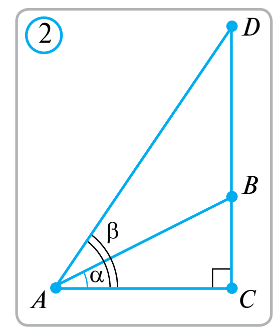
$$\mathrm{tg}\,\alpha = \frac{\sin\alpha}{\cos\alpha} = -\frac{0{,}6}{0{,}8} = -\frac{3}{4}; \qquad \mathrm{ctg}\,\alpha = \frac{1}{\mathrm{tg}\,\alpha} = -\frac{4}{3}.$$

Javob: $\cos\alpha = -0{,}8$; $\mathrm{tg}\,\alpha = -\frac{3}{4}$; $\mathrm{ctg}\,\alpha = -\frac{4}{3}$.

Savol, masala va topshiriqlar

1. $\mathrm{tg}\,\alpha = \frac{\sin\alpha}{\cos\alpha}$, $\mathrm{ctg}\,\alpha = \frac{\cos\alpha}{\sin\alpha}$, $\mathrm{tg}\,\alpha \cdot \mathrm{ctg}\,\alpha = 1$ ayniyatlar α ning qanday qiymatlari uchun o‘rinli?
2. Ifodalarni soddalashtiring:
   1) $1 - \cos^2\alpha$;
   2) $(1 - \sin\alpha)(1 + \sin\alpha)$;
   3) $\sin^4\alpha + 2\sin^2\alpha \cdot \cos^2\alpha + \cos^4\alpha$;
   4) $1 - \sin^4\alpha - \sin^2\alpha \cdot \cos^2\alpha$;
   5) $\mathrm{ctg}^2\alpha(2\sin^2\alpha + \cos^2\alpha - 1)$;
   6) $\mathrm{tg}^2\alpha - \sin^2\alpha \cdot \mathrm{tg}^2\alpha$.
3. Agar a) $\sin\alpha = \frac{4}{5}$ va $90° < \alpha < 180°$ bo‘lsa, $\cos\alpha$ nimaga tengligini toping; b) $\cos\beta = \frac{2}{3}$ va $90° < \beta < 180°$ bo‘lsa, $\sin\beta$ nimaga teng; d) $\cos\alpha = 1$ bo‘lsa, $\sin\alpha$ ning qiymatini hisoblang.
4. O‘tkir burchagi 60° ga, balandligi esa 3 *sm* ga teng rombning yuzini toping.
5. Teng yonli uchburchakning asosi 4,8 *sm*, asosidagi burchagi esa 30°. Uchburchakning balandligi va yon tomonini toping.
6. Agar a) $\cos\alpha = \frac{1}{2}$; b) $\cos\alpha = \frac{2}{3}$; d) $\cos\alpha = -1$ bo‘lsa, $\sin\alpha$ nimaga teng?
7. a) $\sin A = \frac{2}{3}$; b) $\cos A = \frac{3}{4}$; d) $\cos\alpha = \frac{2}{5}$ ekanligi ma’lum, A burchakni yasang.

8*. α va β burchaklar $0° < \alpha < \beta < 90°$ shartni qanoatlantiradi. 2-rasmdan foydalanib isbotlang:
a) $\sin\alpha < \sin\beta$; b) $\cos\alpha > \cos\beta$;
d) $\mathrm{tg}\,\alpha < \mathrm{tg}\,\beta$; e) $\mathrm{ctg}\,\alpha > \mathrm{ctg}\,\beta$.

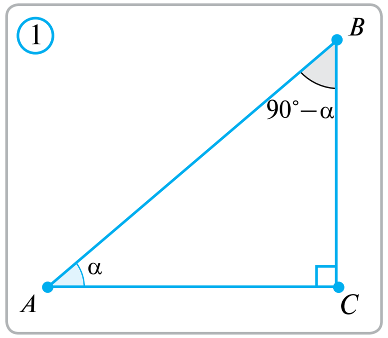


9*. OA nur bilan Ox nur orasidagi burchak α ga teng. Agar a) $OA = 3$, $\alpha = 45°$; b) $OA = 1{,}5$, $\alpha = 90°$; d) $OA = 5$, $\alpha = 150°$; e) $OA = 2$, $\alpha = 180°$; f) $OA = 4$, $\alpha = 30°$ bo‘lsa, A nuqtaning koordinatalarini toping.

28 ASOSIY TRIGONOMETRIK AYNIYATLAR (davomi)

1-teorema. **Har qanday o‘tkir α burchak uchun:**

$$\mathbf{\sin(90^\circ-\alpha)=\cos\alpha, \qquad \cos(90^\circ-\alpha)=\sin\alpha.} \qquad (1)$$



Isbot. A uchidagi o‘tkir burchagi α ga teng bo‘lgan to‘g‘ri burchakli ABC uchburchakni qaraymiz (*1-rasm*). U holda uning B uchidagi o‘tkir burchagi $\beta=90^\circ-\alpha$ ga teng. Ta’rifga ko‘ra,

$$\sin(90^\circ-\alpha)=\sin\beta=\frac{AC}{AB}=\cos\alpha,$$

$$\cos(90^\circ-\alpha)=\cos\beta=\frac{BC}{AB}=\sin\alpha.$$

Teorema isbotlandi.

1-masala. Quyidagi sonlar ichida o‘zaro tenglarini toping: $\sin 10^\circ$, $\cos 10^\circ$, $\sin 80^\circ$, $\cos 80^\circ$.

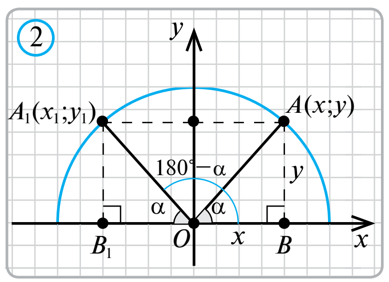
Yechilishi. $80^\circ=90^\circ-10^\circ$ $(\alpha=10^\circ)$ va $50^\circ=90^\circ-40^\circ$ $(\alpha=40^\circ)$ bo‘lgani uchun 1-teoremaga ko‘ra,

$$\sin 80^\circ=\sin(90^\circ-10^\circ)=\cos 10^\circ, \qquad \cos 80^\circ=\cos(90^\circ-10^\circ)=\sin 10^\circ.$$

Javob: $\sin 80^\circ=\cos 10^\circ$, $\cos 80^\circ=\sin 10^\circ$.

2-teorema. **Har qanday α $(0\le\alpha\le180^\circ)$ burchak uchun:**

$$\mathbf{\sin(180^\circ-\alpha)=\sin\alpha, \qquad \cos(180^\circ-\alpha)=-\cos\alpha.} \qquad (2)$$



Isbot. To‘g‘ri burchakli Oxy koordinatalar sistemasida markazi O nuqtada, radiusi 1 ga teng yarim aylanani yasaymiz *(2-rasm)*. Aylananing OA radiusi bilan Ox nur orasidagi burchak α bo‘lsin. Ox nur bilan $180^\circ-\alpha$ ga teng burchak tashkil qiluvchi OA_1 radiusni o‘tkazamiz. OAB va OA_1B_1 to‘g‘ri burchakli uchburchaklar teng. Xususan, $OB=OB_1$ va $AB=A_1B_1$ yoki $x_1=-x$ va $y_1=y$ tengliklarga egamiz. Shunday qilib,

$$\sin(180^\circ-\alpha)=y_1=y=\sin\alpha; \qquad \cos(180^\circ-\alpha)=x_1=-x=-\cos\alpha.$$

Teorema isbotlandi.

(1) va (2) formulalar **keltirish formulalari** deyiladi.

2-masala. $\alpha=120^\circ$ bo‘lsa, $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\,\alpha$ va $\mathrm{ctg}\,\alpha$ qiymatlarni hisoblang.

Yechilishi. a) (2) formulaga ko‘ra,

$$\sin 120^\circ=\sin(180^\circ-60^\circ)=\sin 60^\circ=\frac{\sqrt{3}}{2}; \quad \cos 120^\circ=\cos(180^\circ-60^\circ)=-\cos 60^\circ=-\frac{1}{2}.$$

Unda

$$\mathrm{tg}\,120^\circ=\frac{\sin 120^\circ}{\cos 120^\circ}=-\sqrt{3}; \quad \mathrm{ctg}\,120^\circ=\frac{\cos 120^\circ}{\sin 120^\circ}=-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}.$$

Javob: $\sin 120^\circ=\frac{\sqrt{3}}{2}$; $\cos 120^\circ=-\frac{1}{2}$; $\mathrm{tg}\,120^\circ=-\sqrt{3}$; $\mathrm{ctg}\,120^\circ=-\frac{\sqrt{3}}{3}$.

Savol, masala va topshiriqlar

1. $\mathrm{tg}(90^\circ-\alpha)=\mathrm{ctg}\,\alpha$ $(\alpha\ne0^\circ)$ va $\mathrm{ctg}(90^\circ-\alpha)=\mathrm{tg}\,\alpha$ $(\alpha\ne0^\circ)$ ayniyatlarni isbotlang.
2. $\mathrm{tg}(180^\circ-\alpha)=-\mathrm{tg}\,\alpha$ $(\alpha\ne90^\circ)$ va $\mathrm{ctg}(180^\circ-\alpha)=-\mathrm{ctg}\,\alpha$ $(\alpha\ne0^\circ$ va $\alpha\ne180^\circ)$ ayniyatlarni isbotlang.
3. Jadvalni to‘ldiring.

α	0°	30°	45°	60°	90°	120°	135°	150°	180°
sinα									
cosα									
tgα									
ctgα									

4. Agar $90^\circ<\alpha<180^\circ$ va a) $\sin\alpha=\frac{1}{2}$; b) $\cos\alpha=-\frac{\sqrt{2}}{2}$; d) $\mathrm{tg}\,\alpha=-1$; e) $\mathrm{ctg}\,\alpha=-\sqrt{3}$ bo‘lsa, α burchak kattaligini toping.
5. Hisoblang:
   a) $\sin 180^\circ+2\cos 90^\circ$; b) $4\sin 150^\circ+\sqrt{3}\,\mathrm{tg}\,150^\circ$;
   d) $\cos 40^\circ+\cos 50^\circ-\sin 40^\circ-\sin 50^\circ$; e) $3\cos 120^\circ-2\sqrt{3}\,\mathrm{ctg}\,60^\circ$.
6. Soddalashtiring:
   a) $\cos^2(180^\circ-\alpha)+\cos^2(90^\circ-\alpha)$; b) $\sin^2(180^\circ-\alpha)+\sin^2(90^\circ-\alpha)$;
   d) $\mathrm{tg}\,\alpha\cdot\mathrm{tg}(90^\circ-\alpha)$; e) $\mathrm{ctg}\,\alpha\cdot\mathrm{ctg}(90^\circ-\alpha)$.
7. ABC uchburchakda $\angle A=150^\circ$ va $AC=7$ *sm* bo‘lsa, uchburchakning C uchidan tushirilgan balandligini toping.
8. To‘g‘ri to‘rtburchakning 12 *sm* ga teng diagonali bir tomoni bilan 30° ga teng burchak hosil qiladi. To‘g‘ri to‘rtburchak yuzini toping.
9. Agar a) $\sin\alpha=\frac{\sqrt{3}}{2}$; b) $\sin\alpha=\frac{1}{4}$; d) $\sin\alpha=1$ bo‘lsa, $\cos\alpha$ ni toping.

10*. Agar a) $\sin\alpha=\frac{1}{2}$; b) $\mathrm{tg}\,\alpha=-1$; d) $\cos\alpha=-\frac{\sqrt{3}}{2}$ bo‘lsa, α ni toping.

29 BILIMINGIZNI SINAB KO‘RING

I. Chap ustunda berilgan atamalarga o‘ng ustunda berilgan ta’riflardan to‘g‘risini mos qo‘ying.

1. α burchak sinusi	a) α burchak qarshisidagi katetning gipotenuzaga nisbati;
2. α burchak kosinusi	b) α burchakka yopishgan katetning gipotenuzaga nisbati;
3. α burchak tangensi	d) α burchak qarshisidagi katetning ikkinchi katetga nisbati;
4. α burchak kotangensi	e) α burchakka yopishgan katetning ikkinchi katetga nisbati.

II. Testlar

1. Noto‘g‘ri formulani toping:
A. $\sin(90°-\alpha)=\cos\alpha$; B. $\cos(90°-\alpha)=\sin\alpha$;
D. $\sin(180°-\alpha)=\sin\alpha$; E. $\cos(180°-\alpha)=\cos\alpha$.

2. Agar $90°<\alpha<180°$ bo‘lsa, quyidagilardan qaysi biri musbat?
A. $\sin\alpha$; B. $\cos\alpha$; D. $\mathrm{tg}\alpha$; E. $\mathrm{ctg}\alpha$.

3. To‘g‘ri tenglikni toping:
A. $\sin^2\alpha=1+\cos^2\alpha$; B. $\mathrm{tg}^2\alpha=1+\cos^2\alpha$;
D. $\frac{1}{\cos^2\alpha}=1+\mathrm{tg}^2\alpha\ (\alpha\neq 90°)$; E. $\sin^2 x\cdot\cos^2 x=1$.

4. sin70° nimaga teng?:
A. sin20°; B. − sin20°; D. cos70°; E. cos 20°.

5. $\sin\alpha=\frac{1}{2}$ bo‘lgan α o‘tkir burchakni ko‘rsating:
A. 30°; B. 45°; D. 90°; E. 60°.

6. $\cos\alpha=\frac{1}{2}$ bo‘lsa, α o‘tkir burchakni toping:
A. 30°; B. 45°; D. 90°; E. 60°.

7. $\mathrm{tg}\alpha=1$ bo‘lsa, α o‘tkir burchakni toping:
A. 30°; B. 45°; D. 90°; E. 60°.

8. $\mathrm{ctg}\alpha=1$ bo‘lsa, α o‘tkir burchakni toping:
A. 30°; B. 45°; D. 90°; E. 60°.

9. Qaysi o‘tkir α burchak uchun $\sin\alpha=\cos\alpha$ tenglik o‘rinli?
A. 30°; B. 45°; D. 90°; E. 60°.

10. Agar $\sin B=\frac{2}{5}$ bo‘lsa, cos$B$ ni toping.

A. $\frac{4}{25}$; B. $\frac{\sqrt{29}}{5}$; D. $\frac{\sqrt{21}}{5}$; E. $\frac{\sqrt{10}}{5}$.

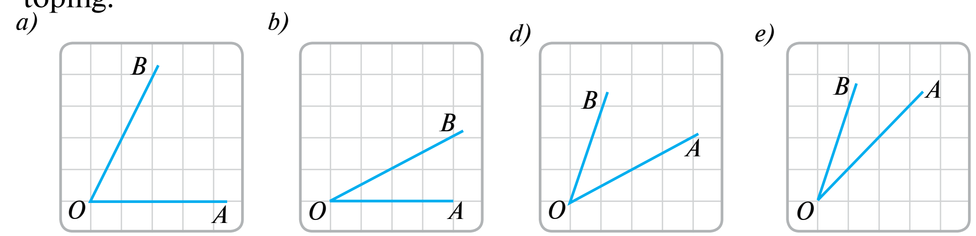
11. Agar $\cos A=0,2$ bo‘lsa, tg$A$ ni toping.
A. $\sqrt{96}$; B. $2\sqrt{6}$; D. $\sqrt{15}$; E. $\frac{\sqrt{6}}{12}$.

12. To‘g‘ri to‘rtburchakning diagonali uning bir tomonidan 2 marta uzun. To‘g‘ri to‘rtburchakning diagonallari orasidagi burchakni toping.
A. 30°; B. 60°; D. 90°; E. 150°.

13. Teng yonli uchburchakning asosiga tushirilgan balandligi 3 sm, asosi esa 8 sm. Uchburchakning asosiga yopishgan burchagi sinusini toping.
A. $\frac{3}{5}$; B. $\frac{3}{4}$; D. $\frac{\sqrt{73}}{73}$; E. $\frac{4}{5}$.

III. Masalalar

1. Rasmda tasvirlangan burchaklarning sinusi, kosinusi, tangensi va kotangensini toping.



2. Ashrafjon uyidan sharq tomonga qarab 800 *m*, so‘ng shimol tomonga qarab 600 *m* yo‘l yurdi. U uyidan necha metr uzoqlikka keldi? Endi u uyiga to‘g‘ri chiziq bo‘ylab yetib olishi uchun g‘arbga nisbatan qanday burchak ostida yurishi kerak?

3. Poyezd har 30 *m* yo‘l yurganda 1 *m* tepaga ko‘tariladi. Temir yo‘lning gorizontga nisbatan ko‘tarilish burchagini toping.

4. Agar balandligi 30 *m* bo‘lgan bino soyasining uzunligi 45 *m* bo‘lsa, quyosh nurining shu bino joylashgan maydonga tushish burchagini toping.

5. To‘g‘ri burchakli uchburchakning bir burchagi 60° ga, katta kateti esa 6 ga teng. Uning kichik kateti va gipotenuzasini toping.

6. O markazli aylananing A nuqtasidan o‘tkazilgan urinmada B nuqta olingan. Agar $AB=9$ *sm*, $\angle ABO=30°$ bo‘lsa, aylana radiusini va BC kesma uzunligini toping.

7. m to‘g‘ri chiziq va uni kesib o‘tmaydigan AB kesma berilgan. Bunda $AB=10$, AB va m to‘g‘ri chiziqlar orasidagi burchak 60°. AB kesma uchlaridan m to‘g‘ri chiziqqa AC va BD perpendikularlar tushirilgan. CD kesmani toping.

8. Rombning o‘tkir burchagi 60° ga, balandligi esa 6 ga teng. Rombning katta diagonali uzunligini va yuzini toping.

9. Radiusi 5 *sm* bo‘lgan aylanaga teng yonli trapetsiya tashqi chizilgan. Agar trapetsiyaning o‘tkir burchagi 30° bo‘lsa, uning yon tomoni va yuzini toping.

10. Agar $ABCD$ to‘g‘ri to‘rtburchakda $AB=4$, $\angle CAD=30°$ bo‘lsa, unga tashqi chizilgan aylana radiusini va to‘g‘ri to‘rtburchak yuzini hisoblang.

11. To‘g‘ri to‘rtburchakning tomonlari 3 *sm* va $\sqrt{3}$ *sm*. Uning bir diagonali bilan tomonlari hosil qilgan burchaklarini toping.

12. Agar a) $\sin A=\frac{4}{7}$; b) $\cos A=\frac{4}{7}$; d) $\cos A=-\frac{4}{7}$ bo‘lsa, A burchakni yasang.

13. To‘g‘ri burchakli uchburchakning bir burchagi 30°, gipotenuzasiga tushirilgan balandligi 6 *sm*. Uchburchak tomonlarini toping.

14. O‘tkir burchagi 30° ga, balandligi esa 4 *sm* ga teng bo‘lgan rombning yuzini hisoblang.

15. Agar $\sin A=\frac{8}{17}$ va $90°<\alpha<180°$ bo‘lsa, $\cos\alpha$, $\operatorname{tg}\alpha$ va $\operatorname{ctg}\alpha$ qiymatini toping.

16. To‘g‘ri burchakli ABC uchburchakning AB gipotenuzasiga CD balandlik tushirilgan. Agar $\angle A=60°$ va $BD=2$ bo‘lsa, BC katetni toping.

17. ABC uchburchakda $\angle A=30°$, $\angle C=45°$. Agar uchburchakning BD balandligi 12 *sm* bo‘lsa, uning AC tomonini va yuzini toping.

IV. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi)

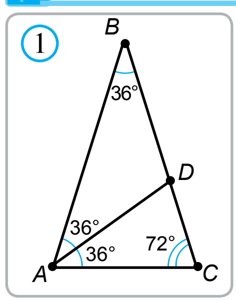
1. Agar $\cos\alpha=-\frac{8}{17}$ va $90°<\alpha<180°$ bo‘lsa, $\sin\alpha$, $\operatorname{tg}\alpha$, $\operatorname{ctg}\alpha$ nimaga teng?

2. To‘g‘ri burchakli uchburchakning gipotenuzasi $c=18$ *sm* va kateti $a=4$ *sm* bo‘lsa, uning ikkinchi kateti va o‘tkir burchaklarini toping.

3. Teng tomonli uchburchakning medianasi uning tomonidan kichik bo‘lishini isbotlang.

4. *(Qo‘shimcha).* To‘rtburchakning har bir tomoni qolgan tomonlarining yig‘indisidan kichik ekanini isbotlang.

Tarixiy lavhalar. **“Oltin uchburchak”**



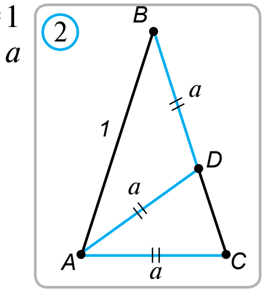
Yunonlar burchaklari 36°, 72° va 72° bo‘lgan teng yonli uchburchakni — “oltin uchburchak” deb atashgan. Sababi u mana bunday ajoyib xossaga ega ekan: asosidagi burchak bissektrisasi AD uni ikkita teng yonli uchburchakka bo‘ladi (*1-rasm*).

Haqiqatan, AD bissektrisa bo‘lgani uchun, BAD va DAC burchaklar ham 36° dan. Demak, ABD uchburchak teng yonli. ADC uchburchakda ADC burchak $180°-36°-72°=72°$ bo‘lib, ACD burchakka teng. Demak, ADC uchburchak ham teng yonli.

Natija. ABC uchburchak ACD uchburchakka o‘xshash va

$$\frac{AC}{AB}=\frac{CD}{AC}. \qquad (1)$$

Agar ABC uchburchakning yon tomonlari $AB=BC=1$ deb olsak, uning asosi quyidagicha topiladi (*2-rasm*): $AC=a$ bo‘lsin. U holda,



1. $AD=a$ bo‘ladi, chunki ΔACD teng yonli.
2. $BD=a$ bo‘ladi, chunki ΔABD teng yonli.
3. $CD=BC-BD=1-a$.

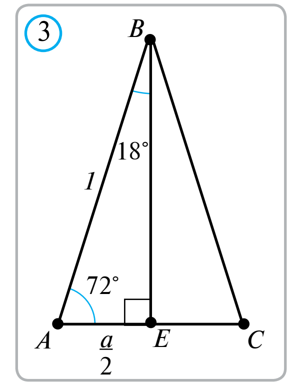
(1) tenglikka ko‘ra:

$$\frac{a}{1}=\frac{1-a}{a}$$

Bundan $a^2+a-1=0$. Bu kvadrat tenglamani yechib, $a=\frac{\sqrt{5}-1}{2}$ ekanligini topamiz.

Masala. $\sin 18°$, $\cos 18°$, $\sin 72°$, $\cos 72°$ qiymatlarni hisoblang.

Yechilishi: Yon tomoni $AB=BC=1$ va asosi $AC=a=\frac{\sqrt{5}-1}{2}$ ga teng bo‘lgan ABC “oltin uchburchak”ni qaraymiz (*3-rasm*). Uning BE balandligini o‘tkazamiz.



To‘g‘ri burchakli ABE uchburchakdan

$$\sin 18°=\frac{AE}{AB}=\frac{a}{2}=\frac{\sqrt{5}-1}{4}$$

Bundan foydalanib, topilishi talab qilingan boshqa qiymatlarni hisoblaymiz:

$$\cos 18°=\sqrt{1-\sin^2 18°}=\frac{\sqrt{5}+1}{4};$$

$$\sin 72°=\sin(90°-18°)=\cos 18°=\frac{\sqrt{5}+1}{4};$$

$$\cos 72°=\sin 18°=\frac{\sqrt{5}-1}{2}.$$

Javob: $\sin 18°=\cos 72°=\frac{\sqrt{5}-1}{4}$; $\cos 18°=\sin 72°=\frac{\sqrt{5}+1}{4}$.

Tarixiy lavhalar

Ulug‘bek (1394 — 1449)

Ulug‘bek (1394–1449) — buyuk o‘zbek olimi va davlat arbobi. Asl ismi Muhammad Tarag‘ay. U sohibqiron Amir Temurning nabirasi. Ulug‘bekning otasi Shohruh ham davlat arbobi bo‘lgan. Ulug‘bek taxminan 1425–1428-yillari Samarqand yaqinidagi Obi Rahmat tepaligida o‘zining mashhur rasadxonasini quradi. Rasadxonaning binosi uch qavatli bo‘lib, uning asosiy asbobi — kvadrantning balandligi 50 metr edi. Ulug‘bekning eng mashhur asari “Ziji ko‘ragoniy” deb ataluvchi astronomik jadvaldir. U 1018 ta yulduzni o‘z ichiga olgan.

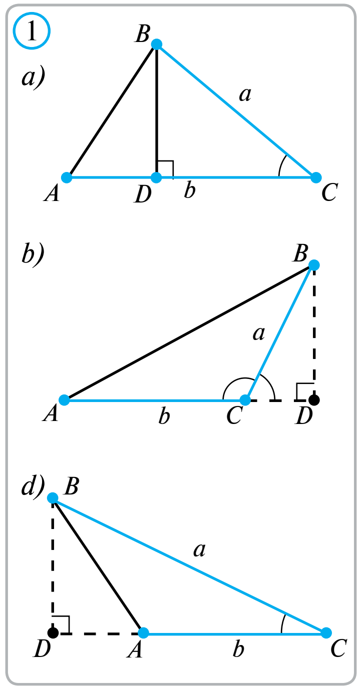
30 UCHBURCHAK YUZINI BURCHAK SINUSI YORDAMIDA HISOBLASH

1-teorema. **Uchburchak yuzi uning ikki tomoni bilan shu ikki tomon orasidagi burchak sinusi ko'paytmasining yarmiga teng.**

$\triangle ABC$, $BC=a$, $AC=b$, $\angle C$ (1-rasm) ⇨ $S_{ABC}=\frac{1}{2}ab\sin C$

Isbot. ABC uchburchakning BD balandligini tushiramiz. U holda 1-rasmda ko'rsatilgan uch hol bo'lishi mumkin.

Birinchi holni qaraymiz. BCD uchburchakda $\sin C=\frac{BD}{BC}$. Bundan $BD=BC\cdot\sin C=a\cdot\sin C$. Shunday qilib,



$$S_{ABC}=\frac{1}{2}\cdot AC\cdot BD=\frac{1}{2}\cdot b\cdot a\cdot\sin C=\frac{1}{2}ab\sin C.$$

Ikkinchi va uchunchi hollarning isbotini mustaqil bajaring. ***Teorema isbotlandi.***

1-teoremaga ko'ra, uchburchak yuzi uchun

$$S_{ABC}=\frac{1}{2}bc\sin A \quad \text{va} \quad S_{ABC}=\frac{1}{2}ac\sin B$$

formulalar ham o'rinli bo'ladi.

1-masala. ABC uchburchakning yuzi 24 sm^2. Agar $AC=8$ *sm* va $\angle A=30°$ bo'lsa, AB tomonni toping.

Yechilishi. Uchburchak yuzini burchak sinusi orqali topish formulasiga ko'ra,

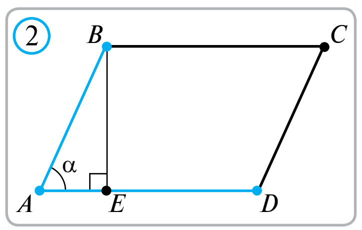
$$S_{ABC}=\frac{1}{2}\cdot AB\cdot AC\cdot\sin A$$

Bundan,

$$AB=\frac{2S_{ABC}}{AC\cdot\sin A}=\frac{2\cdot 24}{8\cdot\sin 30°}=\frac{2\cdot 24}{8\cdot 0,5}=12\ (sm).$$

Javob: 12 *sm*.

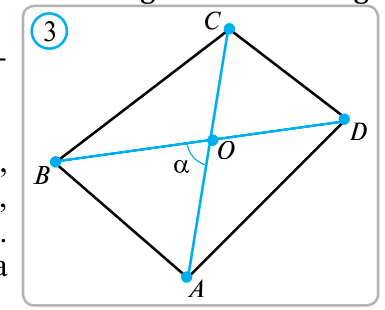
2-masala. Parallelogramm yuzi uning ikkita qo'shni tomoni va shu tomonlar orasidagi burchagi sinusining ko'paytmasiga teng ekanligini isbotlang.



ABCD parallelogramm, $AB=a$, $AD=b$, $\angle A=\alpha$ (2-rasm) ⇨ $S_{ABCD}=ab\sin\alpha$

Yechilishi. BE balandlik tushiramiz. ABE uchburchakda $\sin A=\frac{BE}{AB}$ yoki $BE=AB\sin A=a\sin\alpha$. U holda, $S_{ABCD}=AD\cdot BE=ab\sin\alpha$.

2-teorema. **To'rtburchak yuzi uning diagonallari bilan diagonallar orasidagi burchak sinusi ko'paytmasining yarmiga teng.**



Isbot. Diagonallar kesishishidan hosil bo'lgan burchaklarni qaraymiz (*3-rasm*):

$\angle AOB=\alpha$ ⇐ shartga ko'ra,
$\angle COD=\alpha$ ⇐ $\angle AOB$ ga vertikal bo'lgani uchun,
$\angle BOC=180°-\alpha$ ⇐ $\angle AOB$ ga qo'shni bo'lgani uchun,
$\angle DOA=180°-\alpha$ ⇐ $\angle BOC$ ga vertikal bo'lgani uchun.

Uchburchak yuzini burchak sinusi yordamida hisoblash formulasiga ko'ra:

$$S_{AOB}=\frac{1}{2}AO\cdot OB\sin\alpha;\qquad S_{BOC}=\frac{1}{2}BO\cdot OC\sin(180°-\alpha)=\frac{1}{2}BO\cdot OC\sin\alpha;$$

$$S_{COD}=\frac{1}{2}CO\cdot OD\sin\alpha;\qquad S_{DOA}=\frac{1}{2}DO\cdot OA\sin(180°-\alpha)=\frac{1}{2}DO\cdot OA\sin\alpha.$$

Yuzning xossasiga ko'ra:

$$S_{ABCD}=S_{AOB}+S_{BOC}+S_{COD}+S_{DOA}=$$

$$=\frac{1}{2}AO\cdot OB\sin\alpha+\frac{1}{2}BO\cdot OC\sin\alpha+\frac{1}{2}CO\cdot OD\sin\alpha+\frac{1}{2}DO\cdot OA\sin\alpha=$$

$$=\frac{1}{2}(AO\cdot OB+BO\cdot OC+CO\cdot OD+DO\cdot OA)\sin\alpha=\frac{1}{2}\{(OB\cdot(AO+OC)+$$

$$+OD\cdot(CO+OA)\}\sin\alpha=\frac{1}{2}(OB\cdot AC+OD\cdot AC)\sin\alpha=\frac{1}{2}AC\cdot BD\sin\alpha.$$

Teorema isbotlandi.

Savol, masala va topshiriqlar

1. 1-teoremani 1-b va 1-d rasmda tasvirlangan hollar uchun isbotlang.
2. Agar a) $AB=6$ *sm*, $AC=4$ *sm*, $\angle A=30°$; b) $AC=14$ *sm*, $BC=7\sqrt{3}$ *sm*, $\angle C=60°$; d) $BC=3$ *sm*, $AB=4\sqrt{2}$ *sm*, $\angle B=45°$ bo'lsa, ABC uchburchak yuzini toping.
3. Diagonali 12 *sm* va diagonallari orasidagi burchagi 30° bo'lgan to'g'ri to'rtburchak yuzini toping.
4. Tomoni $7\sqrt{2}$ *sm* va o'tmas burchagi 135° bo'lgan romb yuzini toping.
5. Rombning katta diagonali 18 *sm* va bir burchagi 120°. Romb yuzini toping.
6. Yuzi $6\sqrt{2}$ sm^2 ga teng bo'lgan ABC uchburchakda $AB=9$ *sm*, $\angle A=45°$. Uchburchakning AC tomonini va shu tomonga tushirilgan balandligini toping.

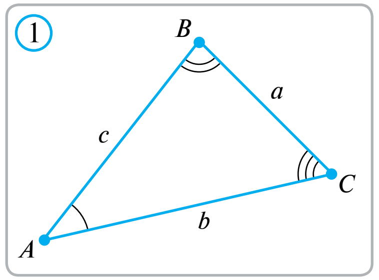
7*. ABC uchburchakda $\angle A=\alpha$, uning B va C uchlaridan tushirilgan balandliklari esa mos ravishda h_b va h_c bo'lsa, uchburchak yuzini toping.

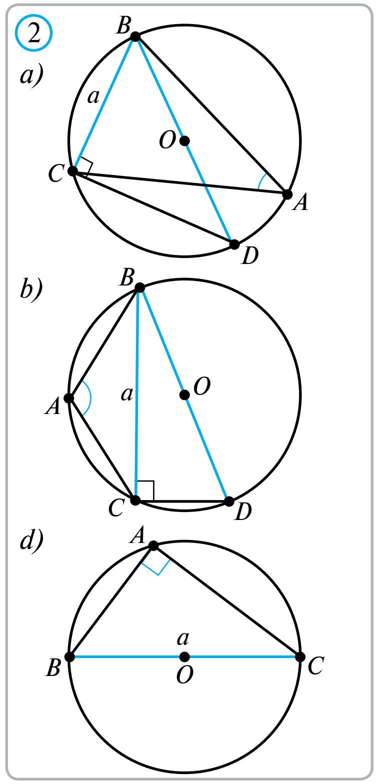
8*. ABC uchburchakda $AB=8$ *sm*, $AC=12$ *sm* va $\angle A=60°$ bo'lsa, uning AD bissektrisasini toping (*ko'rsatma:* $S_{ABC}=S_{ABD}+S_{ADC}$).

31 SINUSLAR TEOREMASI

Teorema. (Sinuslar teoremasi). **Uchburchakning tomonlari qarshisidagi burchaklarning sinuslariga proporsional.**

ΔABC, $AB=c$, $BC=a$, $CA=b$ *(1-rasm)* ⇒ $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$





Isbot. Uchburchak yuzini burchak sinusi orqali topish formulasiga ko‘ra,

$$S=\frac{1}{2}ab\sin C,\quad S=\frac{1}{2}bc\sin A,\quad S=\frac{1}{2}ac\sin B.\quad (❖)$$

Bu tengliklarning dastlabki ikkitasiga ko‘ra,

$$\frac{1}{2}ab\sin C=\frac{1}{2}bc\sin A,\ \text{ demak }\ \frac{a}{\sin A}=\frac{c}{\sin C}.$$

Shuningdek, (❖) tengliklarning birinchi va uchinchidan $\dfrac{c}{\sin C}=\dfrac{b}{\sin B}$ tenglikni hosil qilamiz.

Shunday qilib, $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$.

Teorema isbotlandi.

1-masala. ABC uchburchakda $AB=14$ *dm*, $\angle A=30^\circ$, $\angle C=65^\circ$ (*1-rasm*). BC tomonni toping.

Yechilishi: Sinuslar teoremasiga ko‘ra,

$\dfrac{AB}{\sin C}=\dfrac{BC}{\sin A}$. Undan,

$$BC=\frac{AB\cdot\sin A}{\sin C}=\frac{14\cdot\sin 30^\circ}{\sin 65^\circ}\approx\frac{14\cdot 0{,}5}{0{,}9}\approx 7{,}78\ (dm).$$

Eslatma: Trigonometrik funksiyalarning qiymatlari maxsus kalkulator yoki jadvallar yordamida topiladi. Bu yerda $\sin 65^\circ\approx 0{,}9$ ekanligini darslikning 153-betidagi jadvaldan aniqladik.

Javob: 7,78 dm.

2-masala. Uchburchak tomonining shu tomon qarshisidagi burchagi sinusiga nisbati uchburchakka tashqi chizilgan aylana diametriga teng, ya’ni

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$$

ekanligini isbotlang *(1-rasm).*

Yechilishi: Ravshanki, sinuslar teoremasiga ko‘ra, $\dfrac{a}{\sin A}=2R$ tenglikni isbotlash kifoya. Uch hol bo‘lishi mumkin:

1-hol: $\angle A$ — o‘tkir burchak (*2-a rasm*); 2-hol: $\angle A$ — o‘tmas burchak (*2-b rasm*);

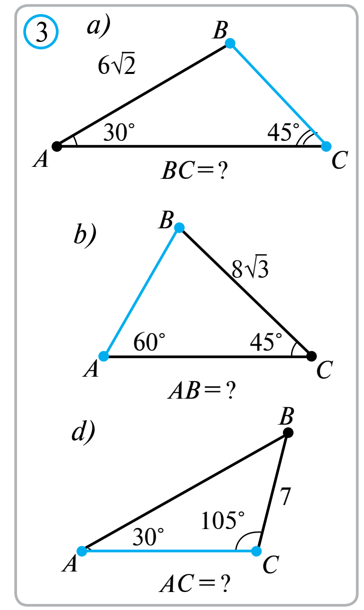
3-hol: $\angle A$ — to‘g‘ri burchak (*2-d rasm*).

1-holni qaraymiz: C va D nuqtalarni tutashtiramiz. BCD — to‘g‘ri burchakli uchburchak, chunki $\angle BCD$ burchak BD diametrga tiralgan. ΔBCD da: $BC=BD\cdot\sin D=2R\sin D$. Lekin, $\angle D=\angle A$, chunki ular bitta BC yoyga tiralgan ichki chizilgan burchaklar. Unda,

$$BC=2R\sin A\qquad \text{yoki}\qquad \frac{a}{\sin A}=2R.$$

Qolgan hollarni mustaqil isbotlang (*ko‘rsatma:* 2-holda $\angle D=180^\circ-\angle A$ ekanligidan, 3-holda $a=2R$ ekanligidan foydalaning).

Savol, masala va topshiriqlar



1. Uchburchak istalgan tomonining shu tomon qarshisidagi burchak sinusiga nisbati uchburchakka tashqi chizilgan aylana diametriga teng ekanligini 2-masalada keltirilgan 2- va 3-hollar uchun isbotlang.

2. 3-rasmda berilganlarga ko‘ra, so‘ralgan kesmalarni toping.

3. Agar ABC uchburchakda:

a) $\sin A=0{,}4$; $BC=6$ *sm* va $AB=5$ *sm* bo‘lsa, $\sin C$ ni;

b) $\sin B=\frac{3}{5}$; $AC=8$ *dm* va $BC=7$ *dm* bo‘lsa, $\sin A$ ni;

d) $\sin C=\frac{4}{7}$; $AB=6$ *m* va $AC=8$ *m* bo‘lsa, $\sin B$ ni toping.

4. Uchburchakning bir burchagi 30° ga teng. Uning qarshisidagi tomon 4,8 *dm*. Uchburchakka tashqi chizilgan aylana radiusini hisoblang.

5. Uchburchakning bir tomoni uchburchakka tashqi chizilgan aylana radiusiga teng. Uchburchakning shu tomoni qarshisidagi burchagini toping. Bunda, ikki holni qarashga to‘g‘ri kelishiga e’tibor qiling.

6. ABC uchburchak uchun $AB:BC:CA=\sin C:\sin A:\sin B$ tenglik o‘rinli bo‘lishini asoslang. $\sin A:\sin B:\sin C=3:5:7$ tenglik to‘g‘ri bo‘lishi mumkinmi?

7. Agar ABC uchburchakda $BC=20$ *m*, $AC=13$ *m* va $\angle A=67^\circ$ bo‘lsa, uchburchakning AB tomonini, B va C burchaklarini toping.

8*. Agar ABC uchburchakda $BC=18$ *dm*, $\angle A=42^\circ$, $\angle B=62^\circ$ bo‘lsa, uchburchakning C burchagini, AB va AC tomonlarini toping.

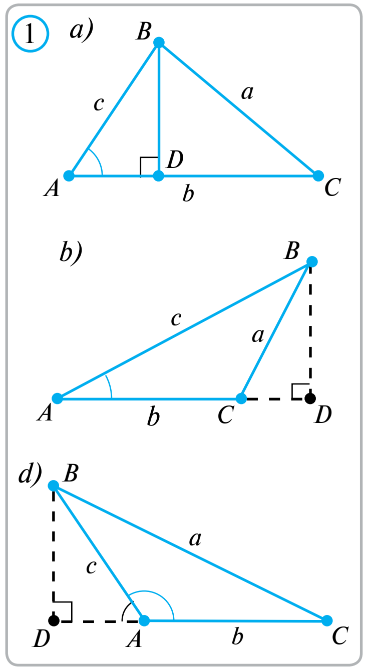
32 KOSINUSLAR TEOREMASI

To‘g‘ri burchakli uchburchakda to‘g‘ri burchak qarshisidagi tomon (gipotenuza) kvadrati qolgan tomonlar (katetlar) kvadratlari yig‘indisiga teng.

Xo‘sh, to‘g‘ri bo‘lmagan burchak uchun-chi? Quyidagi teorema shu xususda.

Teorema. (Kosinuslar teoremasi). **Uchburchak istalgan tomonining kvadrati qolgan ikki tomoni kvadratlari yig‘indisi shu ikki tomon bilan ular orasidagi burchak kosinusi ko‘paytmasining ikkilangani ayirmasiga teng.**

ΔABC, $AB=c$, $BC=a$, $CA=b$ *(1-rasm)* ⇒ $a^2=b^2+c^2-2bc\cos A$



Isbot. ABC uchburchakning BD balandligini o‘tkazamiz. D nuqta AC tomonda *(1-a rasm)* yoki uning davomida *(1-b va 1-d rasmlar)* bo‘lishi mumkin. Birinchi holni qaraymiz. To‘g‘ri burchakli BCD uchburchakda Pifagor teoremasiga ko‘ra,

$$BC^2=BD^2+DC^2.$$

$DC=AC-AD$ bo‘lgani uchun:

$$BC^2=BD^2+(AC-AD)^2=BD^2+AC^2-2\cdot AC\cdot AD+AD^2.$$

To‘g‘ri burchakli ABD uchburchakda $BD^2+AD^2=AB^2$ va $AD=AB\cos A$ ekanligini hisobga olib, oxirgi tenglikdan

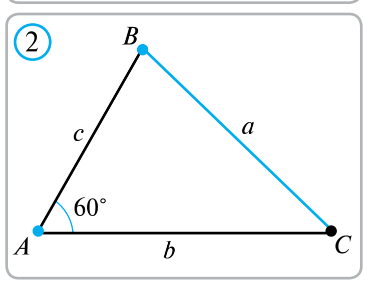
$$BC^2=AB^2+AC^2-2AB\cdot AC\cdot\cos A,$$

ya’ni $a^2=b^2+c^2-2bc\cos A$ tenglikka ega bo‘lamiz.

Teorema isbotlandi.

1-b rasmda tasvirlangan holda $DC=AD-AC$, 1-d rasmda tasvirlangan holda $DC=AD+AC$ va $\cos(180°-A)=-\cos A$ tengliklardan foydalanib, kosinuslar teoremasini mustaqil isbotlang.

Eslatma. Kosinuslar teoremasi Pifagor teoremasining umumlashganidir. $\angle A=90°$ bo‘lganda ($\cos 90°=0$ bo‘lgani uchun) kosinuslar teoremasidan Pifagor teoremasi kelib chiqadi.

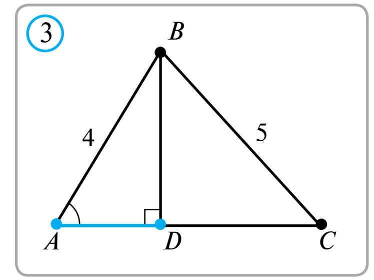


1-masala. ABC uchburchakda $AB=6$ *sm*, $AC=7$ *sm*, $\angle A=60°$ *(2-rasm)*. BC tomonni toping.

Yechilishi. Kosinuslar teoremasiga ko‘ra, $a^2=b^2+c^2-2bc\cos A$ yoki $BC^2=AC^2+AB^2-2AC\cdot AB\cdot\cos A$ bo‘lgani uchun

$$BC^2=7^2+6^2-2\cdot 7\cdot 6\cdot\cos 60°=49+36-84\cdot\frac{1}{2}=43,$$

ya’ni $BC=\sqrt{43}$ *sm*. ***Javob:*** $\sqrt{43}$ *sm*.



Kosinuslar teoremasidan foydalanib, tomonlari ma’lum bo‘lgan uchburchakning burchaklarini topish mumkin:

$$\cos A=\frac{b^2+c^2-a^2}{2bc}. \qquad (1)$$

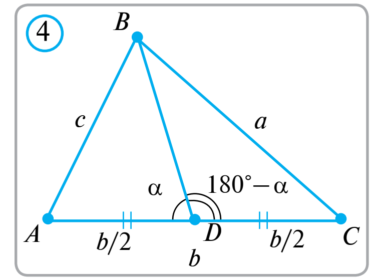
2-masala. ABC uchburchakning tomonlari $a=5$ *m*, $b=6$ *m* va $c=4$ *m*. Kichik tomonning katta tomondagi proyeksiyasini toping *(3-rasm)*.

Yechilishi. (1) formula asosida $\cos A$ ni topamiz:

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{6^2+4^2-5^2}{2\cdot 6\cdot 4}=\frac{9}{16}.$$

To‘g‘ri burchakli ABD uchburchakda $AD=AB\cdot\cos A$ bo‘lgani uchun $AD=4\cdot\frac{9}{16}=2{,}25$ (*m*).

Javob: 2,25 *m*.



Savol, masala va topshiriqlar

1. Kosinuslar teoremasini 1-b va 1-d rasmda tasvirlangan hollarda isbotlang.
2. ABC uchburchakda
   a) $AC=3$ *sm*, $BC=4$ *sm* va $\angle C=60°$ bo‘lsa, AB ni;
   b) $AB=4$ *m*, $BC=4\sqrt{2}$ *m* va $\angle B=45°$ bo‘lsa, AC ni;
   d) $AB=7$ *dm*, $AC=6\sqrt{3}$ *dm* va $\angle A=150°$ bo‘lsa, BC ni toping.
3. Tomonlari 5 *sm*, 6 *sm*, 7 *sm* bo‘lgan uchburchak burchaklari kosinuslarini toping.
4. ABC uchburchakda $AB=10$ *sm*, $BC=12$ *m* va $\sin B=0{,}6$ bo‘lsa, AC tomonni toping.
5. Parallelogrammning diagonallari 10 *sm* va 12 *sm*, ular orasidagi burchagi 60° ga teng. Parallelogramm tomonlarini toping.
6. Tomonlari 5 *sm* va 7 *sm* bo‘lgan parallelogrammning bir burchagi 120° ga teng. Uning diagonallarini toping.
7*. Tomonlari a, b, c bo‘lgan ABC uchburchakning BD medianasi $BD=\frac{1}{2}\sqrt{2a^2+2c^2-b^2}$ formula bilan hisoblanishini isbotlang *(4-rasm)*.
8*. Tomonlari 6 *m*, 7 *m* va 8 *m* bo‘lgan uchburchak medianalarini toping.
9. 3-masaladagi uchburchak bissektrisalarini toping.
10. 3-masaladagi uchburchak balandliklarini toping.

33 SINUSLAR VA KOSINUSLAR TEOREMALARINING BA'ZI TATBIQLARI

Oldingi darslarda isbotlangan sinuslar va kosinuslar teoremalaridan uchburchaklarga oid turli-tuman masalalarni yechishda samarali foydalanish mumkin. Bu darsda bu teoremalarning ba'zi bir tatbiqlariga to'xtalamiz.

1. Kosinuslar teoremasi uchburchak burchaklarini topmasdan, uning burchaklar bo'yicha turini (o'tkir, o'tmas yoki to'g'ri burchakli ekanligini) aniqlashga imkon beradi. Haqiqatan,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

formulada

1) agar $b^2+c^2>a^2$ bo'lsa, $\cos A>0$. Demak, ***A* — o'tkir burchak;**
2) agar $b^2+c^2=a^2$ bo'lsa, $\cos A=0$. Demak, ***A* — to'g'ri burchak;**
3) agar $b^2+c^2<a^2$ bo'lsa, $\cos A<0$. Demak, ***A* — o'tmas burchak.**

$b^2+c^2=a^2$ tenglik yoki $b^2+c^2<a^2$ tengsizlik a — uchburchakning eng katta tomoni bo'lgan holdagina bajariladi. Demak, uchburchakning to'g'ri yoki o'tmas burchagi uning eng katta tomoni qarshisida yotadi.

Uchburchakning eng katta tomoni qarshisidagi burchakning kattaligiga qarab, bu uchburchakning qanday (o'tkir, o'tmas, to'g'ri burchakli) uchburchak ekanligi haqida xulosaga kelish mumkin.

1-masala. Tomonlari 5 *m*, 6 *m* va 7 *m* bo'lgan uchburchak burchaklarini topmasdan uning turini aniqlang.

Yechilishi. Eng katta burchak qarshisida eng katta tomon yotadi. Shuning uchun, agar $a=7$, $b=6$, $c=5$ bo'lsa, $\angle A$ eng katta burchak bo'ladi.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{36+25-49}{2\cdot6\cdot5} = \frac{12}{60} = \frac{1}{5} > 0.$$

Demak, A — o'tkir burchak, berilgan uchburchak esa o'tkir burchakli.

2. Uchburchak yuzini uning ikki tomoni va ular orasidagi burchagi orqali hisoblash formulasi

$$S = \frac{1}{2} bc\sin A$$

va $\sin A = \frac{a}{2R}$ formulalardan uchburchak yuzini hisoblash uchun

$$S = \frac{abc}{4R}$$

formulani va uchburchakka tashqi chizilgan aylana radiusini hisoblash uchun

$$R = \frac{abc}{4S}$$

formulani hosil qilamiz.

2-masala. Tomonlari $a=5$, $b=6$, $c=10$ bo'lgan uchburchakka tashqi chizilgan aylana radiusini toping.

Yechilishi. Geron formulasidan foydalanib, uchburchak yuzini topamiz:

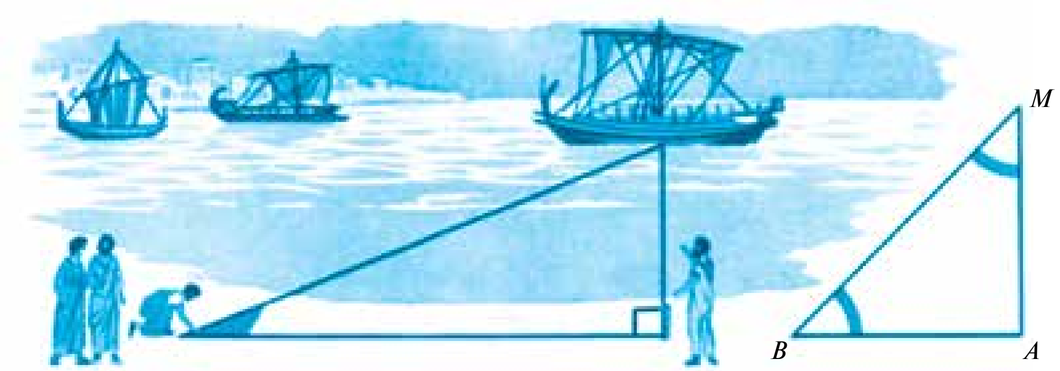
$$p = \frac{a+b+c}{2} = \frac{5+7+10}{2} = 11,$$

$$S=\sqrt{p(p-a)(p-b)(p-c)} = \sqrt{11(11-5)(11-7)(11-10)} = \sqrt{11\cdot6\cdot4} = \sqrt{264} \approx 16,3.$$

Unda, $R = \frac{abc}{4S} \approx \frac{5\cdot7\cdot10}{4\cdot16,3} \approx 5,4.$ ***Javob:*** $\approx 5,4$.

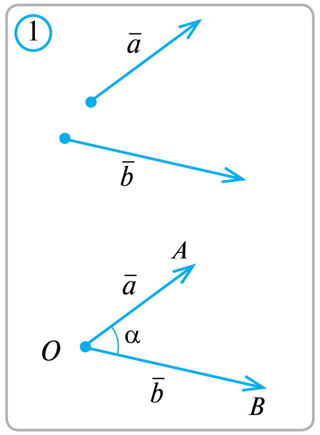
Savol, masala va topshiriqlar

1. Agar $AB=7$ *sm*, $BC=8$ *sm*, $CA=9$ *sm* bo'lsa, ABC uchburchakning eng katta va eng kichik burchagini toping.
2. Agar ABC uchburchakda $\angle A=47°$, $\angle B=58°$ bo'lsa, uchburchakning eng katta va eng kichik tomonlarini aniqlang.
3. Uchburchakning uchta tomoni berilgan:
   a) $a=5$, $b=4$, $c=4$; b) $a=17$, $b=8$, $c=15$; d) $a=9$, $b=5$, $c=6$.
   Uchburchak o'tkir burchakli, to'g'ri burchakli yoki o'tmas burchakli ekanligini aniqlang.
4. Tomonlari a) 13, 14, 15; b) 15, 13, 4; d) 35, 29, 8; e) 4, 5, 7 bo'lgan uchburchakka tashqi chizilgan aylana radiusini toping.
5. ABC uchburchakning AB tomonida D nuqta bergilangan. CD kesma AC va BC kesmalarning kamida bittasidan kichik ekanligini isbotlang.
6. Uchburchakning katta burchagi qarshisida katta tomoni yotishini isbotlang.
7. Uchburchakning katta tomoni qarshisida katta burchagi yotishini isbotlang.
8*. ABC uchburchakning CD medianasi o'tkazilgan. Agar $AC>BC$ bo'lsa, ACD burchak BCD burchakdan kichik bo'lishini isbotlang.
9. Quyidagi rasmga mos masala tuzing.



34 IKKI VEKTOR ORASIDAGI BURCHAKNI HISOBLASH

Vektorlarning skalyar ko‘paytmasi tushunchasi va xossalari bilan 8-sinfda tanishgan edingiz. Ikki vektorning skalyar ko‘paytmasi ularning koordinatalari orqali ifodalangan edi. Quyida kosinuslar teoremasi yordamida vektorlarning skalyar ko‘paytmasi uchun yana bir muhim formula chiqariladi. Bunda skalyar ko‘paytma vektorlarning uzunligi va ular orasidagi burchak orqali ifodalanadi.



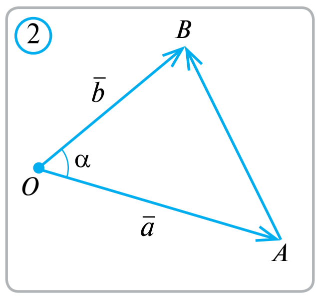
Nol vektordan farqli a va b vektorlar berilgan bo‘lsin. Ixtiyoriy O nuqtadan $\overline{OA}=\bar{a}$ va $\overline{OB}=\bar{b}$ vektorlarni qo‘yamiz. $\bar{a}$ va $\bar{b}$ vektorlar orasidagi burchak deb AOB burchakka aytiladi *(1-rasm)*. Bir xil yo‘nalgan vektorlar orasidagi burchak 0° ga teng deb hisoblanadi.

Agar ikkita vektor orasidagi burchak 90° ga teng bo‘lsa, ular **perpendikular** deyiladi.

Eslatib o‘tamiz:

1. $\bar{a}(a_1;a_2)$ vektorning uzunligi
$$|\bar{a}|=\sqrt{a_1^2+a_2^2}.$$
2. $\bar{a}(a_1;a_2)$ va $\bar{b}(b_1;b_2)$ vektorlarning skalyar ko‘paytmasi
$$\bar{a}\bar{b}=a_1b_1+a_2b_2$$

formulalar bilan aniqlanar edi.



Nokollinear $\bar{a}$ va $\bar{b}$ vektorlarni qaraymiz. Ixtiyoriy O nuqtadan $\overline{OA}=\bar{a}$ va $\overline{OB}=\bar{b}$ vektorlarni qo‘yamiz *(2-rasm)*. $\angle AOB=\alpha$ bo‘lsin. Unda, bir tomondan kosinuslar teoremasiga ko‘ra,

$$AB^2=OA^2+OB^2-2\cdot OA\cdot OB\cos\alpha. \qquad (1)$$

Ikkinchi tomondan

$$AB^2=|\overline{AB}|^2=|\overline{OB}-\overline{OA}|^2=(\overline{OA}-\overline{OB})^2=\overline{OA}^{\,2}+\overline{OB}^{\,2}-2\overline{OA}\cdot\overline{OB}. \qquad (2)$$

Demak, (1) va (2) ga ko‘ra $\overline{OA}\cdot\overline{OB}=OA\cdot OB\cos\alpha$ yoki $\bar{a}\bar{b}=|\bar{a}|\cdot|\bar{b}|\cos\alpha$.

Natija. Nol vektordan farqli $\bar{a}(a_1;a_2)$ va $\bar{b}(b_1;b_2)$ vektorlar orasidagi α burchak uchun

$$\cos\alpha=\frac{\bar{a}\bar{b}}{|\bar{a}|\cdot|\bar{b}|} \quad \text{yoki} \quad \cos\alpha=\frac{a_1b_1+a_2b_2}{\sqrt{a_1^2+a_2^2}\sqrt{b_1^2+b_2^2}}$$

formula o‘rinli.

Masala. $\bar{a}(1;2)$ va $\bar{b}(4;-2)$ vektorlar orasidagi burchakni toping.

Yechilishi. Berilgan vektorlar orasidagi burchakni α deb belgilasak, formulaga ko‘ra,

$$\cos\alpha=\frac{1\cdot4+2\cdot(-2)}{\sqrt{1^2+2^2}\cdot\sqrt{4^2+(-2)^2}}=\frac{4-4}{\sqrt{5}\cdot\sqrt{20}}=0.$$

Demak, α=90°. ***Javob:*** 90°.

Savol, masala va topshiriqlar

1. Agar $\bar{a}$ va $\bar{b}$ vektorlar uchun **a)** $a=4$, $b=5$, $\alpha=30°$; **b)** $a=8$, $b=7$, $\alpha=45°$; d) $a=2{,}4$, $b=10$, $\alpha=60°$; e) $a=0{,}8$, $b=$[illegible], $\alpha=40°$ bo‘lsa, bu vektorlarning skalyar ko‘paytmasini toping (bu yerda α — $\bar{a}$ va $\bar{b}$ vektorlar orasidagi burchak).

2. a) $\bar{a}$([illegible];-1) va $\bar{b}(2;3)$; b) $\bar{a}(-5;6)$ va $\bar{b}(6;5)$; d) $\bar{a}(1{,}5;2)$ va $\bar{b}(4;-2)$ vektorlarning skalyar ko‘paytmasini hisoblang va ular orasidagi burchakni toping.

3. $ABCD$ rombning diagonallari O nuqtada kesishadi va bunda $BD=AB=4$ *sm*.
a) $\overline{AB}$ va $\overline{AD}$; b) $\overline{AB}$ va $\overline{AC}$; d) $\overline{AD}$ va $\overline{DC}$; e) $\overline{AC}$ va $\overline{OD}$
vektorlarning skalyar ko‘paytmasini va bu vektorlar orasidagi burchakni toping.

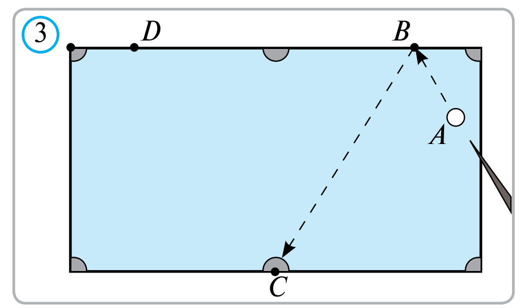
4. Nol vektordan farqli $\bar{a}$ va $\bar{b}$ vektorlar berilgan bo‘lsin. $\bar{a}\cdot\bar{b}=0$ bo‘lganda bu vektorlar perpendikular bo‘lishini va aksincha, $\bar{a}$ va $\bar{b}$ vektorlar perpendikular bo‘lsa, $\bar{a}\cdot\bar{b}=0$ bo‘lishini isbotlang.

5*. x ning qanday qiymatida a) $\bar{a}(4;5)$ va $\bar{b}(x;6)$; **b)** $\bar{a}(x;1)$ va $\bar{b}(3;2)$; **d)** $\bar{a}(0;-3)$ va $\bar{b}(5;x)$ vektorlar o‘zaro perpendikular bo‘ladi?

6. $\bar{a}(3;3)$, $\bar{b}(2;-2)$, $\bar{c}(-1;-4)$ va $\bar{d}(-4;1)$ vektorlar orasidan o‘zaro perpendikular juftlarini toping.

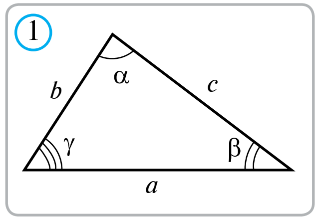
7. $a^2=|a|^2$ tenglikni isbotlang.

8*. Bilyard o‘yinida A nuqtada turgan shar zarbadan keyin bilyard stoli tomoniga B nuqtada urildi va yo‘nalishini o‘zgartirib C nuqtadagi savatchaga tushdi (*3-rasm*). Agar $AB=40$ *sm*, $BC=150$ *sm* va $\angle ABD=120°$ bo‘lsa, $\overline{AB}\cdot\overline{BC}$ skalyar ko‘paytmani toping.



9. $F(-3,\ 4)$ kuch ta’siri ostida nuqta $A(5,-1)$holatdan $B(2,\ 1)$ holatga o‘tdi. Bu jarayonda qanday ish bajarildi?

35 UCHBURCHAKLARNI YECHISH



Uchburchakning tomonlarini *a, b, c* bilan, bu tomonlar qarshisidagi burchaklarni mos ravishda α, β, γ bilan belgilaymiz *(1-rasm)*. Uchburchakning tomonlari va burchaklarini bitta nom bilan — uning **elementlari** deb atashadi.

Uchburchakni aniqlovchi berilgan elementlariga ko‘ra, uning qolgan elementlarini topish **uchburchakni yechish** deb yuritiladi.

1-masala. (Uchburchakni berilgan bir tomoni va unga yopishgan burchaklari bo‘yicha yechish). Agar uchburchakda $a=6$, $\beta=60°$ va $\gamma=45°$ bo‘lsa, uning uchinchi burchagi va qolgan ikki tomonini toping.

Yechilishi. **1.** Uchburchak burchaklari yig‘indisi 180° bo‘lgani uchun

$$\alpha=180°-\beta-\gamma=180°-60°-45°=75°.$$

Sinuslar teoremasidan foydalanib, qolgan ikki tomonni topamiz:

2. $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}$ tenglikdan $b=a\cdot\frac{\sin\beta}{\sin\alpha}=6\cdot\frac{\sin 60°}{\sin 75°}\approx 6\cdot\frac{0{,}8660}{0{,}9659}\approx 5{,}3794\approx 5{,}4.$

(sin60° va sin75° qiymatlari mikrokalkulatorda topib qo‘yildi, ularni darslikning 153-betidagi jadvaldan topishingiz mumkin).

3. $\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$ tenglikdan $c=a\cdot\frac{\sin\gamma}{\sin\alpha}=6\cdot\frac{\sin 45°}{\sin 75°}\approx 6\cdot\frac{0{,}7071}{0{,}9659}\approx 4{,}3924\approx 4{,}4.$

Javob: $\alpha=75°$; $\beta\approx 5{,}4$; $c\approx 4{,}4$.

2-masala. (Uchburchakni berilgan ikki tomoni va ular orasidagi burchagi bo‘yicha yechish). Agar uchburchakda $a=6$, $b=4$ va $\gamma=120°$ bo‘lsa, uning uchinchi tomoni va qolgan burchaklarini toping.

Yechilishi. **1.** Kosinuslar teoremasidan foydalanib, uchburchakning uchinchi *c* tomonini topamiz.

$$c=\sqrt{a^2+b^2-2ab\cos\gamma}=\sqrt{36+16-2\cdot 6\cdot 4\cdot(-0{,}5)}=\sqrt{76}\approx 8{,}7.$$

2. Endi, uchburchakning uchta tomonini bilgan holda, kosinuslar teoremasidan foydalanib, uchburchakning qolgan burchaklarini topamiz:

$$\cos\alpha=\frac{b^2+c^2-a^2}{2bc}=\frac{4^2+76-6^2}{2\cdot 4\cdot\sqrt{76}}\approx 0{,}8046.$$

$\cos\alpha\approx 0{,}8046$ tenglik asosida α burchakning qiymatini 153-betdagi jadvaldan aniqlaymiz (α — o‘tkir burchak): $\alpha\approx 36°$.

3. $\beta=180°-\alpha-\gamma\approx 180°-(36°+120°)=24°$. ***Javob:*** $c\approx 8{,}7$; $\alpha\approx 36°$, $\beta\approx 24°$.

3-masala. (Uchburchakni berilgan uch tomoni bo‘yicha yechish). Agar uchburchakda $a=10$, $b=6$ va $c=13$ bo‘lsa, uning burchaklarini toping.

Yechilishi: **1.** Uchburchak o‘tmas burchakli bo‘lishi yoki bo‘lmasligini katta tomon qarshisidagi burchak kosinusining ishorasiga qarab aniqlaymiz:

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{100+36-169}{2\cdot 10\cdot 6}=-\frac{33}{120}\approx -0{,}275<0.$$

Demak, *C* — o‘tmas burchak ekan. Buni 153-betdagi jadvaldan *C* burchakning kattaligini aniqlashda hisobga olamiz. Jadvaldan kosinusi 0,275 ga teng burchak $\angle C_1=74°$ ekanligini topamiz. Unda $\cos(180°-\alpha)=-\cos\alpha$ formulaga ko‘ra,

$$\angle C=180°-\angle C_1=180°-74°=106°.$$

2. Sinuslar teoremasiga ko‘ra,

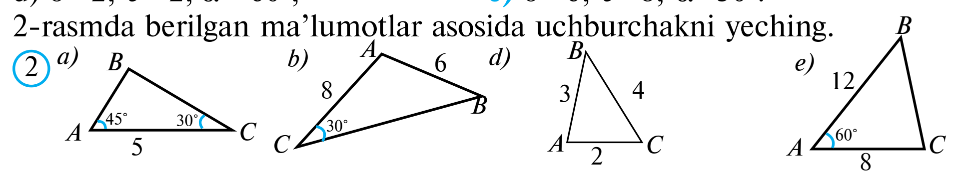
$\frac{a}{\sin A}=\frac{c}{\sin C}$. Bundan, $\sin A=\frac{a\cdot\sin C}{c}=\frac{10\cdot\sin 106°}{13}=\frac{10\cdot\sin 74°}{13}\approx\frac{10\cdot 0{,}9615}{13}\approx 0{,}7396.$

A — o‘tkir burchak bo‘lgani uchun 153-betdagi jadvaldan $\angle A\approx 47°$ ekanligini aniqlaymiz.

3. $\angle B\approx 180°-(106°+47°)=26°$. ***Javob:*** $\angle A\approx 47°$, $\angle B\approx 26°$, $\angle C\approx 106°$.

Savol, masala va topshiriqlar

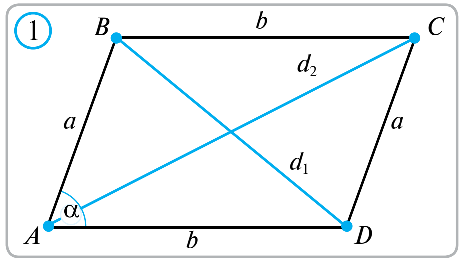
1. Uchburchakning bir tomoni va unga yopishgan ikkita burchagi berilgan:
   a) $a=5$ *sm*, $\beta=45°$, $\gamma=45°$; b) $a=20$ *sm*, $\alpha=75°$, $\beta=60°$;
   d) $a=35$ *sm*, $\beta=40°$, $\gamma=120°$; **e)** $b=12$ *sm*, $\alpha=36°$, $\beta=25°$.
   Uchburchakning uchinchi burchagi va qolgan ikki tomonini toping.
2. Uchburchakning ikki tomoni va ular orasidagi burchagi berilgan:
   a) $a=6$, $b=4$, $\gamma=60°$; b) $a=14$, $b=43$, $\gamma=130°$;
   d) $b=17$, $c=9$, $\alpha=85°$; **e)** $b=14$, $c=10$, $\alpha=145°$.
   Uchburchakning qolgan burchaklarini va uchinchi tomonini toping.
3. Uchburchakning uchta tomoni berilgan:
   a) $a=2$, $b=3$, $c=4$; b) $a=7$, $b=2$, $c=8$;
   d) $a=4$, $b=5$, $c=7$; **e)** $a=15$, $b=24$, $c=18$.
   Uchburchakning burchaklarini toping.
4. Uchburchakning ikki tomoni va bu tomonlardan birining qarshisidagi burchagi berilgan. Uchburchakning qolgan tomoni va burchaklarini toping:
   a) $a=12$, $b=5$, $\alpha=120°$; b) $a=27$, $b=9$, $\alpha=138°$;
   d) $b=2$, $c=2$, $\alpha=60°$; **e)** $b=6$, $c=8$, $\alpha=30°$.
5. 2-rasmda berilgan ma’lumotlar asosida uchburchakni yeching.



36 MASALALAR YECHISH

1-masala. Parallelogramm diagonallari kvadratlarining yig‘indisi tomonlari kvadratlarining yig‘indisiga teng ekanligini isbotlang.

ABCD — parallelogramm, AB = a, AD = b, BD = d_1, AC = d_2 (1-rasm). ⇒ $d_1^2 + d_2^2 = 2(a^2 + b^2)$



Yechilishi. *ABCD* parallelogrammning *A* burchagi α ga teng bo‘lsin. Unda $\angle B = 180° - \alpha$. *ABD* va *ABC* uchburchaklarga kosinuslar teoremasini qo‘llaymiz *(1-rasm)*:

$$d_1^2 = a^2 + b^2 - 2ab\cos\alpha, \qquad (1)$$

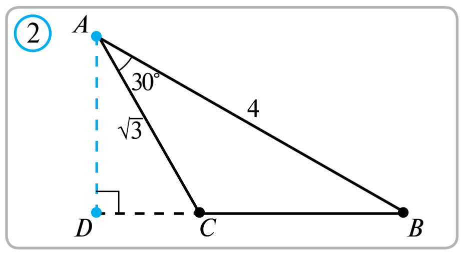
$$d_2^2 = a^2 + b^2 - 2ab\cos(180° - \alpha).$$

$\cos(180° - \alpha) = -\cos\alpha$ tenglikni hisobga olsak,

$$d_2^2 = a^2 + b^2 + 2ab\cos\alpha. \qquad (2)$$

(1) va (2) tengliklarning mos qismlarini qo‘shib, $d_1^2 + d_2^2 = 2(a^2 + b^2)$ tenglikni hosil qilamiz.

2-masala. *ABC* uchburchakda $\angle A = 30°$, $AB = 4$, $AC = \sqrt{3}$ bo‘lsa, uchburchakning *A* uchidan tushirilgan *AD* balandligini toping (*2-rasm*).



Yechilishi. 1) Kosinuslar teoremasidan foydalanib, uchburchakning *BC* tomonini topamiz:

$$BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cdot \cos A =$$
$$= 4^2 + (\sqrt{3})^2 - 2 \cdot 4 \cdot \sqrt{3} \cdot \cos 30° = 7, \quad BC = \sqrt{7}.$$

2) Endi uchburchakning yuzini topamiz:

$$S = \frac{1}{2} \cdot AB \cdot AC \cdot \sin A = \frac{1}{2} \cdot \sqrt{3} \cdot 4 \cdot \sin 30° = \sqrt{3}.$$

3) Topilganlardan foydalanib, uchburchakning *AD* balandligini topamiz:

$S = \frac{1}{2} \cdot BC \cdot AD$ formuladan $AD = \frac{2S}{BC} = \frac{2\sqrt{3}}{\sqrt{7}} = \frac{2\sqrt{21}}{7}$. **Javob:** $\frac{2\sqrt{21}}{7}$.

3-masala. Haydovchi yo‘l harakati qoidasini buzib, soat 12^{00} da shohko‘chaning *A* nuqtasidan Olmazor ko‘chasi tomon burildi va 140 *km/soat* tezlikda harakatini davom ettirdi (*3-rasm*). Soat 12^{00} da DAN xodimi *B* nuqtadan toshloq yo‘l bo‘y-

lab 70 *km/soat* tezlikda qoidabuzar haydovchi yo‘lini kesib chiqish uchun yo‘lga chiqdi. DAN xodimi chorrahada, ya’ni *C* nuqtada qoidabuzar haydovchini to‘xtatib qola oladimi?

Yechilishi: *ABC* uchburchakda

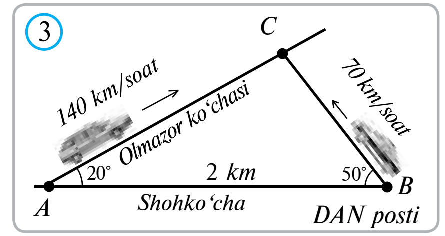
$$\angle C = 180° - (\angle A + \angle B) = 180° - (20° + 50°) = 180° - 70° = 110°.$$

1. Olmazor ko‘chasidagi yo‘lning *AC* qismi uzunligini topamiz: sinuslar teoremasiga ko‘ra, $\frac{AC}{\sin B} = \frac{AB}{\sin C}$. Bu tenglikdan $AC = \frac{AB \cdot \sin B}{\sin C} = \frac{2 \cdot \sin 50°}{\sin 110°} = \frac{2 \cdot \sin 50°}{\sin(90° + 20°)} = \frac{2 \cdot \sin 50°}{\cos 20°} \approx \frac{2 \cdot 0{,}766}{0{,}940} = \frac{1{,}532}{0{,}94} \approx 1{,}630$ (*km*). Bu yo‘lni qoidabuzar haydovchi $\frac{1{,}630\ km}{140\ km/soat} \approx 0{,}0116$ *soat* $= 0{,}012 \cdot 3600$ *sekund* ≈ 42 *sekund*da bosib o‘tadi.

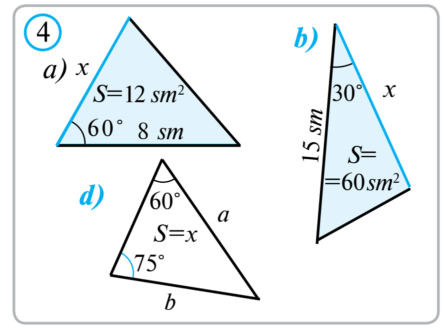
2. Endi toshloq yo‘lning *BC* qismi uzunligini topamiz: sinuslar teoremasiga ko‘ra, $\frac{BC}{\sin A} = \frac{AC}{\sin B}$. Bu tenglikdan $BC = \frac{AC \cdot \sin A}{\sin B} = \frac{2 \cdot \sin 20°}{\sin 50°} = \frac{2 \cdot 0{,}342}{0{,}766} \approx 0{,}893$ (*km*).

Bu yo‘lni DAN xodimi $\frac{0{,}893\ km}{70\ km/soat} \approx 0{,}0128$ *soat* $= 0{,}0128 \cdot 3600$ *sekund* ≈ 46 *sekund*da bosib o‘tadi. Demak, *C* chorrahaga DAN xodimi haydovchidan kechroq yetib kelar ekan.

Javob: Yo‘q.

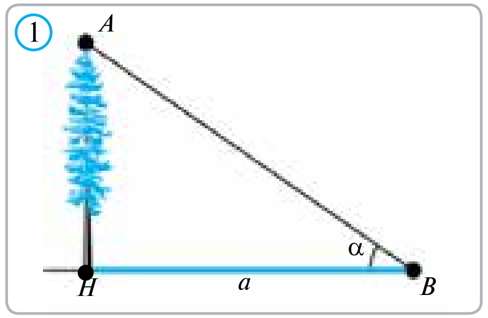


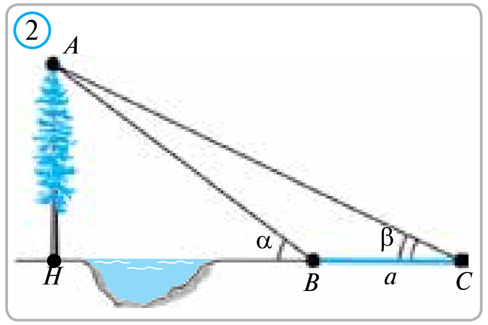
Savol, masala va topshiriqlar

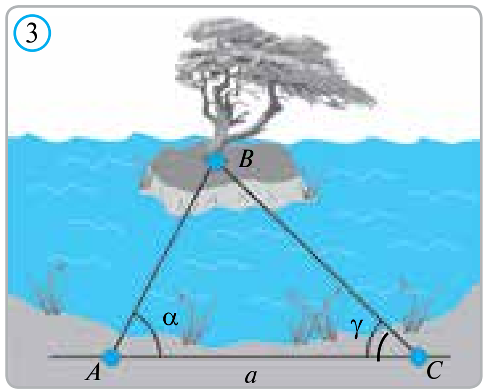


1. 4-rasmdagi ma’lumotlar bo‘yicha *x* ning qiymatini toping.
2. *ABC* uchburchakning *CD* balandligi 4 *m*. Agar $\angle A = 45°$, $\angle B = 30°$ bo‘lsa, uchburchak tomonlarini toping.
3. Bir nuqtaga kattaligi bir xil bo‘lgan ikkita kuch qo‘yilgan. Agar bu kuchlar yo‘nalishlari orasidagi burchak 60° va kuchlarning teng ta’sir etuvchisi 150 *kg* bo‘lsa, bu kuchlar kattaligini toping.
4. Uchburchakning ikki tomoni 7 *dm* va 11 *dm*, uchinchi tomoniga tushirilgan medianasi esa 6 *dm*. Uchburchakning uchinchi tomonini toping.
5. Tomonlari 6 *sm* va 8 *sm* bo‘lgan parallelogrammning bir diagonali 12 *sm* bo‘lsa, uning ikkinchi diagonalini toping.
6. Uchburchakning 18 *sm* ga teng tomoni qarshisidagi burchagi 60° ga teng. Uchburchakka tashqi chizilgan aylana radiusini toping.
7. Teng yonli trapetsiyaning kichik asosi yon tomoniga teng, katta asosi esa 20 *sm*. Agar trapetsiyaning bir burchagi 120° bo‘lsa, uning perimetrini toping.

37 UCHBURCHAKLARNI YECHISHNING AMALIYOTDA QO‘LLANISHI







1. Balandlikni o‘lchash. Aytaylik, nimaningdir (masalan, daraxtning) AH balandligini o‘lchash zarur bo‘lsin (*1-rasm*).

a) Buning uchun B nuqtani belgilaymiz va BH masofa a ni va HBA burchak α ni o‘lchaymiz. Unda, to‘g‘ri burchakli ABH uchburchakda

$$AH = BH\,\mathrm{tg}\alpha = a\,\mathrm{tg}\alpha.$$

b) Agar balandlikning asosi H nuqta borib bo‘lmaydigan nuqta bo‘lsa (*2-rasm*), yuqoridagi usul bilan AH balandlikni aniqlay olmaymiz. Unda quyidagicha yo‘l tutamiz:

1) H nuqta bilan bir to‘g‘ri chiziqda yotgan B va C nuqtalarni belgilaymiz;

2) BC masofani o‘lchab a ni topamiz;

3) ABH va ACH burchaklarni o‘lchab $\angle ABH = \alpha$ va $\angle ACH = \beta$ larni topamiz;

4) ABC uchburchakka sinuslar teoremasini qo‘llasak ($\angle BAC = \alpha - \beta$)

$$\frac{AB}{\sin\beta} = \frac{a}{\sin(\alpha-\beta)}, \quad \text{ya'ni } AB = \frac{a\sin\beta}{\sin(\alpha-\beta)}.$$

5) to‘g‘ri burchakli ABH uchburchakda AH balandlikni topamiz:

$$AH = AB\sin\alpha = \frac{a\sin\alpha\cdot\sin\beta}{\sin(\alpha-\beta)}.$$

2. Borib bo‘lmaydigan nuqtagacha bo‘lgan masofani hisoblash. Aytaylik, A nuqtadan borib bo‘lmaydigan B nuqtagacha bo‘lgan masofani hisoblash kerak (*3-rasm*). Bu masalani uchburchaklarning o‘xshashlik alomatlaridan foydalanib javobini topganimizni eslatib o‘tamiz. Endi bu masalani sinuslar teoremasidan foydalanib yechamiz.

1) A va B nuqtalardan ko‘rinib turgan tekis joyda C nuqtani belgilaymiz.

2) AC masofani o‘lchaymiz: $AC=a$.

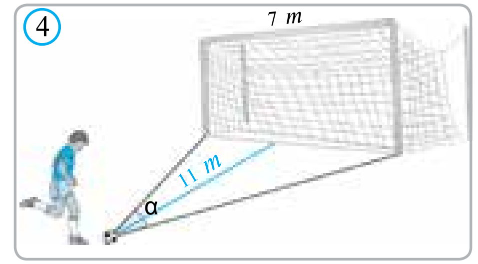
3) Asboblar yordamida ACB va BAC burchaklarni o‘lchaymiz: $\angle BAC=\alpha$, $\angle BAC=\gamma$.

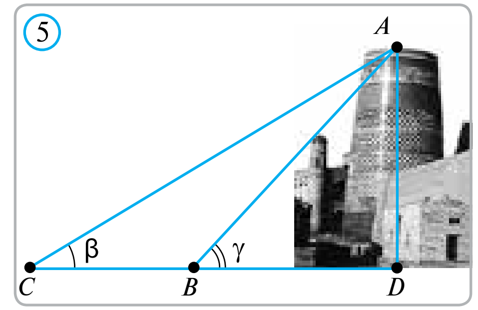
4) ABC uchburchakda $\angle B = 180° - \alpha - \gamma$ bo‘lgani uchun,

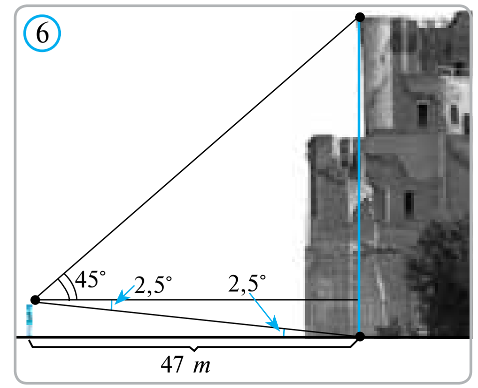
$$\sin B = \sin(180° - \alpha - \gamma) = \sin(\alpha + \gamma).$$

Sinuslar teoremasiga ko‘ra, $\frac{AB}{\sin C} = \frac{AC}{\sin B}$ yoki $AB = \frac{a\sin\gamma}{\sin(\alpha+\gamma)}$.

Savol, masala va topshiriqlar







1. 1-rasmda $a = 12$ *m*, $\alpha = 42°$ bo‘lsa, daraxt balandligini hisoblang.
2. 2-rasmda $a = 8$ *m*, $\alpha = 43°$, $\beta = 32°$ bo‘lsa, daraxt balandligini hisoblang.
3. 3-rasmda $a = 60$ *m*, $\alpha = 62°$, $\gamma = 44°$ bo‘lsa, AB masofani toping.
4. Futbol o‘yinida 11 metrlik jarima to‘pini darvozaga yo‘naltirish burchagi α ni toping (*4-rasm*). Darvozaning kengligi 7 *m*.
5. 5-rasmda Xiva shahridagi Kaltaminor tasvirlangan. Agar $\beta=45°$, $\gamma=24°$, $BC=50$ *m* bo‘lsa, Kaltaminor balandligini toping.
6. Sayohatchi Shahrisabz shahridagi Oqsaroyni undan 47 *m* masofada tomosha qilyapti (*6-rasm*). Agar u Oqsaroy asosini gorizontga nisbatan 2,5° ga teng burchak ostida, tepa qismini esa 45° ga teng burchak ostida ko‘rayotgan bo‘lsa, Oqsaroy balandligini toping.
7. Uchta yo‘l ABC uchburchakni tashkil qiladi. Bu uchburchakda $\angle A = 20°$, $\angle B = 150°$. A nuqtadan yo‘lga chiqqan haydovchi C nuqtaga imkon boricha tezroq yetib bormoqchi. AC va CB yo‘llar toshloq, AB asfalt yo‘l bo‘lib, asfalt yo‘lda toshloq yo‘lga qaraganda 2 baravar tezroq harakatlanish mumkin. Haydovchiga qaysi yo‘ldan yurishni maslahat berasiz?

Qiziqarli masala

Pifagor teoremasining yana bir “isboti”

To‘g‘ri burchakli ABC uchburchakda $a = c\sin\alpha$, $b = c\cos\alpha$. Bu ikki tenglikni kvadratga oshirib, hadma-had qo‘shsak va $\sin^2\alpha + \cos^2\alpha = 1$ ekanligini hisobga olsak,

$$a^2 + b^2 = c^2\sin^2\alpha + c^2\cos^2\alpha = c^2(\sin^2\alpha + \cos^2\alpha) = c^2.$$

Demak, $a^2 + b^2 = c^2$. Bu “isbot” mantiqan noto‘g‘ri ekanligini isbotlang.

38–39 II BOBGA DOIR QO‘SHIMCHA MASALALAR VA MA’LUMOTLAR

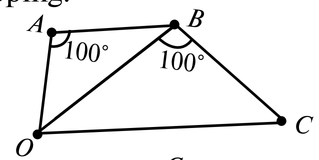
I. Testlar

1. **Tomonlari *a*, *b*, *c*, mos burchaklari α, β, γ, yuzi *S* bo‘lgan uchburchak uchun qaysi tenglik noto‘g‘ri?**
A. $a^2=b^2+c^2-2bc\cos\alpha$; B. $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$;
D. $S=\frac{1}{2}ab\sin\gamma$; E. $S=\frac{1}{2}ab\sin\alpha$.
2. **Noto‘g‘ri tenglikni toping:**
A. $\sin^2\alpha+\cos^2\alpha=1$; B. $\sin(180°-\alpha)=\sin\alpha$;
D. $\cos(180°-\alpha)=\cos\alpha$; E. $\sin(90°-\alpha)=\cos\alpha$.
3. **Uchburchakning uchta tomoni ma’lum bo‘lsa, qaysi teoremadan foydalanib uning burchaklarini topish mumkin?**
A. Sinuslar teoremasi; B. Kosinuslar teoremasi;
D. Fales teoremasi; E. Geron formulasi.
4. **Uchburchakning bir burchagi 137° ga, ikkinchi burchagi 15° ga teng. Agar bu uchburchakning katta tomoni 22 ga teng bo‘lsa, uning kichik tomonini toping.**
A. 8,3; B. 9,3; D. 3,8; E. 6,5.
5. **Uchburchakning 14 va 19 ga teng bo‘lgan tomonlari orasidagi burchagi 26°. Shu uchburchakning uchinchi tomonini toping.**
A. 1,2; B. 5,4; D. 6,9; E. 19,7.
6. **Agar ikki vektorning uzunliklari $|\bar{a}|=2$, $|\bar{b}|=5$ va ular orasidagi burchak 45° bo‘lsa, $\bar{a}$ va $\bar{b}$ vektorlarning skalyar ko‘paytmasini toping.**
A. 52; B. 32 D. 102; E. 2.
7. **$\bar{a}(4; -1)$ va $\bar{b}(2; 3)$ vektorlarning skalyar ko‘paytmasini toping.**
A. 5; B. 3; D. 4; E. 9.
8. **$\bar{a}(-\frac{1}{2}; \frac{\sqrt{3}}{2})$ va $\bar{b}(\sqrt{3}; 1)$ vektorlar orasidagi burchakni toping.**
A. 30°; B. 60°; D. 90°; E. 45°.
9. **Uchburchak burchaklarining nisbati 3:2:1 kabi bo‘lsa, uning tomonlari nisbatini toping.**
A. 3:2:1; B. 1:2:3; D. $2:\sqrt{3}:1$; E. $\sqrt{3}:\sqrt{2}:1$.
10. **Tomoni 3 *sm* bo‘lgan muntazam uchburchakka tashqi chizilgan aylana radiusini toping.**
A. $\sqrt{3}$; B. $\frac{\sqrt{3}}{3}$ D. $2\sqrt{3}$; E. $\frac{\sqrt{3}}{2}$

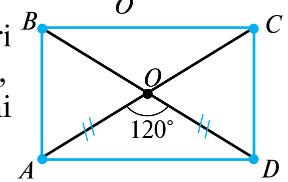
II. Masalalar

1. ABC uchburchakka $AB=6$ *sm*, $\angle A=60°$, $\angle B=75°$. BC tomonni hamda ABC uchburchakka tashqi chizilgan aylananing radiusini toping.

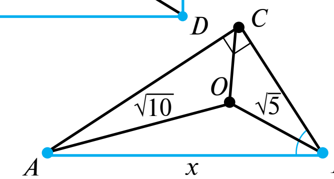
2. Tomonlari 5 *sm*, 6 *sm* va 10 *sm* bo‘lgan uchburchak burchaklarining kosinuslarini toping.
3. ABC uchburchakda $\angle B=60°$, $AB=6$ *sm*, $BC=4$ *sm*. AC tomonni hamda ABC uchburchakka tashqi chizilgan aylananing radiusini toping.
4. Tomonlari 51 *sm*, 52 *sm* va 53 *sm* bo‘lgan uchburchakka tashqi chizilgan aylananing radiusini toping.
5. Uchburchakning ikkita tomoni 14 *sm* va 22 *sm*, uchinchi tomoniga o‘tkazilgan medianasi esa 12 *sm*. Uchburchakning uchinchi tomonini toping.
6. Parallelogrammning diagonallari 4 *sm*, $4\sqrt{2}$ *sm* va ular orasidagi burchak 45°. Parallelogrammning a) yuzini; b) perimetrini; d) balandliklarini toping.
7. Tomonlari 3 va 5 bo‘lgan parallelogrammning bir diagonali 4 ga teng. Uning ikkinchi diagonalini toping.
8. Tomonlari a) 2, 2 va 2,5; b) 24, 7 va 25; d) 9, 5 va 6 bo‘lgan uchburchak turini aniqlang.
9. Parallelogrammning tomonlari $7\sqrt{3}$ va 6 *sm*. Agar uning o‘tmas burchagi 120° bo‘lsa, uning yuzini toping.
10. ABC uchburchakning AB, BC tomonlarida N, K nuqtalar olingan. Unda $BN=2AN$, $3BK=2KC$. Agar $AB=3$, $BC=5$, $CA=6$ bo‘lsa, NK kesmani toping.
11. ABC uchburchakda $\angle A=30°$, $BC=7$ *sm*. Uchburchakka tashqi chizilgan aylana radiusini toping.
12. ABC uchburchakning BE bissektrisasi o‘tkazilgan. E nuqtadan BC tomonga EF perpendikular tushirilgan. Agar $EF=3$, $\angle A=30°$ bo‘lsa, AE ni toping.
13. $ABCD$ to‘g‘ri to‘rtburchak AD tomonining o‘rtasi N nuqtada. Agar $AB=3$, $BC=6$ bo‘lsa, $\overline{NB}\cdot\overline{NC}$ skalyar ko‘paytmani toping.
14. $\bar{a}(2;x)$, $\bar{b}(-4;1)$ bo‘lib, $\bar{a}+\bar{b}$ va $\bar{b}$ vektorlar perpendikular. x ni toping.
15. $\bar{m}(7;3)$ va $\bar{n}(-2;-5)$ vektorlar orasidagi burchakni toping.
16. Rasmda berilganlardan foydalanib, rasmdagi eng katta kesmani aniqlang.



17. $ABCD$ to‘g‘ri to‘rtburchakning diagonallari O nuqtada kesishadi. Agar $AO=12$ *sm*, $\angle AOD=120°$ bo‘lsa, to‘rtburchak perimetrini toping.



18. To‘g‘ri burchakli ABC uchburchak bissektrisalari O nuqtada kesishadi ($\angle C=90°$). Agar $OA=\sqrt{10}$, $OB=\sqrt{5}$ bo‘lsa, AB gipotenuzani toping.



III. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi)

1. Tomonlari a=45, b=70, c=95 bo‘lgan uchburchakning eng katta burchagini toping.
2. Uchburchakda b=5, α=30°, β=50° bo‘lsa, uchburchakni yeching.
3. PKH uchburchakda PK=6, KH=5, $\angle PKH$=100°. HF mediana uzunligini va PFH uchburchak yuzini toping.
4. *(Qo‘shimcha)*. Uchburchakda $a=\sqrt{3}$, b=1, α=135° bo‘lsa, β burchakni toping.

Tarixiy lavhalar. ***Sinus haqida***

Sinus haqidagi ma’lumot dastlab IV–V asrlardagi hind astronomlarining asarlarida uchraydi.

O‘rta Osiyolik olimlar al-Xorazmiy, Beruniy, Ibn Sino, Abdurahmon al-Haziniy (XII asr) sinus uchun ***«al-jayb»*** atamasini ishlatishgan.

Hozirgi sinus belgisini Simpson, Eyler, D’alamber, Lagranj (XVII asr) va boshqalar qo‘llagan.

«Kosinus» atamasi lotincha «komplimenti sinus» atamasining qisqartirilgani, u «qo‘shimcha sinus», aniqrog‘i «qo‘shimcha yoyning sinusi» demakdir.

Kosinuslar teoremasini yunonlar ham bilgan, uning isboti Yevklidning “Negizlar” asarida keltirilgan. Sinuslar teoremasining o‘ziga xos isbotini Abu Rayhon Beruniy bayon qilgan.

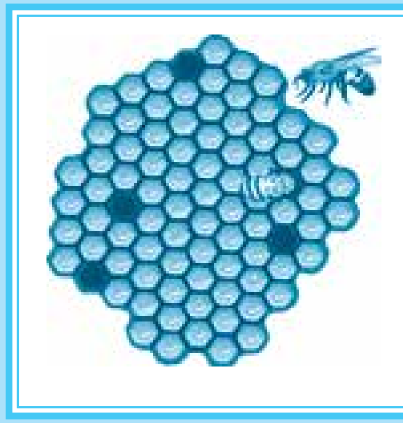
Tarixiy lavhalar. Beruniy (to‘liq ismi – Abu Rayhon Muhammad ibn Ahmad) (973 — 1048) – o‘rta asrning buyuk qomusiy olimi. U Xorazm o‘lkasining Qiyot shahrida tug‘ilgan. Qiyot Amudaryoning o‘ng qirg‘og‘i – hozirgi Beruniy shahrining o‘rnida bo‘lgan, u yaqin vaqtlargacha Shabboz deb atalgan. Beruniyning matematika va fanning boshqa sohalariga qo‘shgan hissasini yozib qoldirgan 150 dan ortiq asaridan ham ko‘rish mumkin. Ulardan eng yiriklari – “Hindiston”, “Yodgorliklar”, “Mas’ud qonunlari”, “Geodeziya”, “Mineralogiya” va “Astronomiya”.


Beruniy (973 — 1048)

Beruniyning shoh asari “Mas’ud qonunlari”, asosan, astronomiyaga oid bo‘lsa ham, uning matematikaga oid anchagina kashfiyotlari shu asarda bayon etilgan.

Bu asarda Beruniy ikki burchak yig‘indisi va ayirmasining sinuslari, ikkilangan va yarim burchakning sinuslari haqidagi teoremalar bilan teng kuchli bo‘lgan vatarlar haqidagi teoremalarni isbotlagan, ikki gradusli yoyning vatarlarini hisoblab topgan, sinuslar va tangenslar jadvallarini tuzgan, **sinuslar teoremasini** o‘ziga xos usulda isbotlagan.

III BOB



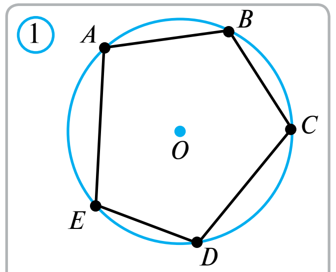
AYLANA UZUNLIGI VA DOIRA YUZI

Ushbu bobni o‘rganish natijasida siz quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

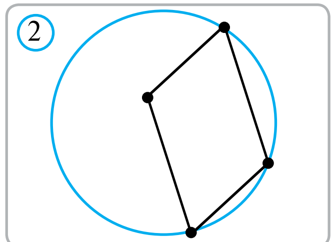
Bilimlar:
- √ ko‘pburchakka tashqi va ichki chizilgan aylanalarning xossalarini bilish;
- √ muntazam ko‘pburchaklarning xossalarini bilish;
- √ muntazam ko‘pburchakning yuzini hisoblash formulalarini bilish;
- √ aylana va uning yoyi uzunligini hisoblash formulalarini bilish;
- √ doira va uning bo‘laklari yuzini topish formulalarini bilish;
- √ burchakning radian o‘lchovini bilish.

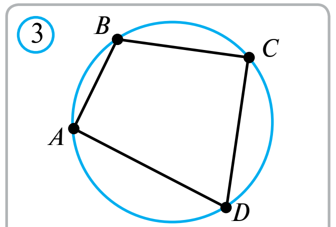
Amaliy ko‘nikmalar:
- √ muntazam ko‘pburchaklarni tasvirlay olish;
- √ muntazam ko‘pburchakka tashqi va ichki chizilgan aylanalarning radiuslarini topa olish;
- √ aylana va yoy uzunligini hisoblay olish;
- √ doira va uning bo‘laklari yuzini hisoblay olish.

40 AYLANAGA ICHKI CHIZILGAN KO‘PBURCHAK

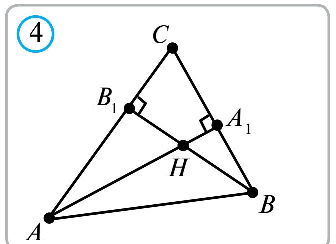


Aylanaga ichki chizilgan beshburchak yoki beshburchakka tashqi chizilgan aylana.





$\angle A+\angle C=180°$
$\angle B+\angle D=180°$



Ta’rif. Agar ko‘pburchakning barcha uchlari aylanada yotsa, bu ko‘pburchak aylanaga **ichki chizilgan**, aylana esa ko‘pburchakka **tashqi chizilgan** deyiladi *(1-rasm)*.

Istalgan uchburchakka tashqi aylana chizish mumkinligi va bu aylana markazi uchburchak tomonlarining o‘rta perpendikularlari kesishgan nuqtada yotishini 8-sinfda o‘rgangansiz.

Agar ko‘pburchak burchaklari soni uchtadan ortiq bo‘lsa, ko‘pburchakka har doim ham tashqi aylana chizib bo‘lavermaydi. Masalan, to‘g‘ri to‘rtburchakdan farqli parallelogramm uchun tashqi chizilgan aylana mavjud emas *(2-rasm)*.

8-sinf geometriya kursidan ma’lumki, to‘rtburchakka qarama-qarshi burchaklari yig‘indisi 180° ga teng bo‘lganda va faqat shu holda unga tashqi aylana chizish mumkin *(3-rasm)*.

1-masala. O‘tkir burchakli ABC uchburchakning AA_1 va BB_1 balandliklari H nuqtada kesishadi. A_1HB_1C to‘rtburchak aylanaga ichki chizilgan ekanligini isbotlang.

Yechilishi. $AA_1 \perp BC$ va $BB_1 \perp AC$ bo‘lgani uchun (*4-rasm*)

$$\angle HB_1C=\angle HA_1C=90°.$$

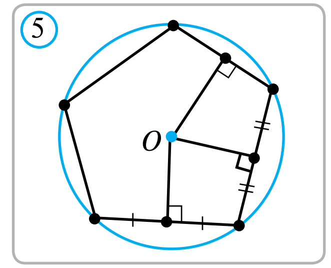
Unda $\angle HB_1C+\angle HA_1C=180°$. To‘rtburchak ichki burchaklari yig‘indisi 360° bo‘lgani uchun:

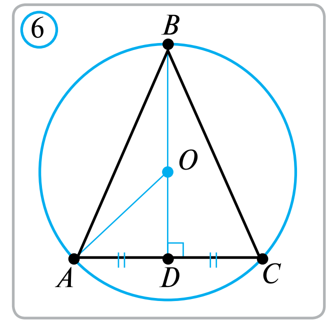
$$\angle B_1CA_1+\angle B_1HC=180°.$$

Demak, A_1HB_1C to‘rtburchakka tashqi aylana chizish mumkin.

Aylanaga ichki chizilgan ko‘pburchak uchlari aylana markazidan teng uzoqlikda yotgani uchun aylana markazi ko‘pburchak tomonlarining o‘rta perpendikularlarida yotadi *(5-rasm)*. Demak, aylanaga ichki chizilgan ko‘pburchak tomonlarining o‘rta perpendikularlari bir nuqtada kesishishi shart.

2-masala. Asosiga tushirilgan balandligi 16 *sm* bo‘lgan teng yonli uchburchak radiusi 10 *sm* bo‘lgan aylanaga ichki chizilgan. Uchburchak tomonlarini toping.





Yechilishi. ABC uchburchakka tashqi chizilgan aylana markazi O nuqta AC tomonning o‘rta perpendikulari bo‘lgan BD balandlikda yotadi (*6-rasm*). Unda,

$$OD=BD-OB=16-10=6\ (sm)$$

bo‘ladi va Pifagor teoremasiga ko‘ra,

$$AD=\sqrt{OA^2-OD^2}=\sqrt{10^2-6^2}=8\ (sm),\ AC=2AD=16(sm).$$

Shuningdek, to‘g‘ri burchakli ABD uchburchakda

$$AB=\sqrt{AD^2+BD^2}=\sqrt{8^2+16^2}=8\sqrt{5}\ (sm).$$

Javob: $8\sqrt{5}$ *sm*, $8\sqrt{5}$ *sm*, 16 *sm*.

Savol, masala va topshiriqlar

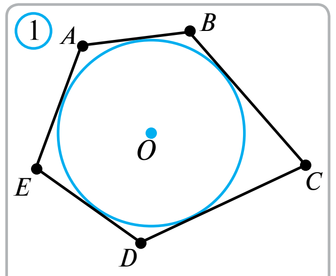
1. Agar ko‘pburchak aylanaga ichki chizilgan bo‘lsa, uning tomonlari o‘rta perpendikularlari bir nuqtada kesishishini isbotlang.
2. Qanday uchburchak aylanaga ichki chizilgan bo‘lishi mumkin? To‘rtburchak-chi?
3. $ABCDE$ beshburchak aylanaga ichki chizilgan bo‘lsa, $\angle ACB=\angle AEB$ bo‘lishini isbotlang.
4. Katetlari 16 *sm* va 12 *sm* bo‘lgan to‘g‘ri burchakli uchburchakka tashqi chizilgan aylana radiusini toping.
5. Radiusi 25 *sm* bo‘lgan aylanaga bir tomoni 14 *sm* bo‘lgan to‘g‘ri to‘rtburchak ichki chizilgan. To‘g‘ri to‘rtburchak yuzini toping.
6. Radiusi 10 *sm* bo‘lgan aylanaga ichki chizilgan a) teng tomonli uchburchak; b) kvadrat; d) teng yonli to‘g‘ri burchakli uchburchak tomonlarini toping.
7. Tomonlari 16 *sm*, 10 *sm* va 10 *sm* bo‘lgan uchburchakka tashqi chizilgan aylana radiusini toping.
8. Aylanaga ichki chizilgan $ABCDEF$ oltiburchakda $\angle BAF+\angle AFB=90°$ bo‘lsa, aylana markazi AF tomonda yotishini isbotlang.
9. Istalgan teng yonli trapetsiya aylanaga ichki chizilishi mumkinligini isbotlang.
10. Teng yonli trapetsiya chizing. Unga tashqi chizilgan aylana yasang.

Qiziqarli masala

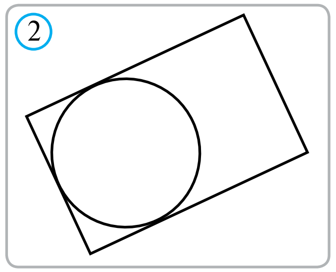
O‘n olti yoshli Galua (E.Galua — fransuz matematigi, 1811—1832) kollejda o‘qib yurgan chog‘larida, unga o‘qituvchisi bir soat ichida uchta masalani yechib berishni so‘ragan. Galua yechimi uncha oson bo‘lmagan bu masalalarni 15 daqiqada yechib, hammani hayron qoldirgan. Mana, shu masalalardan biri. Uni siz ham yechib ko‘ring-chi!

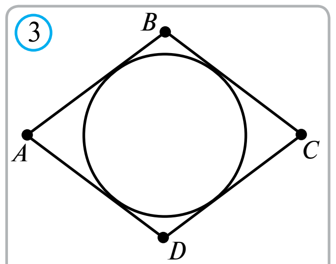
Masala. Aylanaga ichki chizilgan to‘rtburchakning to‘rtta tomoni a, b, c va d ga teng. Uning diagonallarini toping.

41 AYLANAGA TASHQI CHIZILGAN KO‘PBURCHAK

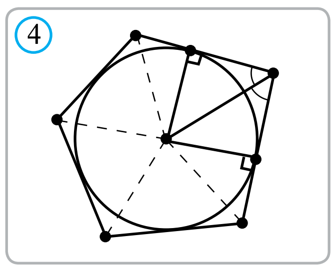


Aylanaga tashqi chizilgan ABCDE beshburchak yoki ABCDE beshburchakka ichki chizilgan aylana.





AB + CD = AD + BC



Ta’rif. Agar ko‘pburchakning barcha tomonlari aylanaga urinsa, u holda ko‘pburchak aylanaga **tashqi chizilgan**, aylana esa ko‘pburchakka **ichki chizilgan** deyiladi *(1-rasm)*.

Istalgan uchburchakka ichki aylana chizish mumkinligi va bu aylana markazi uchburchak bissektrisalari kesishgan nuqtada ekanligi bilan 8-sinfda tanishgansiz.

Agar ko‘pburchak burchaklari soni uchtadan ortiq bo‘lsa, bu ko‘pburchakka har doim ham ichki aylana chizib bo‘lavermaydi. Masalan, kvadratdan farqli to‘g‘ri to‘rtburchakka ichki aylana chizib bo‘lmaydi *(2-rasm)*.

Yana 8-sinf geometriya kursidan ma’lumki, to‘rtburchakka faqat va faqat qarama-qarshi tomonlari yig‘indisi teng bo‘lganda ichki aylana chizish mumkin *(3-rasm)*.

Aylanaga tashqi chizilgan ko‘pburchak tomonlari aylanaga uringani uchun aylana markazi shu ko‘pburchak burchaklari bissektrisasida yotadi *(4-rasm)*. Demak, aylanaga tashqi chizilgan ko‘pburchak burchaklarining bissektrisalari bir nuqtada kesishadi.

Teorema. **Agar r radiusli aylanaga tashqi chizilgan ko‘pburchakning yuzi S, yarim perimetri p bo‘lsa, $S=pr$ bo‘ladi.**

Isbot. Teorema isbotini aylanaga tashqi chizilgan ABCDEF oltiburchak uchun keltiramiz. Aylana markazi O nuqtani ko‘pburchak uchlari bilan tutashtirib, ko‘pburchakni uchburchaklarga ajratamiz. Bu uchburchaklarning balandliklari r ga teng *(5-rasm)*. Unda,

$$S=S_{AOB}+S_{BOC}+...+S_{FOA}=\tfrac{1}{2}AB\cdot r+\tfrac{1}{2}BC\cdot r+...+$$
$$+\tfrac{1}{2}FA\cdot r=\frac{AB+BC+...+FA}{2}\cdot r=pr.$$

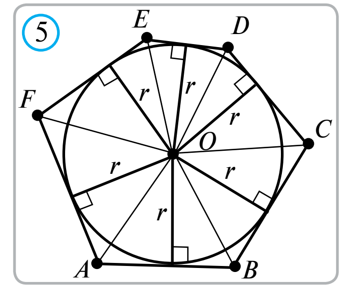
Teorema isbotlandi.

Masala. Aylanaga tashqi chizilgan to‘rtburchakning yuzi 21 sm^2 ga, perimetri esa 7 *sm* ga teng. Aylana radiusini toping.

Yechilishi. $S=pr$ formulaga ko‘ra,

$$r=\frac{S}{p}=\frac{21}{3,5}=6\ (sm).$$

Javob: 6 *sm*.



Savol, masala va topshiriqlar

1. Tomoni 6 *sm* bo‘lgan a) teng tomonli uchburchakka; b) kvadratga ichki chizilgan aylana radiusini toping.

2. Radiusi 5 *sm* bo‘lgan aylanaga tashqi chizilgan ko‘pburchak yuzi 18 sm^2. Ko‘pburchak perimetrini toping.

3. 6-rasmdagi to‘rtburchaklarning perimetrini toping.

4. 7-rasmdagi ma’lumotlar asosida so‘ralgan kesmani toping.

5. Aylanaga tashqi chizilgan parallelogramm romb bo‘lishini isbotlang.

6. To‘g‘ri burchakli uchburchakka ichki chizilgan aylana radiusi katetlar yig‘indisi bilan gipotenuza ayirmasining yarmiga tengligini isbotlang.

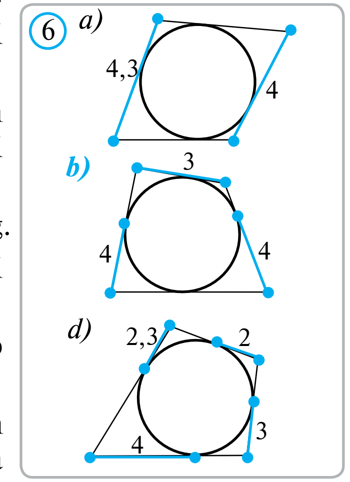
7. Aylanaga tashqi chizilgan teng yonli trapetsiyaning o‘rta chizig‘i uning yon tomoniga teng ekanligini isbotlang.

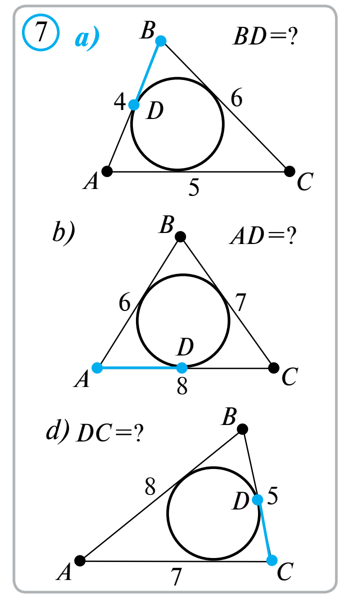
8. Asoslari 9 *sm* va 16 *sm* bo‘lgan teng yonli trapetsiya aylanaga tashqi chizilgan. Aylana radiusini toping.

9*. *ABCD* to‘rtburchak *O* markazli aylanaga tashqi chizilgan. *AOB* va *COD* uchburchaklar yuzlarining yig‘indisi to‘rtburchak yuzining yarmiga tengligini isbotlang.

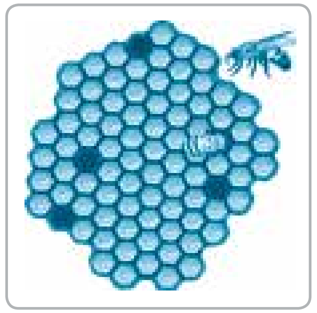
10*. Aylanaga tashqi chizilgan trapetsiyaning asoslari a va b bo‘lsa, uning balandligi $\sqrt{ab}$ ga teng ekanligini isbotlang.

11*. Uchlari *ABCD* to‘rtburchak bissektrisalarining kesishgan nuqtalarda bo‘lgan *EFPQ* to‘rtburchakka tashqi aylana chizish mumkinligini isbotlang.





42 MUNTAZAM KO‘PBURCHAKLAR

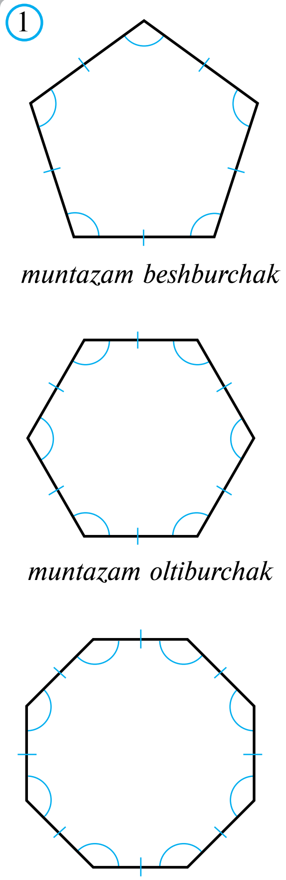


Faollashtiruvchi mashq

1. Qanday shakllar ko‘pburchak deyiladi?
2. Ko‘pburchak burchaklari, qo‘shni tomonlari, diagonallari deb nimaga aytiladi?
3. Qavariq ko‘pburchak deb qanday ko‘pburchakka aytiladi?
4. Qavariq ko‘pburchak ichki burchaklari yig‘indisi haqidagi teoremani ayting.

Ta’rif. Hamma tomonlari teng va hamma burchaklari teng bo‘lgan qavariq ko‘pburchak ***muntazam ko‘pburchak*** deyiladi.

Teng tomonli uchburchak, kvadrat muntazam ko‘pburchakka misol bo‘ladi. 1-rasmda muntazam beshburchak, oltiburchak va sakkizburchaklar tasvirlangan.



1

muntazam beshburchak

muntazam oltiburchak

muntazam sakkizburchak

Teorema. **Muntazam *n* burchakning har bir burchagi $\frac{n-2}{n}\cdot 180°$ ga teng.**

Isbot. Muntazam *n* burchakning burchaklari yig‘indisi $(n-2)\cdot 180°$ ga teng (8-sinf). Demak, uning har bir burchagi $\frac{n-2}{n}\cdot 180°$ga teng. ***Teorema isbotlandi.***

Masala. Muntazam $A_1A_2A_3A_4A_5$ beshburchakda A_1A_3 va A_1A_4 diagonallari teng ekanligini ko‘rsating (*2-rasm*).

$A_1A_2A_3A_4A_5$ — *muntazam beshburchak* ⇒ $A_1A_3 = A_1A_4$

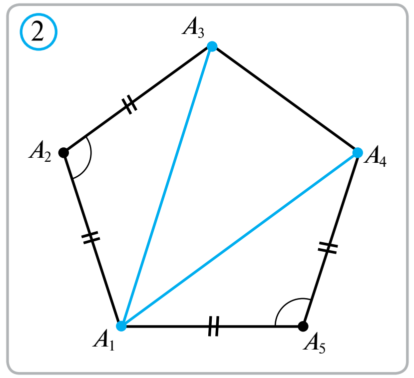
Yechilishi. Uchburchaklar tengligining *TBT* alomatiga ko‘ra, $A_1A_2A_3$ va $A_1A_5A_4$ uchburchaklar o‘zaro teng. Haqiqatan ham, muntazam ko‘pburchakning tomonlari teng va burchaklari teng bo‘lgani uchun,

$$A_1A_2 = A_1A_5,\ A_2A_3 = A_5A_4 \text{ va } \angle A_1A_2A_3 = \angle A_1A_5A_4.$$

Demak, $\Delta A_1A_2A_3 = \Delta A_1A_5A_4$. Bundan

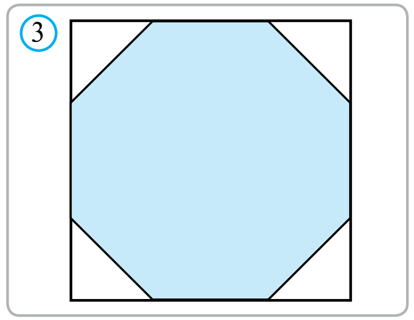
$A_1A_3 = A_1A_4$ ekanligi kelib chiqadi.

Natija. Muntazam beshburchakning barcha diagonallari o‘zaro teng.



Savol, masala va topshiriqlar

1. Muntazam bo‘lmagan ko‘pburchaklarga misollar ayting va nima uchun muntazam emasligini tushuntiring.
2. Quyidagi tasdiqlardan to‘g‘rilarini toping:
   a) barcha tomonlari teng bo‘lgan uchburchak muntazam bo‘ladi;
   b) barcha tomonlari teng to‘rtburchak muntazam bo‘ladi;
   d) barcha burchaklari teng to‘rtburchak muntazam bo‘ladi;
   e) barcha burchaklari teng romb muntazam bo‘ladi;
   f) barcha tomonlari teng to‘g‘ri to‘rtburchak muntazam bo‘ladi.
3. Agar a) $n=3$; b) $n=5$; d) $n=6$; e) $n=10$; f) $n=18$ bo‘lsa, muntazam *n* burchak burchaklarini toping.
4. Muntazam *n* burchakning tashqi burchagi nimaga teng bo‘ladi? Agar a) $n=3$; b) $n=5$; d) $n=6$; e) $n=10$; f) $n=12$ bo‘lsa, muntazam *n* burchakning tashqi burchagini toping.
5. Muntazam *n* burchakning har uchidan bittadan olingan tashqi burchaklari yig‘indisi 360° ga teng ekanligini isbotlang.
6. Agar muntazam ko‘pburchakning har bir burchagi a) 60°; b) 90°; d) 135°; e) 150° bo‘lsa, bu ko‘pburchak tomonlari sonini toping.
7. Muntazam *ABCDEF* oltiburchak berilgan.
   a) *AC* va *BD* diagonallar tengligini isbotlang.
   b) *ACE* — muntazam uchburchak bo‘lishini isbotlang.
   d) *AD*, *BE* va *CF* diagonallar o‘zaro tengligini isbotlang.
8. Tomoni 10 *sm* bo‘lgan muntazam a) beshburchakning; b) oltiburchakning; d) sakkizburchakning; e) o‘n ikkiburchakning; f) o‘n sakkizburchakning kichik diagonalini hisoblang.



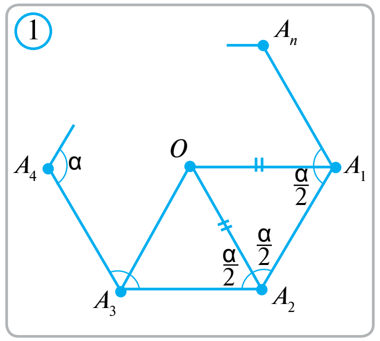
9. Muntazam to‘rtburchakning kvadrat bo‘lishini isbotlang.

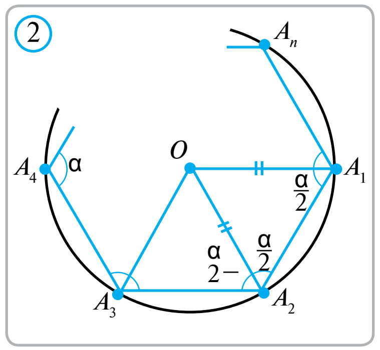
10*. Kvadratning tomoni *a* ga teng. Uning tomonlariga har bir uchidan boshlab diagonalining yarmiga teng kesmalar qo‘yildi. Natijada, 3-rasmda tasvirlangan sakkizburchak hosil bo‘ldi. Uning turini aniqlang va yuzini toping.

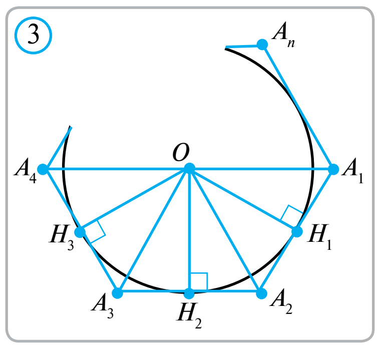
43 MUNTAZAM KO‘PBURCHAKKA ICHKI VA TASHQI CHIZILGAN AYLANALAR

Faollashtiruvchi mashq

1. Qanday ko‘pburchak aylanaga ichki chizilgan ko‘pburchak deyiladi?
2. Qanday ko‘pburchak aylanaga tashqi chizilgan ko‘pburchak deyiladi?
3. Istalgan ko‘pburchak aylanaga ichki (tashqi) chizilgan bo‘lishi mumkinmi?







Teorema. **Har qanday muntazam ko‘pburchakka ichki aylana ham, tashqi aylana ham chizish mumkin.**

Isbot. Aytaylik, $A_1A_2 \dots A_n$ — muntazam ko‘pburchak, O — A_1 va A_2 burchaklari bissektrisalarining kesishish nuqtasi bo‘lsin. Bu muntazam ko‘pburchakning burchagini α bilan belgilaylik.

1. $OA_1=OA_2=\ldots=OA_n$ ekanligini isbotlaymiz (*1-rasm*). Burchak bissektrisasining ta’rifiga ko‘ra,

$$\angle OA_1A_2=\angle OA_2A_1=\frac{\alpha}{2}.$$

Demak, A_1OA_2 — teng yonli uchburchak. Bundan, $OA_1=OA_2$ kelib chiqadi. ΔA_1A_2O va ΔA_3A_2O uchburchaklar tengligining TBT alomatiga ko‘ra teng, chunki $A_1A_2=A_3A_2$, A_2O — tomon umumiy hamda

$$\angle OA_1A_2=\angle OA_2A_1=\frac{\alpha}{2}.$$

Shuning uchun $OA_3=OA_1$. Xuddi shunday yo‘l tutib, $OA_4=OA_2$, $OA_5=OA_3$ va hokazo tengliklar o‘rinli bo‘lishi ko‘rsatiladi.

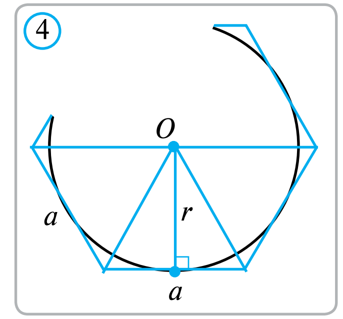
Shunday qilib, $OA_1=OA_2= \ldots =OA_n$, ya’ni markazi O va radiusi OA_1 bo‘lgan aylana ko‘pburchakka tashqi chizilgan aylanadan iborat bo‘ladi (*2-rasm*).

2. Yuqorida aytilganlarga ko‘ra, teng yonli A_1OA_2, A_2OA_3, ... A_nOA_1 uchburchaklar teng. Shuning uchun bu uchburchaklarning O uchidan tushirilgan balandliklari ham teng bo‘ladi (*3-rasm*):

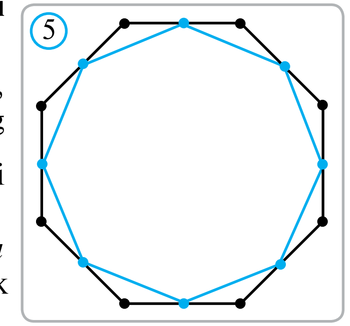
$$OH_1=OH_2= \ldots =OH_n.$$

Demak, O markazli va radiusi OH_1 kesmaga teng bo‘lgan aylana ko‘pburchakning barcha tomonlariga urinadi. Ya’ni, bu aylana ko‘pburchakka ichki chizilgan aylana bo‘ladi. ***Teorema isbotlandi.***

Natija. ***Muntazam ko‘pburchakka ichki chizilgan va tashqi chizilgan aylanalarning markazlari bitta nuqtada bo‘ladi.***



Bu nuqta muntazam ko‘pburchakning **markazi** deyiladi. Muntazam ko‘pburchak markazini uning ikki qo‘shni uchlari bilan tutashtiruvchi nurlardan iborat burchak (1-rasmdagi A_1OA_2, A_2OA_3 ... burchaklar) uning **markaziy burchagi** deyiladi. Muntazam ko‘pburchakning markazidan tomonlariga tushirilgan perpendikularlar (3-rasmdagi OH_1, OH_2, ... kesmalar) uning **apofemasi** deyiladi.



Masala. Agar muntazam n burchakning tomoni a, unga ichki chizilgan aylananing radiusi r bo‘lsa, uning S yuzini $S=\frac{1}{2}nar$ formula bilan hisoblash mumkinligini isbotlang (4-rasm).

Yechilishi. Ko‘pburchakning yarim perimetri $p=\frac{1}{2}na$ bo‘lgani uchun, aylanaga tashqi chizilgan ko‘pburchak yuzini topish formulasi $S=pr$ ga ko‘ra, $S=\frac{1}{2}nar$ bo‘ladi.

Savol, masala va topshiriqlar

1. Yuzi 36 sm^2 bo‘lgan kvadratga ichki va tashqi chizilgan aylanalar radiuslarini toping.
2. Perimetri 18 *sm* bo‘lgan muntazam uchburchakka ichki va tashqi chizilgan aylanalar radiuslarini hisoblang.
3. Muntazam oltiburchakka tashqi chizilgan aylana radiusi uning tomoniga teng bo‘lishini isbotlang.
4. Muntazam ko‘pburchak tomonlarining o‘rtalari boshqa bir muntazam ko‘pburchak uchlari bo‘lishini isbotlang (*5-rasm*).
5. Muntazam uchburchakka ichki chizilgan aylana radiusi tashqi chizilgan aylana radiusidan ikki marta kichik ekanligini isbotlang.

6*. Muntazam ko‘pburchakning istalgan ikkita tomonining o‘rta perpendikularlari bir nuqtada kesishishi yoki bir to‘g‘ri chiziqda yotishini isbotlang.

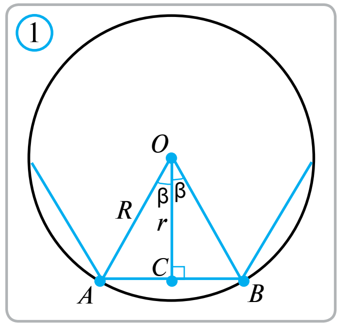
7. Aylanaga ichki chizilgan muntazam ko‘pburchakning bir tomoni aylanadan a) 60°; b) 30°; d) 36°; e) 18°; f) 72° ga teng yoy ajratadi. Ko‘pburchakning nechta tomoni bor?
8. Qog‘ozdan oltita teng muntazam uchburchak qirqib oling. Ulardan foydalanib, muntazam oltiburchak yasang. Tomonlari teng bo‘lgan muntazam oltiburchak va uchburchak yuzlari nisbatini toping.

44 MUNTAZAM KO‘PBURCHAKNING TOMONI BILAN TASHQI VA ICHKI CHIZILGAN AYLANALAR RADIUSLARI ORASIDAGI BOG‘LANISH

Faollashtiruvchi mashq

To‘g‘ri burchakli uchburchak o‘tkir burchagining a) sinusi; b) kosinusi; d) tangensi deb nimaga aytiladi?

Tomoni a_n ga teng bo‘lgan muntazam n burchakka tashqi chizilgan aylananing R radiusi va ichki chizilgan aylananing r radiusini hisoblash uchun formulalar topamiz. Buning uchun to‘g‘ri burchakli ACO uchburchakdan foydalanamiz. Bu yerda O — ko‘pburchakning markazi, C — ko‘pburchakning AB tomoni o‘rtasi (*1-rasm*). Unda,



$$\beta=\angle AOC=\frac{1}{2}\angle AOB=\frac{1}{2}\cdot\frac{360^\circ}{n}=\frac{180^\circ}{n};$$

$$R=OA=\frac{AC}{\sin\beta}=\frac{a_n}{2\sin\frac{180^\circ}{n}};\quad r=OC=\frac{AC}{\mathrm{tg}\,\beta}=\frac{a_n}{2\,\mathrm{tg}\frac{180^\circ}{n}};$$

$$r=OC=OA\cdot\cos\beta=R\cos\frac{180^\circ}{n}.$$

Bu formulalardan foydalanib, ayrim muntazam ko‘pburchaklar tomoni, ichki va tashqi chizilgan aylanalar radiuslari orasidagi bog‘lanishlarni topamiz.

1. Muntazam uchburchak uchun ($n=3$):

$$\beta=\frac{180^\circ}{3}=60^\circ;\quad R=\frac{a_3}{2\sin 60^\circ}=\frac{a_3}{\sqrt{3}};\quad r=\frac{a_3}{2\,\mathrm{tg}\,60^\circ}=\frac{a_3}{2\sqrt{3}};\quad R=2r.$$

2. Kvadrat uchun ($n=4$):

$$\beta=\frac{180^\circ}{4}=45^\circ;\quad R=\frac{a_4}{2\sin 45^\circ}=\frac{a_4}{\sqrt{2}};\quad r=\frac{a_4}{2\,\mathrm{tg}\,45^\circ}=\frac{a_4}{2};\quad R=r\sqrt{2}.$$

3. Muntazam oltiburchak uchun ($n=6$):

$$\beta=\frac{180^\circ}{6}=60^\circ;\quad R=\frac{a_6}{2\sin 30^\circ}=a_6;\quad r=\frac{a_6}{2\,\mathrm{tg}\,30^\circ}=\frac{a_6\sqrt{3}}{2};\quad R=\frac{2r}{\sqrt{3}}.$$

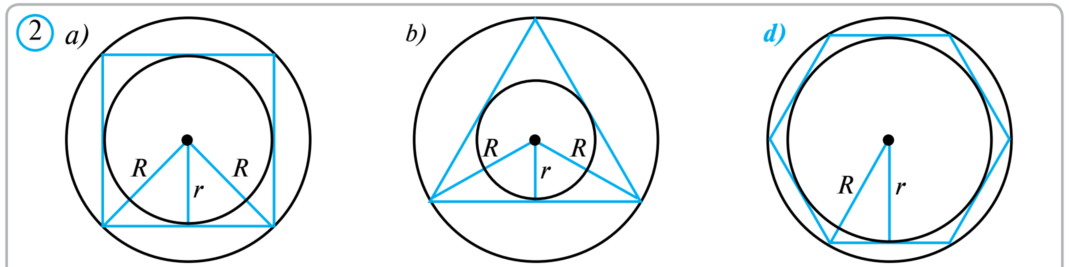
Masala. Muntazam n burchakning a_n tomonini shu ko‘pburchakka tashqi chizilgan aylananing R radiusi va ichki chizilgan aylananing r radiusi orqali ifodalang.

Yechilishi. $R=\frac{a_n}{2\sin\frac{180^\circ}{n}}$ va $r=\frac{a_n}{2\,\mathrm{tg}\frac{180^\circ}{n}}$ formulalardan $a_n=2R\sin\frac{180^\circ}{n}$ va $a_n=2r\,\mathrm{tg}\frac{180^\circ}{n}$ formulalarni hosil qilamiz. Xususan, $n=3$ bo‘lsa, $a_3=R\sqrt{3}=2r\sqrt{3}$.

Savol, masala va topshiriqlar

1. Tomoni 15 *sm* bo‘lgan a) muntazam uchburchakka; b) muntazam to‘rtburchakka; d) muntazam oltiburchakka ichki va tashqi chizilgan aylanalar radiuslarini hisoblang.

2. 2-rasmda R radiusli aylanaga ichki chizilgan kvadrat, muntazam uchburchak va muntazam oltiburchak tasvirlangan. Daftaringizga berilgan jadvallarni ko‘chirib, uning bo‘sh kataklarini to‘ldiring (a_n — ko‘pburchak tomoni, P — ko‘pburchak perimetri, S — uning yuzi, r — unga ichki chizilgan aylana radiusi).



	R	r	a_4	P	S
1.			6		
2.		2			
3.	4				
4.				28	
5.					16

	R	r	a_3	P	S
1.	3				
2.					10
3.		2			
4.			5		
5.				6	

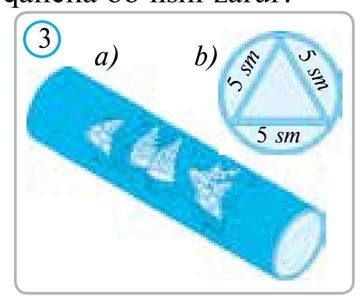
	R	r	a_6	P	S
1.	4				
2.		5			
3.			6		
4.				42	
5.					48

3. Radiusi 8 *sm* bo‘lgan aylanaga ichki chizilgan muntazam o‘n ikkiburchakning bir uchidan chiqqan diagonallarini toping.

4. Aylanaga ichki chizilgan muntazam uchburchak perimetri 24 *sm*. Shu aylanaga ichki chizilgan kvadrat tomonini toping.

5. Silindr shaklidagi yog‘ochdan asosining tomoni 20 *sm* bo‘lgan: a) kvadrat; b) muntazam oltiburchak bo‘lgan prizma shaklidagi ustun tayyorlash kerak. Yog‘och ko‘ndalang kesimining diametri kamida qancha bo‘lishi zarur?

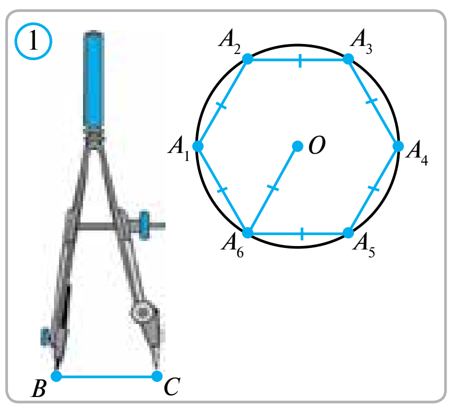
6. 3-*a* rasmda tasvirlangan, rang-barang naqshlarni tomosha qilsa bo‘ladigan “Kaleydoskop” deb nomlangan o‘yinchoq sizga tanish bo‘lsa kerak. O‘yinchoq quvur va 3 ta oyna bo‘laklaridan iborat. 3-*b* rasmda uning ko‘ndalang kesimi tasvirlangan va o‘lchamlari berilgan. Kaleydoskop ko‘ndalang kesimining radiusini toping.



45 BILIMINGIZNI SINAB KO‘RING

I. Testlar

1. **Quyidagi ko‘pburchaklarning qaysi biriga ichki chizilgan aylana mavjud emas?**
A) Uchburchakka; D) Kvadratdan farqli rombga;
B) Kvadratga; E) Rombdan farqli to‘g‘ri to‘rtburchakka.
2. **Quyidagi ko‘pburchaklarning qaysi biriga tashqi chizilgan aylana mavjud emas?**
A) Uchburchakda; D) Kvadratdan farqli rombda;
B) Kvadratda; E) Rombdan farqli to‘g‘ri to‘rtburchakda.
3. **Aylanaga ichki chizilgan barcha ABCD to‘rtburchaklar uchun noto‘g‘ri tenglikni toping.**
A) $\angle A+\angle B+\angle C+\angle D=360°$; D) $AB+CD=BC+AD$;
B) $\angle A+\angle C=180°$; E) $\angle B+\angle D=180°$.
4. **Aylanaga tashqi chizilgan barcha ABCD to‘rtburchaklar uchun noto‘g‘ri tenglikni toping.**
A) $\angle A+\angle B+\angle C+\angle D=360°$; D) $AB+CD=BC+AD$;
B) $\angle A+\angle C=180°$; E) $AB-BC=AD-CD$.
5. **Tomonlari 5 sm va 12 sm bo‘lgan to‘g‘ri to‘rtburchakka tashqi chizilgan aylana radiusini toping.**
A) 6 *sm*; B) 6,5 *sm*; D) 7 *sm*; E) 7,5 *sm*.
6. **Muntazam 24 burchakning ichki burchagini toping.**
A) 120°; B) 135°; D) 150°; E) 165°.
7. **Har bir tashqi burchagi 60° bo‘lgan muntazam ko‘pburchakning ichki burchaklari yig‘indisini toping.**
A) 540°; B) 360°; D) 90°; E) 720°.

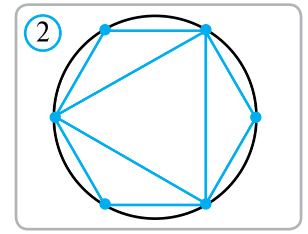


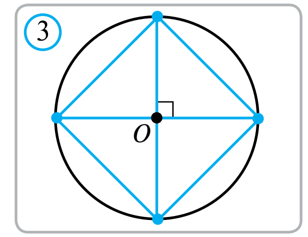
II. Yasashga doir masalalar.

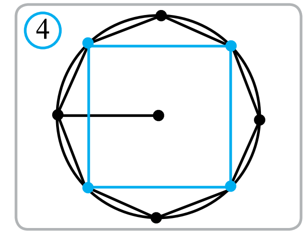
1. Tomoni berilgan kesmaga teng muntazam oltiburchak yasang. Bunda muntazam oltiburchakka tashqi chizilgan aylananing radiusi oltiburchakning tomoniga teng ekanligidan va 1-rasmdan foydalaning.

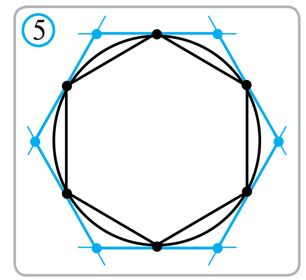
2. 2-4-rasmlardagi ma’lumotlardan foydalanib, berilgan aylanaga ichki chizilgan a) muntazam uchburchak; b) kvadrat; d) muntazam sakkizburchak yasang.

3. 5-rasmdan foydalanib, berilgan aylanaga tashqi chizilgan muntazam oltiburchak yasang (5-rasmda tasvirlangan aylanaga tashqi chizilgan oltiburchak tomonlari shu aylanaga ichki chizilgan muntazam oltiburchak uchlaridan aylanaga o‘tkazilgan urinmalarda yotadi).









III. Hisoblashga doir masalalar.

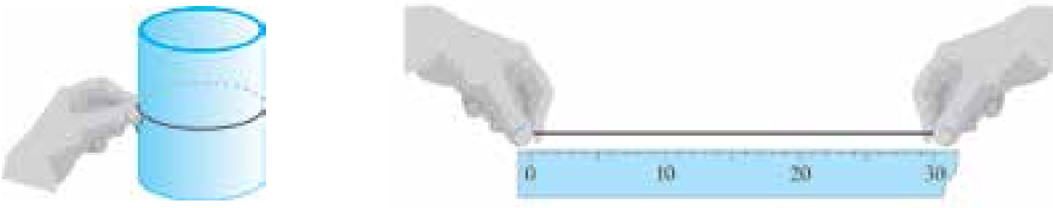
1. Muntazam uchburchak, kvadrat va muntazam oltiburchaklarning tomonlari bir-biriga teng. Ularning yuzlari nisbatini toping.
2. Bitta aylanaga ichki chizilgan muntazam oltiburchak va tashqi chizilgan oltiburchak yuzlari nisbatini toping.
3. Muntazam a) oltiburchak; b) sakkizburchak; d) o‘n ikkiburchakning parallel tomonlari orasidagi masofa 10 *sm* ga teng. Ko‘pburchak tomonini toping.
4. Radiusi R bo‘lgan aylanaga $A_1A_2 \ldots A_8$ muntazam sakkizburchak ichki chizilgan. $A_3A_4A_7A_8$ to‘rtburchakning to‘g‘ri to‘rtburchak ekanini isbotlang va uning yuzini toping.
5. Aylanaga tashqi chizilgan to‘g‘ri burchakli uchburchakning gipotenuzasi shu aylanaga urinish nuqtasida 4 *sm* va 6 *sm* uzunlikdagi kesmalarga bo‘linadi. Uchburchak yuzini toping.
6. Muntazam o‘nburchakning bir uchidan chiqqan eng katta va eng kichik diagonallari orasidagi burchakni toping.

IV. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi).

1. Katetlari 10 *sm* va 24 *sm* bo‘lgan to‘g‘ri burchakli uchburchakka ichki chizilgan va tashqi chizilgan aylanalarning radiuslarini toping.
2. Radiusi 5 *sm* bo‘lgan aylanaga tashqi chizilgan rombning bir burchagi 150° ga teng. Rombning a) perimetrini; b) diagonallarini; d) yuzini toping.
3. Tomoni 4 *sm* bo‘lgan muntazam oltiburchakning bir uchidan chiqqan diagonallarini toping.
4. *(Qo‘shimcha).* Radiusi 3 *sm* bo‘lgan aylanaga ichki chizilgan muntazam oltiburchak va muntazam uchburchaklar yuzlarining ayirmasini toping.

Tarixiy lavhalar. Istalgan muntazam ko‘pburchakni ham sirkul va chizg‘ich yordamida yasab bo‘lavermaydi. Buni 1801-yilda nemis matematigi Karl Gauss (1777-1855) algebraik usulda isbotlagan. U agar n sonning $2^m p_1 p_2 \ldots p_n$ yoyilmasidagi p_1, p_2, … p_n turli tub sonlar $2^{2^k}+1$ ko‘rinishida bo‘lsagina muntazam n burchakni sirkul va chizg‘ich yordamida yasash mumkinligini isbotladi. Bu yerda m va k manfiy bo‘lmagan butun sonlar.

46 AYLANA UZUNLIGI



Faollashtiruvchi mashq

1. Odatda quvur bo‘lagining ko‘ndalang kesimi aylanadan iborat bo‘ladi. Ingichka ipni bir uchidan boshlab, quvurga bir marta o‘rang. Bir marta o‘rashga ketgan ip bo‘lagi quvur ko‘ndalang kesimi, ya’ni aylananing uzunligi bo‘ladi. Uni rasmda ko‘rsatilgandek qilib chizg‘ich yordamida o‘lchang.
2. Yuqoridagi usul bilan quvur ko‘ndalang kesimi diametrini aniqlang.
3. Aniqlangan aylana uzunligini uning diametriga nisbatini hisoblang.
4. Yuqorida keltirilgan o‘lchash va hisoblash ishlarini yana bir nechta turli o‘lchamdagi quvur bo‘laklari uchun ham bajarib, aylana uzunligini uning diametriga nisbatini toping.
5. Mashq natijasiga ko‘ra, aylana uzunligining uning diametriga nisbati haqida qanday xulosa chiqarish mumkin?

Teorema. **Aylana uzunligining aylana diametriga nisbati aylana radiusiga bog‘liq emas, ya’ni har qanday aylana uchun bu nisbat o‘zgarmas sondir.**

Isbot. Ikkita ixtiyoriy aylana olamiz. Ularning radiuslari R_1 va R_2, uzunliklari esa mos ravishda C_1 va C_2 bo‘lsin. $\frac{C_1}{2R_1}=\frac{C_2}{2R_2}$ tenglikni isbotlashimiz kerak. Har ikki aylanaga ichki muntazam n-burchakni chizamiz. Ularning perimetrlarini mos ravishda P_1 va P_2 deb belgilaylik. Unda,

$P_1=n\cdot 2R_1\sin\frac{180^\circ}{n}$, $P_2=n\cdot 2R_2\sin\frac{180^\circ}{n}$ bo‘lgani uchun $\frac{P_1}{P_2}=\frac{2R_1}{2R_2}$ (*) bo‘ladi.

Bu tenglik istalgan n uchun to‘g‘ri. n soni kattalashib borsa, berilgan aylanaga ichki chizilgan n-burchak perimetri P_1 shu aylana uzunligi C_1 ga yaqinlashib boradi. Shu singari P_2 ham C_2 ga yaqinlashib boradi.

Shuning uchun $\frac{P_1}{P_2}$ nisbat $\frac{C_1}{C_2}$ nisbatga teng bo‘ladi (buning to‘liq isboti matematikaning yuqori bosqichlarida o‘rganiladi). Shunday qilib, (*) tenglikdan $\frac{C_1}{C_2}=\frac{2R_1}{2R_2}$, bundan esa $\frac{C_1}{2R_1}=\frac{C_2}{2R_2}$ tenglik kelib chiqadi. ***Teorema isbotlandi.***

Aylana uzunligini uning diametriga nisbatini yunon alifbosining π harfi bilan belgilash qabul qilingan *(“pi” deb o‘qiladi)*. Aylana uzunligining uning diametriga nisbatini “π” harfi bilan belgilashni buyuk matematik Leonard Eyler (1707—1783) fanga kiritgan. Yunonchada “aylana” so‘zi shu harf bilan boshlanadi. π irratsional son bo‘lib, amaliyotda uning 3,1416 ga teng bo‘lgan taqribiy qiymatidan foydalaniladi.

Shunday qilib, $\frac{C}{2R}=\pi$. Bu tenglikdan radiusi R ga teng aylana uzunligi uchun $C=2\pi R$ formulani hosil qilamiz.

Masala. Tomoni 6 *sm* bo‘lgan muntazam uchburchakka tashqi chizilgan aylana uzunligini toping.

Yechilishi. Muntazam uchburchakka tashqi chizilgan aylana radiusini topish formulasi $R=\frac{a}{\sqrt{3}}$ ga ko‘ra, $R=\frac{6}{\sqrt{3}}=2\sqrt{3}$ (*sm*). Endi, aylana uzunligini topish formulasidan

$C=2\pi R=2\pi.2\sqrt{3}=4\pi\sqrt{3}$ (*sm*). ***Javob:*** $4\pi\sqrt{3}$ *sm*.

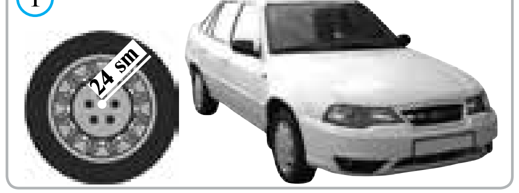
Savol, masala va topshiriqlar

1. Qanday son π bilan belgilanadi? Radiusi R ga teng aylana uzunligini topish formulasidan foydalanib, jadvalni to‘ldiring (π≈3,14 deb hisoblang).

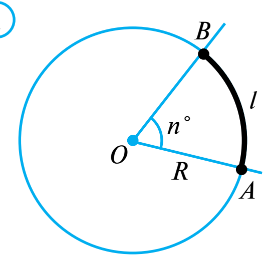
C			82	18π		6,28	
R	4	3			0,7		101,5

2. Agar aylana radiusi a) 3 marta oshsa; b) 3 *sm* ga oshsa; d) 3 marta kamaysa; e) 3 *sm* ga kamaysa, aylana uzunligi qanchaga o‘zgaradi?
3. Agar Yer shari ekvatorining 40 milliondan bir qismi 1 *m* ga teng bo‘lsa, Yer sharining radiusini toping.
4. a) Tomoni a ga teng bo‘lgan muntazam uchburchakka; b) katetlari a va b bo‘lgan to‘g‘ri burchakli uchburchakka; d) asosi a va yon tomoni b bo‘lgan teng yonli uchburchakka tashqi chizilgan aylana uzunligini toping.
5. a) Tomoni a ga teng kvadratga; b) gipotenuzasi c ga teng bo‘lgan teng yonli to‘g‘ri burchakli uchburchakka; d) gipotenuzasi c, o‘tkir burchagi α bo‘lgan to‘g‘ri burchakli uchburchakka ichki chizilgan aylana uzunligini toping.
6. Teplovoz 1413 *m* yo‘l yurdi. Bunda uning g‘ildiragi 300 marta aylandi. Teplovoz g‘ildiragining diametrini toping.
7. “Nexia” avtomobili g‘ildiragi aylanasining radiusi 24 *sm* ga teng. Avtomobil 100 *km* yo‘l yursa, uning g‘ildiragi necha marta aylanadi (*1-rasm*)?

1



47 AYLANA YOYI UZUNLIGI. BURCHAKNING RADIAN O‘LCHOVI



$l=\frac{\pi R}{180^\circ}\cdot n^\circ$

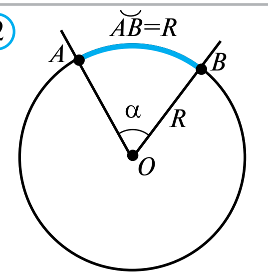
1. n° li markaziy burchak tiralgan yoy uzunligi.

Aytaylik, radiusi R ga teng bo‘lgan aylanada n° li AOB markaziy burchak berilgan bo‘lsin (*1-rasm*). Bunda aylananing AOB markaziy burchakka tiralgan AB yoyining gradus o‘lchovini n° yoki n° li yoy deb yuritilishini eslatib o‘tamiz.

Radiusi R ga teng butun aylana, ya’ni o‘lchovi 360° bo‘lgan yoy uzunligi $2\pi R$ ga teng bo‘lgani uchun,

1° li yoy uzunligi $\frac{2\pi R}{360}=\frac{\pi R}{180}$ ga teng bo‘ladi.

U holda, n° li yoy uzunligi $l=\frac{\pi R}{180^\circ}\cdot n^\circ$ formula bilan aniqlanadi *(1-rasm)*.



$\alpha=1$ *radian* $\approx 57^\circ17'45''$

2. Burchakning radian o‘lchovi.

Burchakning gradus o‘lchovi bilan bir qatorda uning radian o‘lchovi ham ishlatiladi.

Aylana yoyi uzunligining radiusga nisbatini yuqoridagi formulaga asosan: $\frac{l}{R}=\frac{\pi}{180^\circ}\cdot n^\circ$ ga teng. Demak, aylana yoyi uzunligining radiusiga nisbati faqat shu yoyga tiralgan markaziy burchak kattaligiga bog‘liq ekan. Bu xossadan foydalanib, burchakning radian o‘lchovi sifatida xuddi shu nisbatni olamiz:

$$\alpha=\frac{l}{R}=\frac{\pi}{180^\circ}\cdot n^\circ.$$

Odatda, radian so‘zi yozilmaydi. Masalan: 5 radian o‘rniga 5 deb yoziladi. Bir radian $\frac{180^\circ}{\pi}$ gradusga teng: 1 radian$=\frac{180^\circ}{\pi}\approx 57^\circ17'45''$. Burchakning gradus o‘lchovidan radian o‘lchoviga o‘tish uchun

$$\alpha=\frac{\pi}{180^\circ}\cdot n^\circ$$

formuladan foydalaniladi.

Shunday qilib, n° li burchakning radian o‘lchovini topish uchun uning gradus o‘lchovini $\frac{\pi}{180^\circ}$ ga ko‘paytirish kifoya ekan. Xususiy holda, 180° li burchakning radian o‘lchovi π ga teng, 90° li, ya’ni to‘g‘ri burchakning radian o‘lchovi $\frac{\pi}{2}$ ga teng bo‘ladi.

α radianga teng markaziy burchakka mos yoyining uzunligi $l=\alpha R$ formula bilan hisoblanadi.

Masala. Ikkita burchagi mos ravishda 30° va 45° bo‘lgan uchburchak burchaklarining radian o‘lchovlarini toping.

Yechilishi. Uchburchakning 30° li burchagi radian o‘lchovi $30^\circ\cdot\frac{\pi}{180^\circ}=\frac{\pi}{6}$, 45° li burchagi radian o‘lchovi $45^\circ\cdot\frac{\pi}{180^\circ}=\frac{\pi}{4}$. Uchburchak ichki burchaklari yig‘indisi 180° ga, ya’ni π ga tengligi haqidagi teoremaga asosan uchburchakning uchinchi burchagining radian o‘lchovini topamiz

$$\pi-\frac{\pi}{6}-\frac{\pi}{4}=\frac{7\pi}{12}.$$

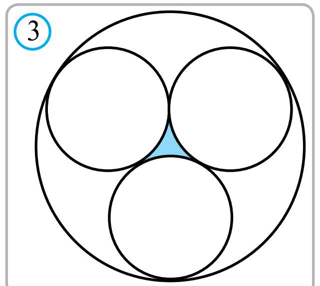
Javob: $\frac{\pi}{6}$; $\frac{\pi}{4}$ va $\frac{7\pi}{12}$.

Savol, masala va topshiriqlar

1. Radiusi 6 *sm* bo‘lgan aylananing gradus o‘lchovi a) 30°; b) 45°; d) 90°; e) 120° bo‘lgan yoyi uzunligini toping.
2. a) 40°; b) 60°; d) 75° ga teng burchakning radian o‘lchovini toping.
3. a) 1,2; b) $\frac{2\pi}{3}$; d) $\frac{5\pi}{6}$ radianga teng burchakning gradus o‘lchovini toping.
4. Agar aylana radiusi 5 *sm* bo‘lsa, uning a) $\frac{\pi}{8}$; b) $\frac{2\pi}{5}$; d) $\frac{3\pi}{4}$ radianga teng markaziy burchagi tiralgan yoy uzunligini toping.
5. Radiusi 12 *sm* bo‘lgan aylanaga ABC uchburchak ichki chizilgan. Agar a) $\angle A=30^\circ$; b) $\angle A=120^\circ$ bo‘lsa, A nuqtani o‘z ichiga olmagan BC yoy uzunligini toping.
6. Aylananing teng vatarlari aylanadan teng yoylar ajratishini isbotlang.

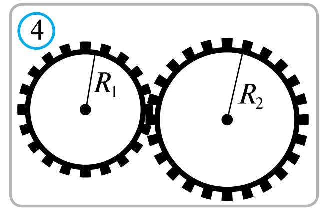
7*. Ikkita aylana bir-birining markazidan o‘tadi. Bu aylanalarning umumiy vatari har ikki aylanadan ajratgan yoylar uzunliklari nisbatini toping.

8*. Radiuslari teng bo‘lgan uchta aylanalar bir-biriga tashqaridan va radiusi R ga teng bo‘lgan aylanaga ichkaridan urinadi (*3-rasm*): a) aylanalar radiusini toping; b) bo‘yalgan shaklni chegaralovchi yoylar uzunliklari yig‘indisini toping.



Qiziqarli masala

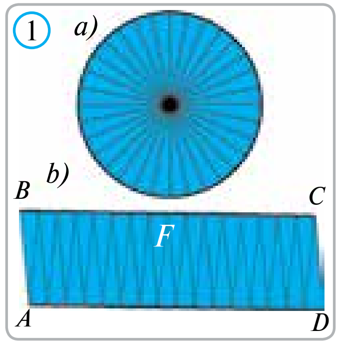
4-rasmda tasvirlangan ikkita tishli g‘ildiraklar bir-biriga “tishlatilgan”. G‘ildiraklar radiusi R_1 va R_2. Birinchi g‘ildirak n marta aylanganda ikkinchi g‘ildirak necha marta aylanadi?

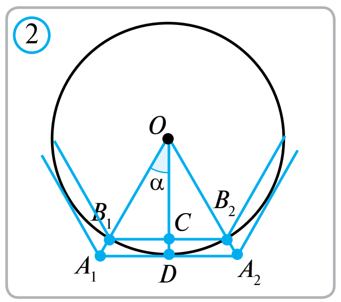


48 DOIRA YUZI

Ta'rif. Tekislikning berilgan O nuqtasidan berilgan R masofadan katta bo'lmagan masofada yotuvchi barcha nuqtalaridan tashkil topgan shaklga **doira** deb ataladi.

Bunda O nuqta doiraning markazi, R esa doiraning radiusi deb ataladi. Mazkur doiraning chegarasi markazi O nuqtada, radiusi esa R ga teng bo'lgan aylanadan iborat bo'ladi.





Faollashtiruvchi mashq

Bir varaq qog'ozga yo'g'on chiziq bilan aylana chizing va 1-*a* rasmda ko'rsatilgandek, uning bir nechta diametrlarini o'tkazib, doirani teng bo'laklarga bo'ling. So'ng bu bo'laklarni qiyib oling va 1-b rasmda ko'rsatilgandek terib, F shaklni hosil qiling. Agar doira istalgancha ko'p teng bo'laklarga bo'linib, bu bo'laklar rasmda ko'rsatilgan tartibda terilsa, natijada to'g'ri to'rtburchakka juda yaqin F shakl paydo bo'ladi.

a) F shaklni to'g'ri to'rtburchak shakliga juda yaqinligini hisobga olib, uning AB tomoni taxminan nimaga teng bo'lishini toping (ko'rsatma: AB tomonni doira radiusi bilan taqqoslang).

b) F shaklning BC "tomoni" taxminan nimaga teng bo'ladi? (Ko'rsatma: BC va AD tomonlar yo'g'on chiziq bilan chizilganiga, ya'ni aylana yoychalaridan iborat ekanligiga e'tibor bering)

d) F shaklning $ABCD$ to'g'ri to'rtburchak shakliga juda yaqin ekanligini hisobga olib, uning yuzini taqriban hisoblang. F shakl yuzi doira yuziga juda yaqin ekanligini nazarda tutib, doira yuzi haqida xulosa chiqaring.

Teorema. **Radiusi R ga teng bo'lgan doiraning yuzi πR^2 ga teng.**

Isbot. Radiusi R va markazi O nuqtada bo'lgan aylanani qaraymiz.

Aylanaga tashqi chizilgan $A_1A_2 \dots A_n$ va ichki chizilgan $B_1B_2 \dots B_n$ muntazam n burchaklarning yuzlari mos ravishda S_n^I va S_n^{II} bo'lsin *(2-rasm)*.

A_1OA_2 va B_1OB_2 uchburchaklar yuzlarini topamiz:

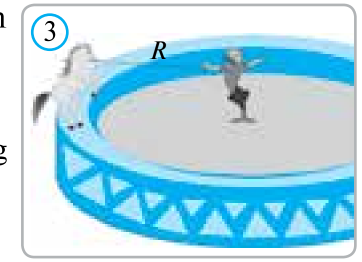
$$S_{A_1OA_2}=\frac{1}{2}A_1A_2\cdot OD=\frac{1}{2}A_1A_2\cdot R;\quad S_{B_1OB_2}=\frac{1}{2}B_1B_2\cdot OC=\frac{1}{2}B_1B_2\cdot OB_1\cos\alpha=\frac{1}{2}B_1B_2\cdot R\cos\alpha.$$

$$\text{Unda, } S_n^I=n\cdot\frac{1}{2}AA_1\cdot R=\frac{1}{2}P_n^IR,\quad S_n^{II}=n\cdot\frac{1}{2}B_1B_2\cdot R\cos\alpha=\frac{1}{2}P_n^{II}R\cos\alpha \qquad (1)$$

Bu yerda P_n^I va P_n^{II} mos ravishda $A_1A_2 \dots A_n$ va $B_1B_2 \dots B_n$ ko'pburchaklarning perimetrlari. $\alpha=\frac{180^\circ}{n}$ bo'lgani uchun n ning yetarlicha katta qiymatlarida $\cos\alpha$ ning qiymati birdan, P_n va P_n larning qiymatlari aylana uzunligi, ya'ni $2\pi R$ dan istalgancha kam farq qiladi. Unda, (1) tengliklarga ko'ra, n ning yetarlicha katta qiymatlarida ko'pburchaklarning yuzi πR^2 ga yaqinlashib boradi. Bundan, doiraning yuzi uchun $S=\pi R^2$ formula kelib chiqadi.

Teorema isbotlandi.

Masala. Sirk arenasi aylanasining uzunligi 41 m. Arena radiusi va yuzini toping.



Yechilishi. 1) Aylana uzunligini topish formulasidan radiusni topamiz *(3-rasm)*:

$$R=\frac{C}{2\pi}\approx\frac{41}{2\cdot 3{,}14}\approx 6{,}53\ (m).$$

2) Doira yuzini hisoblash formulasidan arenaning yuzini topamiz: $S=\pi R^2\approx 3{,}14\cdot 6{,}53^2\approx 133{,}84\ (m^2)$.

Javob: $R\approx 6{,}53\ m$; $S\approx 133{,}84\ m^2$.

Savol, masala va topshiriqlar

1. Doira yuzini hisoblash formulasini asoslang.

2. Radiusi R ga teng bo'lgan doiraning S yuzini topish formulasidan foydalanib jadvalni to'ldiring ($\pi=3{,}14$ deb oling).

R	2	5		$\frac{2}{7}$		54,3		6,25
S			9		49π		$\sqrt{3}$	

3. Agar doira radiusi a) k marta oshsa; b) k marta kamaysa, doira yuzi qanday o'zgaradi?

4. Tomoni 5 *sm* bo'lgan kvadratga ichki chizilgan va tashqi chizilgan doiralarning yuzini toping.

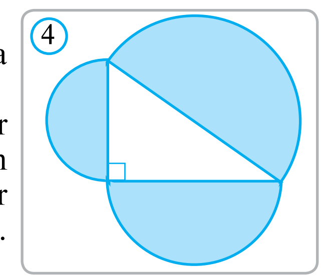
5. Tomoni $3\sqrt{3}$ *sm* bo'lgan muntazam uchburchakka ichki va tashqi chizilgan doiralarning yuzini toping.

6. Radiusi R bo'lgan doiradan eng katta kvadrat qirqib olindi. Doiraning qolgan qismi yuzini toping.

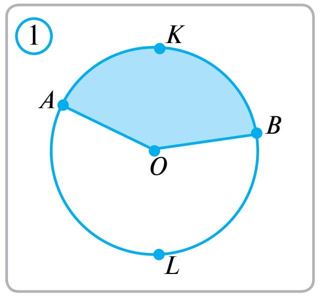
7. Tomonlari 6 *sm* va 7 *sm* bo'lgan to'g'ri to'rtburchakka tashqi chizilgan doira yuzini toping.

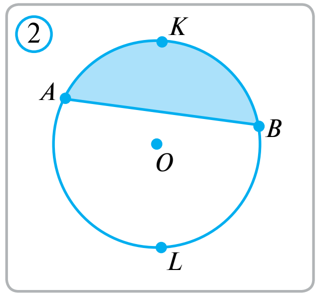
8. Tomoni 10 *sm* va o'tkir burchagi 60° bo'lgan rombga ichki chizilgan doira yuzini toping.

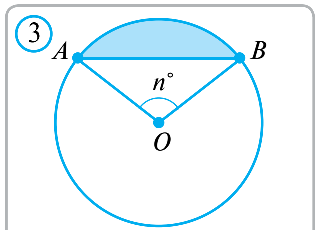
9*. To'g'ri burchakli uchburchak tomonlarini diametr qilib yarim doiralar chizilgan. Gipotenuzaga chizilgan yarim doira yuzi katetlarga chizilgan yarim doiralar yuzlari yig'indisiga teng bo'lishini ko'rsating *(4-rasm)*.



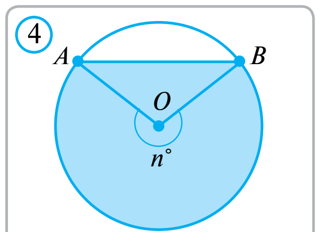
49 DOIRA BO‘LAKLARI YUZI







$S=\frac{\pi R^2}{360}\cdot n-S_{AOB}$



$S=\frac{\pi R^2}{360}\cdot n+S_{AOB}$

Ta’rif. Doiraning yoyi va bu yoy oxirlarini doira markazi bilan tutashtiruvchi ikkita radiusi bilan chegaralangan qismi **sektor** deyiladi. Sektorning chegarasi bo‘lgan yoy **sektor yoyi** deyiladi.

1-rasmda *AKB* va *BLA* yoyli ikkita sektor tasvirlangan (ulardan birinchisi bo‘yalgan).

Radiusi *R* ga va yoyining gradus o‘lchovi $n°$ ga teng bo‘lgan sektorning *S* yuzini topish uchun formula keltirib chiqaramiz. Yoyi 1° ga teng sektorning yuzi doira (ya’ni yoyi 360° ga teng sektor) yuzining $\frac{1}{360}$ qismiga teng bo‘lgani uchun, yoyi *n* gradus bo‘lgan sektorning yuzi

$$S=\frac{\pi R^2}{360}\cdot n \text{ yoki } S=\frac{1}{2}Rl$$

formula orqali topiladi. Bu yerda l — $n°$ li sektor yoyining uzunligi.

Ta’rif. Doiraning yoyi va bu yoy oxirlarini tutashtiruvchi vatari bilan chegaralangan qismi **segment** deyiladi.

2-rasmda *AKB* va *BLA* yoyli ikkita segment tasvirlangan (ulardan birinchisi bo‘yalgan). Yarim doiradan farqli segmentning *S* yuzi

$$S=S_{sektor}\pm S_{\Delta}=\frac{\pi R^2}{360}\cdot n\pm S_{AOB}$$

formula bo‘yicha hisoblanadi *(3- va 4-rasmlarga qarang)*.

Masala. Yoyning gradus o‘lchovi 72° bo‘lgan sektorning yuzi 45π ga teng. Sektor radiusini toping.

Yechilishi. Sektor yuzini topish formulasiga ko‘ra,

$$\frac{\pi R^2}{360}\cdot 72=45\pi.$$

Bundan, $R^2=\frac{45\pi\cdot 360}{72\pi}=225$, demak, $R=15$.

Javob: 15.

Savol, masala va topshiriqlar

1. Sektor yuzini topish formulasini keltirib chiqaring.
2. Segment yuzini topish formulasini keltirib chiqaring.
3. Yoyining gradus o‘lchovi a) 30°; b) 45°; d) 120°; e) 90° va radiusi 7 *sm* bo‘lgan sektor va segment yuzlarini toping.
4. 5-rasmda tomoni *a* ga teng bo‘lgan muntazam uchburchak, kvadrat va muntazam oltiburchak tasvirlangan. Bo‘yalgan shakllar yuzini toping. Bunda sektorlarning radiuslari ko‘pburchak tomonining yarmiga teng.
5. Nishonda radiuslari 1, 2, 3, 4 ga teng bo‘lgan to‘rtta aylana bor. Eng kichik doira yuzini va har bir halqa yuzini toping *(6-rasm)*.
6. Radiusi 10 *sm* ga teng bo‘lgan doirada radiusga teng vatar o‘tkazilgan. Hosil bo‘lgan segmentlar yuzini hisoblang.
7. Radiuslari 15 *sm* dan bo‘lgan ikkita doira markazlari orasidagi masofa 15 *sm*. Doiralar umumiy qismining yuzini toping.
8. Radiusi 10 *sm* bo‘lgan doiraga ichki va tashqi chizilgan muntazam o‘n ikkiburchaklar yuzini hisoblang. Natijalarni doira yuzi bilan solishtiring.

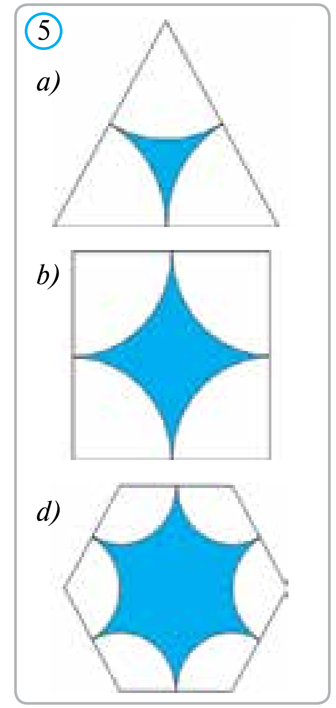
Qiziqarli masala

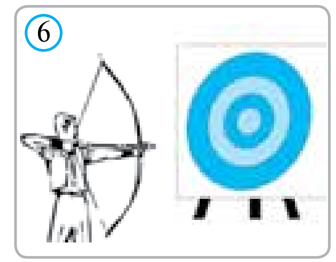
7-rasmda tasvirlangan guldon rasmini uchta to‘g‘ri chiziq bilan:

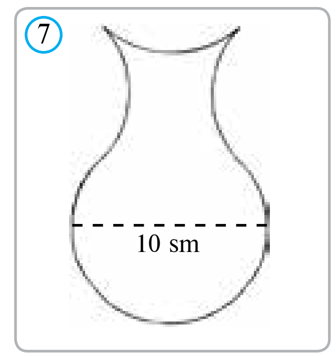
a) shunday to‘rt bo‘lakka bo‘lingki, ulardan to‘g‘ri to‘rtburchak yig‘ish mumkin bo‘lsin;

b) ikkita to‘g‘ri chiziq bilan shunday uch qismga bo‘lingki, ulardan kvadrat yig‘ish mumkin bo‘lsin.

Tarixiy lavhalar. Uzoq vaqtlar mobaynida dunyoning ko‘plab matematiklari “doira kvadraturasi” deb nom olgan quyidagi masalani yechishga harakat qilganlar: sirkul va chizg‘ich yordamida yuzi berilgan doira yuziga teng bo‘lgan kvadrat yasash. Faqat XIX asrning oxirida bu masala yechimga ega emasligi isbotlangan.

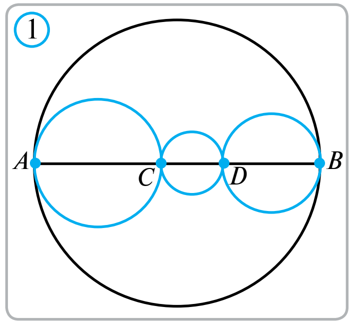






50 MASALALAR YECHISH

1-masala. *C* va *D* nuqtalar aylananing *AB* diametrini uchta *AC*, *CD* va *DB* kesmalarga ajratadi. *AC*, *CD* va *DB* diametrli aylanalar uzunliklarining yig‘indisi *AB* diametrli aylana uzunligiga teng ekanligini isbotlang *(1-rasm)*.



Yechilishi. Aylana uzunligini topish formulasidan foydalanib, *AC*, *CD* va *DB* diametrli aylanalarning C_1, C_2, C_3 uzunliklari yig‘indisini topamiz:

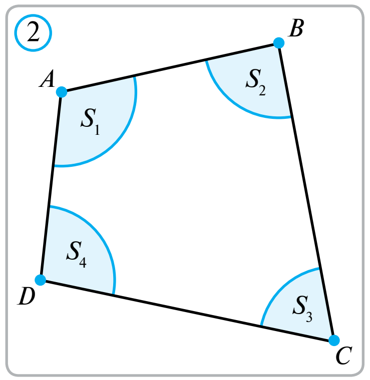
$$C_1+C_2+C_3=AC\cdot\pi+CD\cdot\pi+DB\cdot\pi=\pi(AC+CD+DB).$$

$AC+CD+DB=AB$ va *AB* diametrli aylananing *C* uzunligi $AB\cdot\pi$ ga teng bo‘lgani uchun

$$C_1+C_2+C_3=C.$$

Shu tenglikni isbotlash talab qilingan edi.

2-masala. *ABCD* to‘rtburchakning uchlarini markaz qilib bir xil radiusli sektorlar yasalgan *(2-rasm)*. Bu sektorlardan ixtiyoriy ikkitasi umumiy nuqtaga ega emas hamda barchasining radiusi 1 *sm*. Sektorlar yuzlarining yig‘indisini toping.



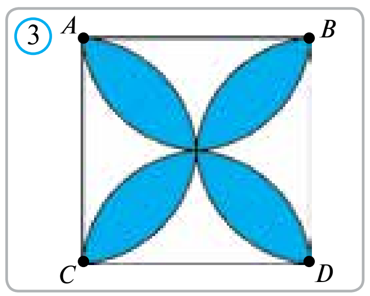
Yechilishi. 1) To‘rtburchakning *A*, *B*, *C*, *D* burchaklari mos ravishda α_1, α_2, α_3, α_4 bo‘lsin. Unda, ko‘pburchak ichki burchaklarining yig‘indisi haqidagi teoremaga ko‘ra,

$$\alpha_1+\alpha_2+\alpha_3+\alpha_4=360°.$$

2) Sektor yuzini topish formulasiga ko‘ra ($R=1$ *sm*),

$$S_1=\frac{\pi}{360°}\cdot\alpha_1,\quad S_2=\frac{\pi}{360°}\cdot\alpha_2,\quad S_3=\frac{\pi}{360°}\cdot\alpha_3,\quad S_4=\frac{\pi}{360°}\cdot\alpha_4.\ (1)$$

3) (1) tengliklarning mos qismlarini qo‘shamiz. Unda,



$$S_1+S_2+S_3+S_4=\frac{\pi}{360°}(\alpha_1+\alpha_2+\alpha_3+\alpha_4)=\frac{\pi}{360°}\cdot 360°=$$
$=\pi$ (sm^2). ***Javob:*** π sm^2.

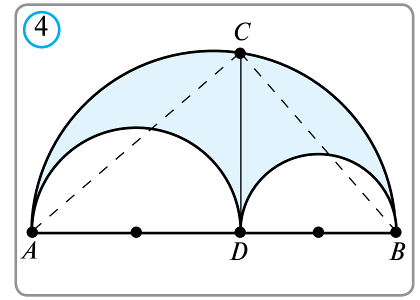
Savol, masala va topshiriqlar

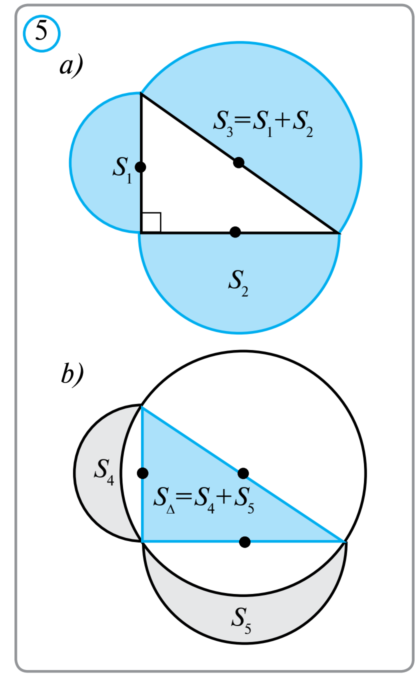
1. Perimetri 1 *m* bo‘lgan kvadrat va uzunligi 1 *m* bo‘lgan aylana berilgan. Bu aylana bilan chegaralangan doira yuzi bilan kvadrat yuzini taqqoslang.

2. Radiusi 8 *sm* bo‘lgan doiradan 60° li sektor qirqib olingan. Doiraning qolgan qismi yuzini toping.
3. Diagonallari 6 *sm* va 8 *sm* bo‘lgan rombga ichki chizilgan doira yuzini hisoblang.
4. 3-rasmda bo‘yab ko‘rsatilgan shakl yuzini toping. Unda *ABCD* — kvadrat, $AB=4$ *sm*.

5*. 4-rasmda “Arximed pichog‘i” deb ataluvchi shakl bo‘yab ko‘rsatilgan. Uning yuzi $\frac{\pi\cdot CD^2}{4}$ formula bilan hisoblanishini isbotlang (bunda, $\angle ACB=90°$ va $CD^2=AD\cdot DB$ ekanligidan foydalaning).

6. Agar $AD=6$ *sm*, $BD=4$ *sm* bo‘lsa, 4-rasmda bo‘yab ko‘rsatilgan shaklning yuzi va perimetrini (uni o‘rab turgan yoylar uzunligi yig‘indisini) toping.





Tarixiy lavhalar. Gippokrat oychalari.

Gippokrat oychasi – ikki aylana yoylari bilan chegaralangan va quyidagi xossaga ega bo‘lgan shakldir: agar aylanalar radiuslari va oycha yoylari tiralgan vatar berilgan bo‘lsa, oychaga tengdosh kvadrat yasash mumkin.

Pifagor teoremasi qo‘llanilsa, 5-a rasmda tasvirlangan gipotenuzaga qurilgan yarim doira yuzi katetlarga qurilgan yarim doiralar yuzlari yig‘indisiga teng bo‘ladi (107-betdagi 9*-masalaga qarang). Shuning uchun 5-b rasmdagi oychalar yuzlarining yig‘indisi uchburchak yuziga teng (mushohada qilib ko‘ring!). Agar rasmdagi uchburchak o‘rniga teng yonli to‘g‘ri burchakli uchburchak olsak, hosil bo‘lgan ikki oychadan har birining yuzi uchburchak yuzining yarmiga teng bo‘ladi. Doira kvadraturasi haqidagi masalani yechishga urinib, yunon matematigi Gippokrat (miloddan avvalgi V asr) ko‘pburchak bilan tengdosh bir necha xil oychalarni ixtiro qilgan.

Gippokrat oychalarining to‘la jadvali faqat XIX–XX asrlarda tuzilgan.

51 III BOBGA DOIR QO‘SHIMCHA MASALALAR VA MA’LUMOTLAR

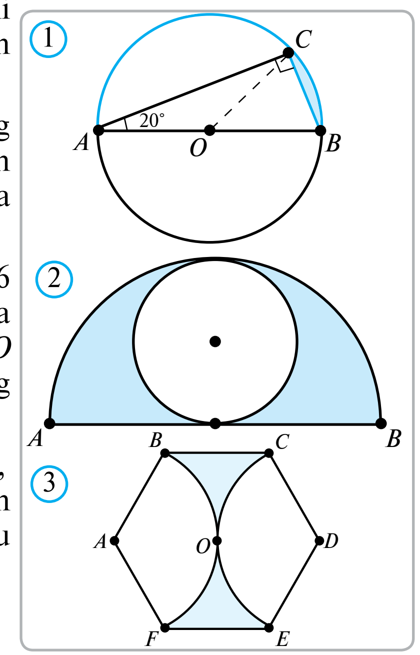
I. Testlar

1. **45 gradusli burchakning radian o‘lchovi nimaga teng?**
   A. 1 ga teng; B. $\frac{\pi}{2}$ ga teng; D. $\frac{\pi}{4}$ ga teng; E. $\sqrt{2}$ ga teng.
2. **Radiusi 3 *sm* bo‘lgan aylananing gradus o‘lchovi 150° bo‘lgan markaziy burchagi tiralgan yoy uzunligini toping.**
   A. $\frac{5\pi}{2}$ *sm*; B. $\frac{5\pi}{3}$ *sm*; D. $\frac{10\pi}{3}$ *sm*; E. $\frac{5\pi}{4}$ *sm*.
3. **Radiusi 6 *sm* bo‘lgan aylanada $\frac{5\pi}{4}$ radianga teng markaziy burchak tiralgan yoy uzunligini toping.**
   A. $\frac{15\pi}{2}$ *sm*; B. $\frac{5\pi}{6}$ *sm*; D. $\frac{4\pi}{3}$ *sm*; E. $\frac{5\pi}{2}$ *sm*.
4. **Tomoni 5 *sm* ga teng bo‘lgan kvadratga tashqi chizilgan aylana uzunligini toping.**
   A. $5\sqrt{2}\pi$; B. $\sqrt{2}\pi$; D. $3\sqrt{2}\pi$; E. 5π.
5. **Diametri 6 ga teng doira yuzini toping.**
   A. 9π; B. 6π; D. $3\sqrt{2}\pi$; E. 12π.
6. **Yoyining gradus o‘lchovi 150°, radiusi 6 *sm* bo‘lgan doiraviy sektorning yuzini toping.**
   A. 15π *sm*²; B. 6π *sm*²; D. $30\sqrt{2}\pi$ *sm*²; E. 24π *sm*².
7. **Yoyining uzunligi 12 *sm* va radiusi 6 *sm* bo‘lgan doiraviy sektorning yuzini toping.**
   A. 15π *sm*²; B. 6π *sm*²; D. $30\sqrt{2}\pi$ *sm*²; E. 24π *sm*².
8. **Yoyining gradus o‘lchovi 120°, radiusi 3 ga teng bo‘lgan doiraviy segmentning yuzini toping.**
   A. $6\pi - 4\sqrt{3}$; B. $6\pi + 4\sqrt{3}$; D. $3\pi - 4\sqrt{3}$; E. $3\pi + 4\sqrt{3}$.

II. Masalalar

1. *ABCDEFKL* muntazam sakkizburchakning tomoni 6 *sm*. Uning *AC* diagonalini toping.
2. Kvadrat radiusi 4 *dm* bo‘lgan aylanaga ichki chizilgan. Kvadrat qo‘shni tomonlarining o‘rtalaridan o‘tuvchi vatarni aylanadan ajratgan yoylarning uzunligini toping.
3. Aylananing 90° li yoyi uzunligi 15π *sm*. Aylana radiusini toping.
4. Radiusi 20 ga teng aylanadan uzunligi 10π ga teng yoy ajratildi. Bu yoyga mos markaziy burchakni toping.
5. Ikkita doiraning umumiy vatari bu doiralarni chegaralovchi aylanalardan 60° li va 120° li yoylarni ajratadi. Doiralar yuzlarining nisbatini toping.
6. Tomonlari 3, 4, 5 bo‘lgan uchburchakka ichki va tashqi chizilgan doiralar yuzlarini toping.

7. Doira vatari 60° li yoyni tortib turadi. Bu vatar ajratgan segmentlar yuzlari nisbatini toping.
8. Muntazam oltiburchak yuzining unga ichki chizilgan doira yuziga nisbatini toping.
9. Tomoni *a* ga teng bo‘lgan *ABCDEF* muntazam oltiburchak berilgan. Markazi *A* nuqtada va radiusi *a* bo‘lgan aylana bu oltiburchakni ikki qismga ajratadi. Har bir qism yuzini toping.
10. To‘g‘ri burchakli *ABC* uchburchakda $\angle A=72°$, $\angle C=90°$, $BC=15$ *sm*. *BC* diametrli aylananing *ABC* uchburchak ichida yotgan yoyi uzunligini toping.
11. Doiraga ichki chizilgan muntazam sakkizburchak berilgan. Uning ikki qo‘shni uchlariga o‘tkazilgan radiuslar doirani ikkita sektorga ajratadi. Bu sektorlar yuzlarining nisbatini toping.
12. To‘g‘ri burchakli *ABC* uchburchakda $\angle A=20°$, $\angle C=90°$, $AB=18$ *sm*. *BC* kesma uchburchakka tashqi chizilgan doirani ikki segmentga ajratadi. Bo‘yab ko‘rsatilgan segment yuzini toping (*1-rasm*).
13. Kichik aylana katta aylanaga hamda uning *AB* diametriga urinadi. Agar diametrga urinish nuqtasi aylana markazi va $AB=4$ bo‘lsa, rasmda bo‘yalgan shakl yuzini toping (*2-rasm*).
14. Muntazam *ABCDEF* oltiburchakning tomoni 6 ga teng va markazi *O* nuqtada. Markazlari *A* va *D* nuqtada va radiuslari teng bo‘lgan aylanalar *O* nuqtada urinadi. Bo‘yalgan soha yuzini toping (*3-rasm*).
15. To‘g‘ri burchakli *ABC* uchburchakda $\angle C=90°$, $AC=4$, $CB=2$. Markazi gipotenuzada bo‘lgan aylana uchburchak katetlariga urinadi. Bu aylana uzunligini toping.



III. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi)

1. Tomoni 6 sm bo‘lgan kvadratga tashqi chizilgan aylana uzunligini va ichki chizilgan doira yuzini toping.
2. Tomoni 24 *sm* bo‘lgan muntazam ko‘pburchakka ichki chizilgan aylana radiusi $4\sqrt{3}$ *sm* ga teng bo‘lsa, unga tashqi chizilgan aylana radiusini toping.

3. 240° li aylana yoyining uzunligi 24 *sm* bo‘lsa,
a) aylana radiusini; b) yoyi 240° bo‘lgan sektor yuzini;
d) yoyi 240° bo‘lgan segmentning yuzini toping.

Qiziqarli masala

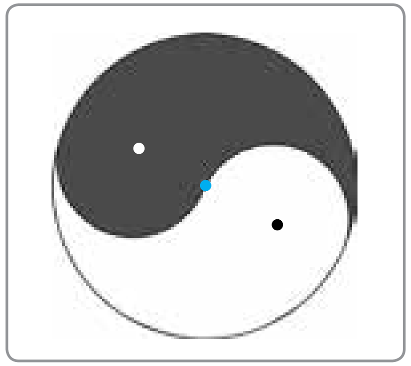
In va Yan

Rasmda tabiatdagi qarama-qarshiliklarni ifodalovchi “In va Yan” deb nomlangan xitoy ramzi tasvirlangan.

a) In va Yan ramzlari yuzlari tengligini ko‘rsating;

b) bitta to‘g‘ri chiziq bilan bu ramzlarning har birini yuzlari teng bo‘lgan ikki bo‘lakka bo‘ling.

d) In va Yan ramzlar perimetrlarini (ularni o‘rab turgan yoylar uzunliklari yig‘indisini) toping.



Tarixiy lavhalar. Aylana uzunligini hisoblash juda qadimdan dolzarb muammo bo‘lgan. Aylana uzunligini unga ichki chizilgan ko‘pburchak perimetriga almashtirish usuli keng tarqalgan.

O‘rta Osiyolik matematiklar ham doiraga ichki chizilgan muntazam ko‘pburchaklarni yasash, ularning tomonlarini doiraning radiusi orqali ifodalash masalalari bilan shug‘ullanganlar. Abu Rayhon Beruniy “Qonuni Mas’udiy” asarida doiraga ichki chizilgan ko‘pburchaklarning tomonini aniqlash bilan shug‘ullanib, ichki chizilgan beshburchak, oltiburchak, yettiburchak,..., o‘nburchak tomonlarini aniqlash usulini ko‘rsatadi. Bu hisoblash natijasida u $\pi \approx 3,14$ qiymatga ega ekanligini aniqlaydi.

Qadimgi Bobil va Misr qo‘lyozmalari va mixxatlarida π uchga teng deb olingan. Bu o‘sha davr aniqlik talabi uchun yetarli bo‘lgan. Keyinchalik rimliklar π uchun 3,12 ni ishlatishgan. π soni uchun Arximed bergan qiymat 3,14 bo‘lib, bu amaliy masalalarni hal qilishda juda ma’qul edi.

Xitoy matematiklarida $\pi \approx 3,155$ ... va 22/7. Hindlarning “Sulva Sutra” (“Arqon qoidasi”) asarida π uchun 3,008 va 3,1416 ... va $\sqrt{10} \approx 3,162$ ... qiymatlar uchraydi.

Mirzo Ulug‘bekning “Astronomiya maktabi” namoyandalaridan biri Jamshid G‘iyosiddin al-Koshiy 1424-yilda yozgan “Aylana uzunligi haqida kitob” nomli risolasida aylanaga ichki va tashqi chizilgan muntazam ko‘pburchak tomonlari sonini ikkilantirish yo‘li bilan $3 \cdot 2^{28} = 800\,335\,168$ tomonli muntazam ko‘pburchaklar perimetrini hisoblab, π uchun $\pi = 3,1\,415\,826\,535\,897\,932$ qiymatni hosil qilgan. Bu 16 ta o‘nli raqamgacha aniqdir.

Ammo al-Koshiyning asari uzoq vaqtgacha Yevropada noma’lum bo‘lgan. Yevropaliklardan belgiyalik Van Romen 1597- yilda 2^{30} tomonli muntazam ko‘pburchakka Arximed usulini tatbiq etib, π uchun 17 ta o‘nli raqamlari aniq bo‘lgan qiymat topgan. Gollandiyalik Rudolf van Seylon (1540−1610) bu aniqlikni 35 ta o‘nli raqamlargacha olib borgan. Hozirgi davrda elektron hisoblash mashinalari yordamida π uchun milliondan ortiq o‘nli raqamlari aniq bo‘lgan qiymatlar topilgan. Kundalik hisoblashlar uchun 3,14 qiymat, matematik hisoblashlar uchun 3,1416 qiymat, hatto astronomiya va kosmonavtika uchun 3,1415826 qiymat kifoyadir.

IV BOB

UCHBURCHAK VA AYLANADAGI METRIK MUNOSABATLAR

Ushbu bobni o‘rganish natijasida siz quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

Bilimlar:

√ *proporsional kesmalarning xossalarini bilish;*

√ *to‘g‘ri burchakli uchburchakda gipotenuzaga tushirilgan balandlikning xossalarini bilish;*

√ *o‘zaro kesishuvchi vatarlar kesmalari to‘g‘risidagi hamda aylanani kesuvchi to‘g‘ri chiziq kesmalari to‘g‘risidagi xossalarni bilish.*

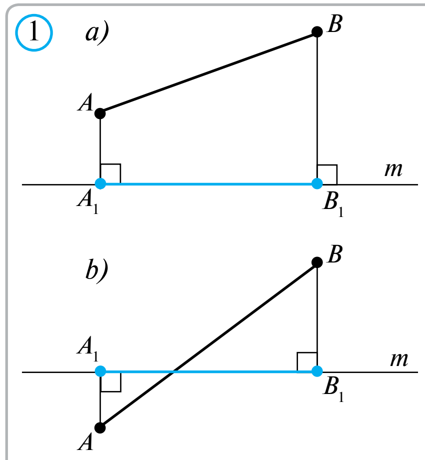
Ko‘nikmalar:

√ *kesmalarning nisbati va proporsional kesmalarga doir masalalarni yecha olish;*

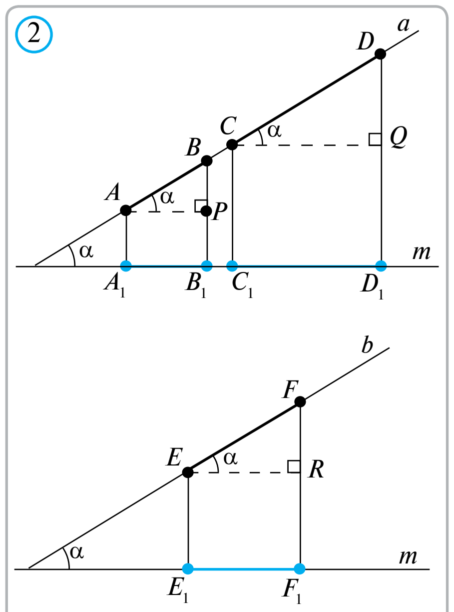
√ *to‘g‘ri burchakli uchburchakda gipotenuzaga tushirilgan balandlikning xossalaridan foydalanib, masalalar yecha olish;*

√ *kesuvchi vatarlar kesmalarining va kesuvchi to‘g‘ri chiziq kesmalarining xossalaridan foydalanib, masalalar yechish.*

52 KESMALAR PROYEKSIYASI VA PROPORSIONALLIK



A_1 — A nuqtaning,
B_1 — B nuqtaning,
A_1B_1 — AB kesmaning ***m* to‘g‘ri chiziqdagi proyeksiyasi**



Faollashtiruvchi mashq

1. Kesmalar nisbati nimani anglatadi?
2. Qanday kesmalar proporsional deyiladi?
3. Fales teoremasini ayting.

Tekislikda m to‘g‘ri chiziq va AB kesma berilgan bo‘lsin. A va B nuqtalardan m to‘g‘ri chiziqqa AA_1 va BB_1 perpendikularlar tushiramiz (*1-rasm*). A_1B_1 kesma AB kesmaning m to‘g‘ri chiziqdagi **proyeksiyasi (soyasi)** deyiladi.

AB kesmaning m to‘g‘ri chiziqdagi A_1A_1 proyeksiyasini qurish amali AB kesmani m to‘g‘ri chiziqqa **proyeksiyalash** deyiladi.

Teorema. **Bir to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotadigan kesmalar berilgan bo‘lsin. Ularning ayni bir to‘g‘ri chiziqqa proyeksiyalari berilgan kesmalarga proporsional bo‘ladi.**

$a \| b$,
A_1B_1 — AB ning,
C_1D_1 — CD ning,
E_1F_1 — EF ning
m to‘g‘ri chiziqdagi proyeksiyalari (2-rasm)

⇒

$$\frac{A_1B_1}{AB}=\frac{C_1D_1}{CD}=\frac{E_1F_1}{EF} \quad (1)$$

Isbot. a) Agar a va b to‘g‘ri chiziqlar m to‘g‘ri chiziqqa parallel bo‘lsa, $AB=A_1B_1$, $CD=C_1D_1$, $EF=E_1F$ bo‘lishi hamda (1) tengliklar o‘rinli ekanligi ravshan.

b) Bordi-yu a va b to‘g‘ri chiziqlar m to‘g‘ri chiziqqa perpendikular bo‘lsa, A_1 va B_1, C_1 va D_1, E_1 va F_1 nuqtalar ustma-ust tushadi. Shuning uchun A_1B_1, C_1D_1, E_1F_1 kesmalarning uzunligi nolga teng bo‘ladi va (1) tengliklar bajariladi.

d) Endi boshqa holni qaraymiz. 2-rasmda tasvirlanganidek, to‘g‘ri burchakli ABP, CDQ, EFR uchburchaklarni quramiz. Unda $a \| b$ bo‘lgani uchun, $\angle BAP=\angle DCQ=\angle FER$. Demak, ABP, CDQ va EFR to‘g‘ri burchakli uchburchaklar o‘xshash. Bundan $\frac{A_1B_1}{AB}=\frac{C_1D_1}{CD}=\frac{E_1F_1}{EF}$ tengliklarni hosil qilamiz. *Teorema isbotlandi.*

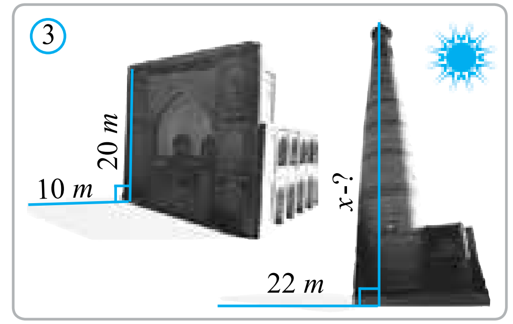
Masala. AB va CD kesmalar parallel to‘g‘ri chiziqlarda yotadi. Agar $AB=12$ *sm*, $CD=15$ *sm* va AB kesmaning biror m to‘g‘ri chiziqdagi proyeksiyasi 8 *sm* bo‘lsa, CD kesmaning shu m to‘g‘ri chiziqdagi proyeksiyasini toping.

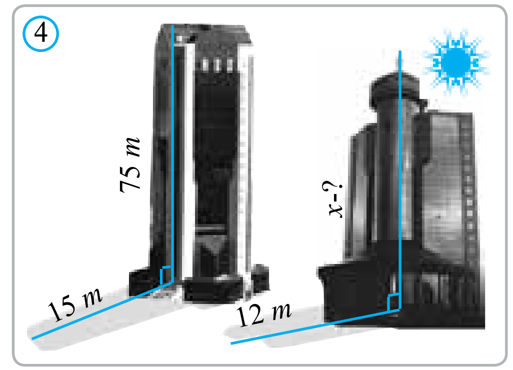
Yechilishi. CD kesmaning m to‘g‘ri chiziqdagi proyeksiyasi x bo‘lsin. Unda, isbotlangan teorema va masala shartidan foydalanib, proporsiya tuzamiz:

$$\frac{x}{15}=\frac{8}{12}.$$

Bu tenglikdan $x=10$ bo‘lishini topamiz.

Javob: 10 *sm*.





Savol, masala va topshiriqlar

1. Kesmaning berilgan to‘g‘ri chiziqdagi proyeksiyasi nima?
2. Bir to‘g‘ri chiziqda yoki parallel to‘g‘ri chiziqlarda yotgan kesmalarning ayni boshqa bir to‘g‘ri chiziqqa proyeksiyalari berilgan kesmalarga proporsional ekanligini isbotlang.
3. a va b to‘g‘ri chiziqlar orasidagi burchak 45° ga teng. a to‘g‘ri chiziqda uzunligi 10 *sm* bo‘lgan AB kesma olingan. AB kesmaning b to‘g‘ri chiziqdagi proyeksiyasini toping.
4. AB kesmaning uchlari l to‘g‘ri chiziqdan 9 *sm* va 14 *sm* uzoqlikda yotadi. Agar AB kesma l to‘g‘ri chiziqni kesib o‘tmasa va $AB=13$ *sm* bo‘lsa, AB kesmaning l to‘g‘ri chiziqdagi proyeksiyasini toping.
5. 3- va 4-rasmlardagi ma’lumotlar asosida binolarning balandliklarini toping.
6. To‘g‘ri chiziq va unga parallel bo‘lmagan kesma chizing. Kesmaning to‘g‘ri chiziqdagi proyeksiyasini yasang.
7. Koordinatalar tekisligida $A(2;3)$ va $B(3;-4)$ nuqtalar belgilangan. AB kesmaning koordinata o‘qlaridagi proyeksiyalarining uzunliklarini toping.
8. a va b to‘g‘ri chiziqlar orasidagi burchak α ekanligi ma’lum. a to‘g‘ri chiziqda AB kesma olingan. AB kesmaning b to‘g‘ri chiziqdagi proyeksiyasini toping.

9*. AB va CD kesmalarning l to‘g‘ri chiziqdagi proyeksiyalari o‘zaro teng. AB va CD kesmalarning uzunliklari haqida nima deyish mumkin? Misollar keltiring.

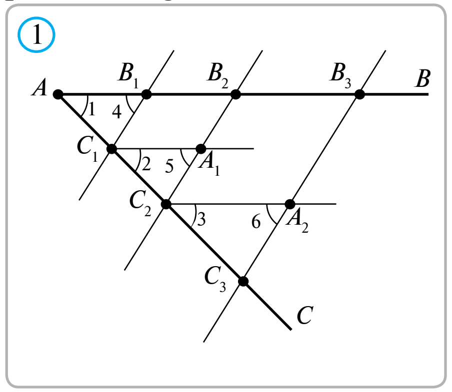
53 PROPORSIONAL KESMALARNING XOSSALARI

Fales teoremasining umumlashmasi bo‘lgan muhim xossani isbotlaymiz.

Teorema. **Burchakning har ikkala tomonini kesib o‘tgan parallel to‘g‘ri chiziqlar uning tomonlaridan proporsional kesmalar ajratadi.**

$\angle BAC, B_1C_1 || B_2C_2 || B_3C_3$ *(1-rasm)* ⇒ $\frac{AB_1}{AC_1}=\frac{B_1B_2}{C_1C_2}=\frac{B_2B_3}{C_2C_3}$

Isbot. C_1 va C_2 nuqtalardan AB ga parallel C_1A_1 va C_2A_2 to‘g‘ri chiziqlarni o‘tkazamiz. U holda, birinchidan, $\angle 1=\angle 2=\angle 3$ bo‘ladi, chunki ular o‘zaro parallel bo‘lgan AB, C_1A_1 va C_2A_2 to‘g‘ri chiziqlarni AC to‘g‘ri chiziq kesganda hosil bo‘lgan mos burchaklardir. Ikkinchidan, $\angle 4=\angle 5=\angle 6$, chunki ular tomonlari parallel bo‘lgan burchaklardir.



Demak, uchburchaklar o‘xshashligining BB alomatiga ko‘ra, $\triangle AB_1C_1 \sim \triangle C_1A_1C_2 \sim \triangle C_2A_2C_3$ bo‘ladi.

U holda, $$\frac{AB_1}{AC_1}=\frac{C_1A_1}{C_1C_2}=\frac{C_2A_2}{C_2C_3} \quad (1)$$

tengliklarni hosil qilamiz.

Bundan tashqari, $B_1C_1A_1B_2$ va $B_2C_2A_2B_3$ to‘rtburchaklar parallelogramm, chunki

$B_1C_1 || B_2C_2 || B_3C_3$ — shartga ko‘ra;

$AB || C_1A_1 || C_2A_2$ — yasashga ko‘ra.

Shuning uchun, bu parallelogrammlarning qarama-qarshi tomonlari o‘zaro teng bo‘ladi:

$$C_1A_1=B_1B_2 \quad \text{va} \quad C_2A_2=B_2B_3. \quad (2)$$

(1) va (2) tengliklardan $\frac{AB_1}{AC_1}=\frac{B_1B_2}{C_1C_2}=\frac{B_2B_3}{C_2C_3}$ bo‘lishi kelib chiqadi.

Teorema isbotlandi.

Amaliy mashq. **Kesmani berilgan nisbatda bo‘lish.**

Berilgan a kesmani to‘rt bo‘lakka shunday bo‘lingki, bo‘laklarning o‘zaro nisbati $m:n:k:l$ kabi bo‘lsin.

Buning uchun quyidagilarni qadam-baqadam bajaramiz:

1-qadam. Ixtiyoriy o‘tkir burchak chizib, uning bir tomoniga uzunliklari $OA=m$, $AB=n$, $BC=l$ va $CD=k$ ga teng bo‘lgan kesmalarni 2-rasmda ko‘rsatilgandek qilib, ketma-ket qo‘yib chiqamiz.

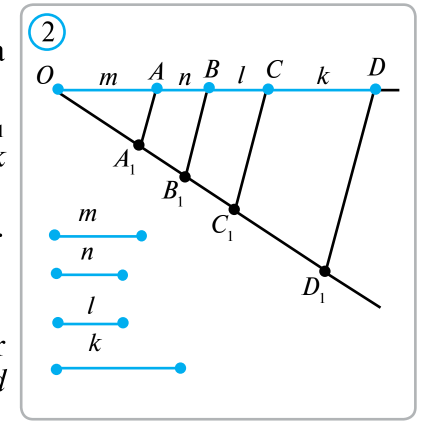
2-qadam. Burchakning ikkinchi tomoniga berilgan a kesmaga teng OD_1 kesmani qo‘yamiz.

3-qadam. D va D_1 nuqtalarni tutashtiramiz.

4-qadam. A, B, C nuqtalar orqali DD_1 ga parallel AA_1, BB_1 va CC_1 kesmalarni o‘tkazamiz.

Yuqoridagi teoremaga ko‘ra, berilgan $a=OD_1$ kesma A_1, B_1, C_1 va D_1 nuqtalar bilan $m:n:l:k$ nisbatda bo‘lingan bo‘ladi.

Topshiriq: Bu tasdiqni mustaqil ravishda asoslang.



Amaliy topshiriq. **To‘rtinchi proporsional kesmani yasash.**

a, b va c kesmalar berilgan. a va b kesmalar c va d kesmalarga proporsional, ya’ni $a:b=c:d$ ekanligi ma’lum. d kesmani yasang *(3-rasm)*.

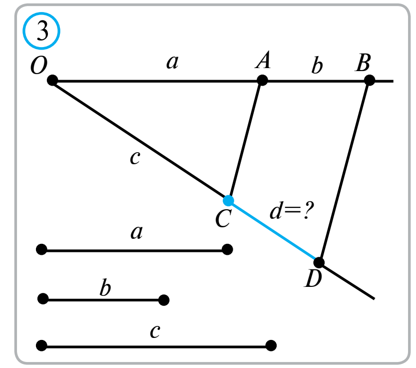
1-qadam. Ixtiyoriy o‘tkir burchak chizib, uning bir tomoniga $OA=a$ va $AB=b$ kesmalarni 3-rasmda ko‘rsatilgandek qo‘yamiz.

2-qadam. Ikkinchi tomoniga esa $OC=c$ kesmani qo‘yamiz.

3-qadam. A va C nuqtalarni tutashtiramiz.

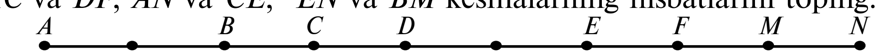
4-qadam. B nuqtadan AC ga parallel BD to‘g‘ri chiziq o‘tkazamiz.

Topshiriq: CD izlanayotgan d kesma bo‘lishini asoslang.



Savol, masala va topshiriqlar

1. Uzunligi 42 *sm* bo‘lgan kesma berilgan. Uni a) 5:2; **b)** 3:4:7; d) 1:5:1:7 nisbatdagi bo‘lakchalarga bo‘ling.
2. Rasmda har bir bo‘lak birlik kesmadan iborat bo‘lsa, AB va CD, EF va MN, AC va DF, AN va CE, EN va BM kesmalarning nisbatlarini toping.



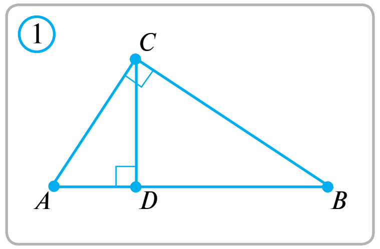
3. m, n kesmalar l va k kesmalarga proporsional. Agar **a)** m=4 *sm*, n=3 *sm* va l=8 *sm*; b) m=2 *sm*, n=3 *sm* va l=7 *sm* bo‘lsa, k — to‘rtinchi kesmani quring va uzunligini toping.
4. To‘rtburchakning perimetri 54 *sm* va tomonlari 3:4:5:6 kabi nisbatda bo‘lsa, uning har bir tomonini aniqlang.
5. To‘rtburchakning burchaklari o‘zaro 3:4:5:6 kabi nisbatda bo‘lsa, uning kichik burchagi nimaga tengligini toping.
6. Uzunligi 4, 5 va 6 bo‘lgan kesmalar berilgan. Uzunligi 4,8 ga teng kesma yasang.

7*. Perimetri 60 *sm* bo‘lgan to‘rtburchakning bir tomoni 15 *sm*, qolgan tomonlari esa 2:3:4 nisbatda ekanligi ma’lum. Uning katta tomonini toping.

54 TO'G'RI BURCHAKLI UCHBURCHAKDAGI PROPORSIONAL KESMALAR

Xossa. To'g'ri burchakli uchburchakning to'g'ri burchagi uchidan tushirilgan balandligi uni o'ziga o'xshash ikkita uchburchakka ajratadi.

$\Delta ABC, \angle C = 90^\circ$, CD — *balandlik* (*1-rasm*) ⇒ $\Delta ABC \sim \Delta ACD$, $\Delta ABC \sim \Delta CBD$



Isbot. ABC va ACD uchburchaklar to'g'ri burchakli bo'lib, A burchak esa ular uchun umumiy. Demak, $\Delta ABC \sim \Delta ACD$. Shu singari, ΔABC va ΔCBD to'g'ri burchakli bo'lib, ular uchun $\angle B$ umumiy. Demak, $\Delta ABC \sim \Delta CBD$.

1-rasmda tasvirlangan AD va DC kesmalar mos ravishda AC va BC katetlarning gipotenuzadagi proeksiyalari deb yuritiladi.

Ta'rif. Agar a, b va c kesmalar uchun $a:b = b:c$ bo'lsa, b kesma a va c kesmalar orasidagi **o'rta proporsional kesma** deb ataladi.

O'rta proporsionallik shartini $b^2 = ac$ yoki $b = \sqrt{ac}$ ko'rinishda yozish mumkin.

Yuqorida isbotlangan xossaga asoslanadigan bo'lsak, o'rta proporsional kesmalar haqidagi quyidagi teoremalar osonlikcha isbotlanadi:

1-teorema. To'g'ri burchakli uchburchakning to'g'ri burchagi uchidan tushirilgan balandlik katetlarning gipotenuzadagi proyeksiyalari orasida o'rta proporsional bo'ladi.

Haqiqatan ham, isbotlangan xossaga ko'ra, $\Delta ACD \sim \Delta CBD$. Bundan,

$$\frac{AD}{CD} = \frac{CD}{BD} \Rightarrow CD^2 = AD \cdot BD \Rightarrow CD = \sqrt{AD \cdot BD}.$$

2-teorema. To'g'ri burchakli uchburchakning kateti gipotenuza bilan shu katetning gipotenuzadagi proyeksiyasi orasida o'rta proporsionaldir (*1-rasm*).

Haqiqatan ham, isbotlangan xossaga ko'ra, $\Delta ABC \sim \Delta ACD$. Bundan,

$$\frac{AB}{AC} = \frac{AC}{AD} \Rightarrow AC^2 = AB \cdot AD \Rightarrow AC = \sqrt{AB \cdot AD}.$$

Xuddi shunga o'xshash $BC = \sqrt{BD \cdot AB}$ ekanligini isbotlash mumkin.

Masala. Katetlari 15 *sm* va 20 *sm* bo'lgan to'g'ri burchakli uchburchak kichik katetining gipotenuzadagi proyeksiyasini toping.

$\Delta ABC, \angle C = 90^\circ$, CD — *balandlik*, $AC = 15$ *sm*, $BC = 20$ *sm* (*1-rasm*) ⇒ $AD = ?$

Yechilishi. 1) Pifagor teoremasidan foydalanib, uchburchak gipotenuzasini topamiz: $AC^2 = AC^2 + BC^2 = 15^2 + 20^2 = 625$, ya'ni $AB = 25$ *sm*.

2) Ikkinchi teoremadan foydalanib, AD ni topamiz:

$$AC^2 = AB \cdot AD \Rightarrow AD = \frac{AC^2}{AB} = \frac{15^2}{25} = 9 \ (sm).$$

Javob: 9 *sm*.

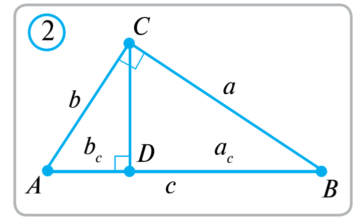
Ikkinchi teoremadan natija sifatida Pifagor teoremasining **Pifagorning o'zi yozib qoldirgan isboti** kelib chiqadi *(1-rasm)*. 2- teoremaga ko'ra,

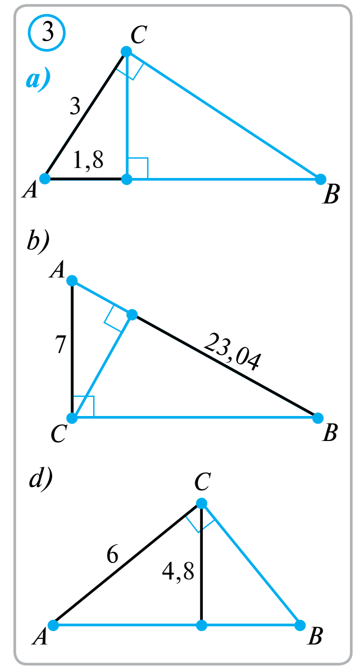
$$\left.\begin{array}{l} AC^2 = AD \cdot AB \\ BC^2 = BD \cdot AB \end{array}\right\} \Rightarrow AC^2 + BC^2 = AD \cdot AB + BD \cdot AB = AB \cdot (\underbrace{AD + BD}_{AB}) = AB \cdot AB = AB^2.$$

Shunday qilib, $AC^2 + BC^2 = AB^2$.

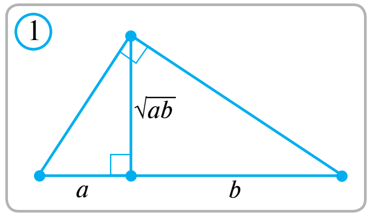
Savol, masala va topshiriqlar

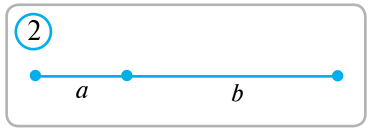
1. Isbotlang *(2-rasm)*:
   a) $\Delta ACD \sim \Delta CBD \sim \Delta ABC$;
   b) $b^2 = b_c \cdot c$, $a^2 = a_c \cdot c$; d) $h_c^2 = a_c \cdot b_c$.
2. To'g'ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligi gipotenuzani 9 *sm* va 16 *sm* ga teng kesmalarga bo'ladi. Uchburchak tomonlarini toping.
3. To'g'ri burchakli uchburchakning gipotenuzasi 15 *sm* ga, bir kateti esa 9 *sm* ga teng. Ikkinchi katetning gipotenuzadagi proyeksiyasini toping.
4. 3-rasmdagi ma'lumotlar asosida ABC uchburchakning tomonlarini toping.
5*. Katetlarining nisbati 4:5 kabi bo'lgan to'g'ri burchakli uchburchak katetlarining gipotenuzadagi proyeksiyalari nisbatini toping.
6*. Katetlarining nisbati 3:2 kabi bo'lgan to'g'ri burchakli uchburchak berilgan. Katetlarning gipotenuzasidagi proyeksiyalaridan biri ikkinchisidan 6 *sm* ga uzun. Uchburchak yuzini toping.
7. Katetlarining gipotenuzasidagi proyeksiyalari 2 *sm* va 18 *sm* bo'lgan to'g'ri burchakli uchburchak yuzini toping.
8*. ABC uchburchakda $\angle C = 90^\circ$, CD — balandlik, CE — bissektrisa va $AE:EB = 2:3$. a) $AC:BC$; b) $S_{ACE}:S_{BCE}$; d) $AD:BD$ nisbatlarni toping.

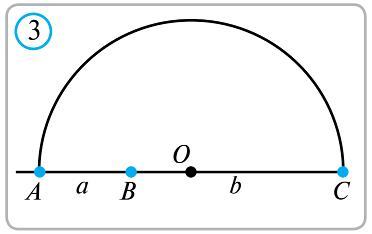


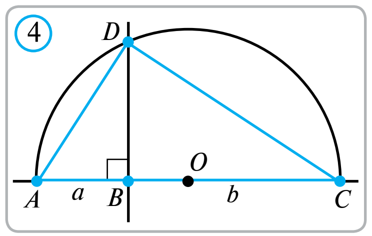


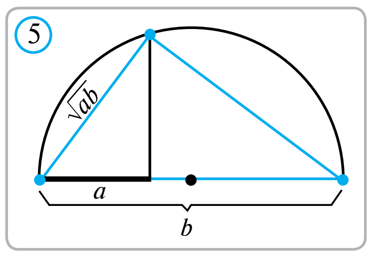
55 BERILGAN IKKITA KESMAGA O'RTA PROPORSIONAL KESMANI YASASH











To'g'ri burchakli uchburchakning to'g'ri burchagidan tushirilgan balandligi gipotenuzani a va b kesmalarga bo'lsa, balandlik $\sqrt{ab}$ ga teng bo'lishini ko'rgan edik (*1-rasm*).

Demak, berilgan ikki kesmaga o'rta proporsional kesmani yasash uchun:

1) gipotenuzasining uzunligi $a+b$ ga teng (*2-rasm*);

2) to'g'ri burchagidan tushirilgan balandligi shu gipotenuzani a va b bo'laklarga bo'ladigan to'g'ri burchakli uchburchak yasash kifoya.

Buning uchun to'g'ri burchakli uchburchakka tashqi chizilgan aylana markazi gipotenuzaning o'rtasida joylashganidan foydalanamiz (*3-rasm*).

Yasash:

1) To'g'ri chiziq chizamiz va unda $AB=a$ va $BC=b$ bo'ladigan qilib A, B va C nuqtalarni belgilaymiz (*3-rasm*).

2) AC kesmaning o'rtasi — O nuqtani topamiz. Markazi O nuqtada bo'lgan AC diametrli yarim aylana yasaymiz *(3-rasm)*.

3) B nuqtadan AC to'g'ri chiziqqa perpendikular to'g'ri chiziq o'tkazamiz *(4-rasm)*. Bu to'g'ri chiziq yarim aylanani D nuqtada kesib o'tgan bo'lsin. Unda ΔADC — to'g'ri burchakli uchburchak, $BD=\sqrt{ab}$ — biz yasashimiz zarur bo'lgan kesma bo'ladi.

Yasash bajarildi.

O'rta proporsional kesmani yasashda to'g'ri burchakli uchburchakning kateti gipotenuza bilan shu katetning gipotenuzadagi proyeksiyasi orasida o'rta proporsional ekanligidan foydalanish ham mumkin *(5-rasm)*.

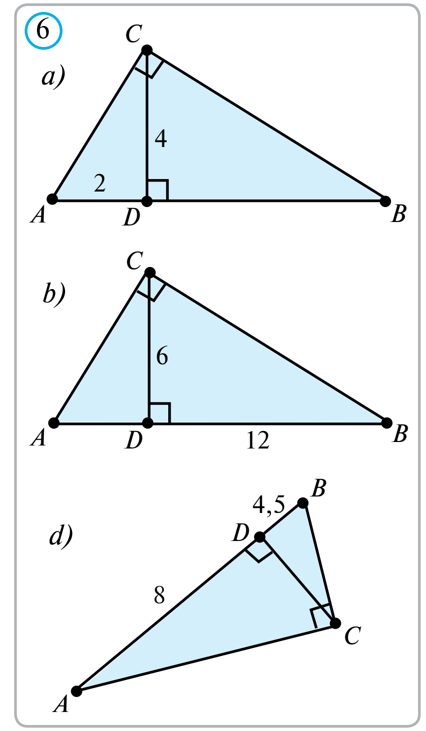
Savol, masala va topshiriqlar

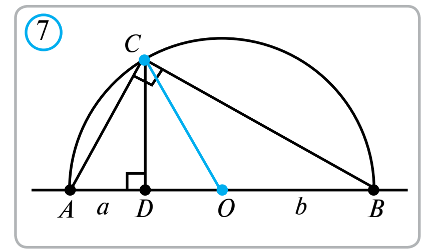
1. Uzunliklari a va b bo'lgan kesmalar berilgan. Uzunligi $\sqrt{ab}$ bo'lgan kesmani yasang.
2. Uzunligi a va b ga teng kesmalar berilgan. Pifagor teoremasidan foydalanib, uzunligi
   a) $\sqrt{a^2+b^2}$; b) $\sqrt{a^2-b^2}$
   bo'lgan kesmalarni yasang.
3. Uzunligi 1 ga teng kesma berilgan. Uzunligi a) $\sqrt{2}$; b) $\sqrt{3}$; d) $\sqrt{5}$; e) $\sqrt{6}$; f) $\sqrt{18}$; g) $\sqrt{30}$ bo'lgan kesmalarni yasang.
4. 6-rasmdagi ma'lumotlar asosida ABC uchburchakning yuzini toping.
5. Aylanadagi C nuqtadan AB diametrga CD perpendikular tushirilgan. Agar $CD=12$ *sm*, $AD=24$ *sm* bo'lsa, doira yuzini toping.
6. Oldingi masaladagi ABC uchburchak yuzini toping.
7. To'g'ri burchakli uchburchak to'g'ri burchagining bissektrisasi gipotenuzani 5:3 kabi nisbatda bo'ladi. To'g'ri burchak uchidan tushirilgan balandlikning gipotenuzadan ajratgan kesmalari nisbatini toping.
8. Radiusi 8 *sm* ga teng doiraga bir burchagi 30° bo'lgan to'g'ri burchakli uchburchak ichki chizilgan. Doiraning uchburchakdan tashqaridagi qismi 3 ta segmentdan iborat. Ana shu segmentlar yuzlarini toping.

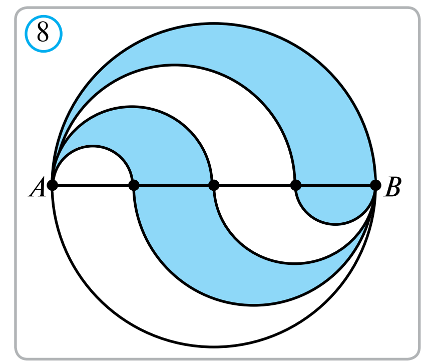
9*. 7-rasmda $AD=a$, $DB=b$, demak, $OC=\frac{a+b}{2}$ (O— aylana markazi). Rasmdan foydalanib, $\frac{a+b}{2}\geq\sqrt{ab}$ tengsizlikni isbotlang.

Qiziqarli masala

Aylananing AB diametri to'rtta teng bo'lakka bo'lindi va 8-rasmda ko'rsatilgandek yarim aylanalar yasaldi. Agar $AB=d$ bo'lsa, rasmda bo'yab ko'rsatilgan har bir shakl yuzini hisoblang.

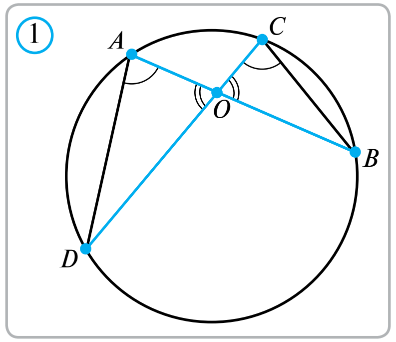






56 AYLANADAGI PROPORSIONAL KESMALAR

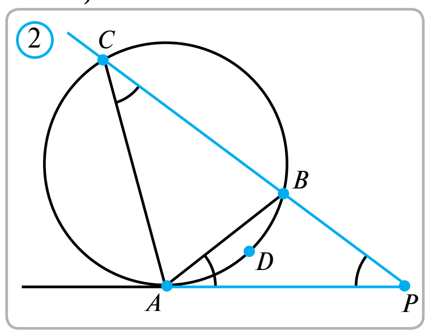
1-teorema. Aylananing AB va CD vatarlari O nuqtada kesishsa, $AO\cdot OB=$ $=CO\cdot OD$ tenglik o‘rinli bo‘ladi.



Isbot. AB va CD vatarlar (*1-rasm*) ko‘rsatilgan tartibda joylashgan bo‘lsin. Uchlarini AD va BC vatarlar bilan tutashtiramiz. Shunda BAD va BCD burchaklar bitta yoyga tiraladi, demak, $\angle BAD=\angle BCD$. Yana ravshanki, $\angle AOD=\angle BOC$. Bu ikki tenglikdan, BB alomatga ko‘ra, AOD va COB uchburchaklarning o‘xshashligi kelib chiqadi. O‘xshash uchburchaklar mos tomonlari esa proporsional: $\frac{OD}{OB}=\frac{AO}{CO}$ yoki $AO\cdot OB=CO\cdot OD$.

Teorema isbotlandi.

2-teorema. Aylana tashqi sohasidagi P nuqtadan aylanaga PA urinma (A — urinish nuqtasi) va aylanani B va C nuqtalarda kesib o‘tuvchi to‘g‘ri chiziq o‘tkazilgan bo‘lsa, $PA^2=PB\cdot PC$ bo‘ladi.



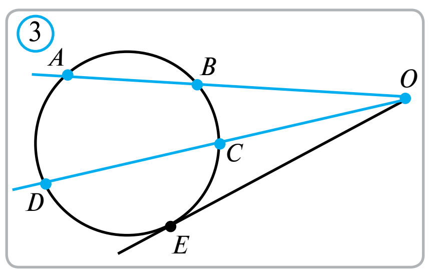
Isbot. ABP va CPA uchburchaklarni qaraymiz *(2-rasm)*. Unda,

$\angle C=\frac{\overset{\frown}{AOB}}{2}=\angle BAP$ hamda $\angle P$ — bu uchburchaklar uchun umumiy burchak. Demak, ABP va CPA uchburchaklar ikki burchagi bo‘yicha o‘xshash.

Bundan, $\frac{PA}{PC}=\frac{PB}{PA}$ yoki $PA^2=PB\cdot PC$.

Teorema isbotlandi.

Masala. A, B, C va D nuqtalar aylanani AB, BC, CD va AD yoylarga ajratadi. Agar AB va DC nurlar O nuqtada kesishsa, u holda $OA\cdot OB=OC\cdot OD$ tenglik o‘rinli bo‘lishini isbotlang.



Yechilishi. Masala shartiga mos chizma chizamiz *(3-rasm)* va O nuqtadan OE urinma o‘tkazamiz. Unda, 2-teoremaga ko‘ra,

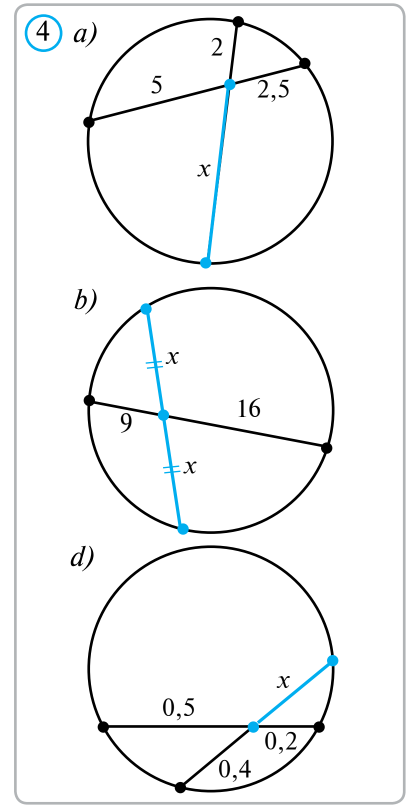
$$\left.\begin{array}{l} OB\cdot OA=OE^2 \\ OC\cdot OD=OE^2 \end{array}\right\} \Rightarrow OA\cdot OB=OC\cdot OD.$$

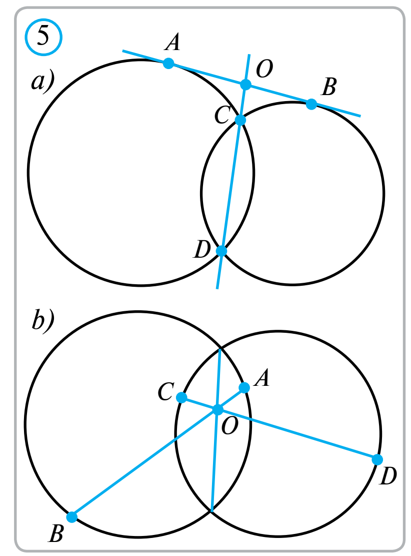
Savol, masala va topshiriqlar

1. 4-rasmda x bilan belgilangan noma’lum kesmani toping.
2. A nuqtadan aylanaga AB urinma (B — urinish nuqtasi) va aylanani C va D nuqtalarida kesadigan kesuvchi o‘tkazilgan. Agar
   a) $AB=4$ *sm*, $AC=2$ *sm* bo‘lsa, AD kesmani;
   b) $AB=5$ *sm*, $AD=10$ *sm* bo‘lsa, AC kesmani;
   d) $AC=3$ *sm*, $AD=2{,}7$ *sm* bo‘lsa, AB kesmani toping.
3. Aylanaga $ABCD$ to‘rtburchak ichki chizilgan. AB va DC nurlar O nuqtada kesishadi. Agar
   a) $AO=10$ *dm*, $BO=6$ *dm*, $DO=15$ *dm* bo‘lsa, OC kesmani;
   b) $CD=10$ *dm*, $OD=8$ *dm*, $AB=4$ *dm* bo‘lsa, OB kesmani toping.
4. Aylananing AB diametri va bu diametrga perpendikular CD vatari E nuqtada kesishadi. Agar $AE=2$ *sm*, $EB=8$ *sm* bo‘lsa, CD vatarni toping.
5. AB va CD kesmalar O nuqtada kesishadi. Agar $AO\cdot OB=BO\cdot OD$ bo‘lsa, A, B, C va D nuqtalarning bir aylanada yotishini isbotlang.
6. Radiusi 13 *dm* bo‘lgan aylana markazidan 5 *dm* uzoqlikda P nuqta olingan. P nuqtadan uzunligi 25 *dm* bo‘lgan AB vatar o‘tkazilgan. AP va PB kesmalarni toping.
7. 3-rasmda $AO\cdot OB=CO\cdot OD$ tenglikni AOD va BOC uchburchaklarning o‘xshash ekanligidan foydalanib isbotlang.

8*. 5-rasmlardagi ma’lumotlar asosida $AO\cdot OB=$ $=CO\cdot OD$ tenglikni isbotlang.

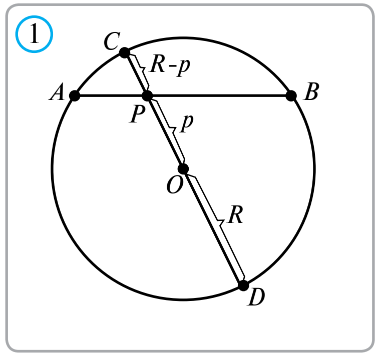
9*. Ikki aylana C nuqtada urinadi. AB to‘g‘ri chiziq birinchi aylanaga A nuqtada, ikkinchi aylanaga esa B nuqtada urinadi. $\angle ACB=90°$ ekanligini isbotlang.





57 MASALALAR YECHISH

Oldingi darsda aylana kesuvchilari va vatarlarining xossalarini isbotlagan edik. Endi shu xossalarning ayrim xususiy hollari bilan tanishamiz.



1-masala. R radiusli aylananing ichki sohasidagi P nuqta aylana markazidan p masofada joylashgan bo‘lsin. Unda P nuqtadan o‘tuvchi ixtiyoriy AB vatar uchun

$$AP \cdot PB = R^2 - p^2 \qquad (1)$$

tenglik o‘rinli bo‘lishini isbotlang.

Yechilishi. P nuqta orqali aylananing CD diametrini o‘tkazamiz. Unda, $PC = R - p$, $PD = R + p$ (*1-rasm*). Kesuvchi vatarlar haqidagi teoremaga ko‘ra,

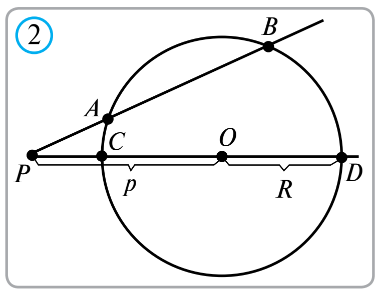
$$AP \cdot PB = CP \cdot PD = (R - p)(R + p) = R^2 - p^2.$$

(1) tenglik isbotlandi.

2-masala. Radiusi 6 *sm* bo‘lgan aylananing O markazidan 4 *sm* uzoqlikda P nuqta olindi. P nuqta orqali AB vatar o‘tkazildi. Agar $AP = 2$ *sm* bo‘lsa, PB kesmani toping.

Yechilishi. Masala shartiga ko‘ra, $R = 6$ *sm*, $d = 4$ *sm*, $AP = 2$ *sm*. U holda (1) tenglikka ko‘ra, $2 \cdot PB = 6^2 - 4^2 = 36 - 16 = 20$. Bundan, $PB = 10$ *sm*.

Javob: $PB = 10$ *sm*.



3-masala. R radiusli aylananing tashqi sohasidagi P nuqta aylana markazidan p masofada joylashgan bo‘lsin. Unda P nuqta orqali o‘tuvchi va aylanani A va B nuqtalarda kesuvchi ixtiyoriy to‘g‘ri chiziq uchun

$$PA \cdot PB = p^2 - R^2 \qquad (2)$$

tenglik o‘rinli bo‘lishini isbotlang.

Yechilishi. Aylananing O markazi orqali o‘tuvchi PO to‘g‘ri chiziq aylana bilan C va D nuqtalarda kesishsin (*2-rasm*). Unda, shartga ko‘ra, $PC = p - R$, $PD = p + R$. Aylana tashqi sohasidagi nuqtadan o‘tkazilgan kesuvchilar haqidagi teoremaga ko‘ra,

$$PA \cdot PB = PC \cdot PD = (p - R)(p + R) = p^2 - R^2.$$

Shunday qilib (2) tenglik isbotlandi.

4-masala. Radiusi 7 *sm* bo‘lgan aylananing markazidan 13 *sm* uzoqlikdagi P nuqtadan o‘tuvchi to‘g‘ri chiziq aylanani A va B nuqtalarda kesadi. Agar $PA = 10$ *sm* bo‘lsa, AB vatarni toping.

Yechilishi. Shartga ko‘ra, $R = 7$ *sm*, $p = 13$ sm. U holda (2) formulaga ko‘ra,

$$PA \cdot PB = p^2 - R^2 = 13^2 - 7^2 = 169 - 49 = 120.$$

Bundan, $PB = \dfrac{120}{PA} = \dfrac{120}{10} = 12$ (*sm*). Demak,

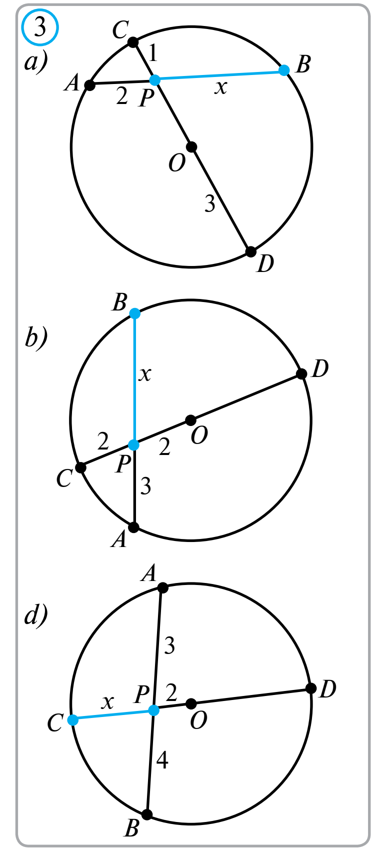
$AB = PB - PA = 12 - 10 = 2$ (*sm*). ***Javob:*** 2 *sm*.

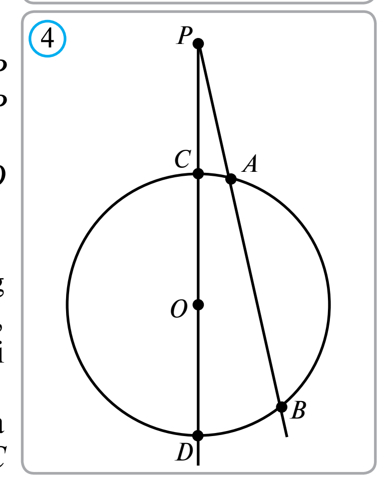
Savol, masala va topshiriqlar

1. Radiusi 5 *sm* bo‘lgan aylana markazidan 3 *sm* uzoqlikda P nuqta olingan. AB vatar P nuqta orqali o‘tadi. Agar $PA = 2$ *sm* bo‘lsa, AB vatar uzunligini toping.
2. Radiusi 5 *m* bo‘lgan aylana markazidan 7 *m* uzoqlikda P nuqta olingan. P nuqta orqali o‘tuvchi to‘g‘ri chiziq aylanani A va B nuqtada kesadi. Agar $PA = 4\,m$ bo‘lsa, AB vatar uzunligini toping.
3. 3-rasmdagi ma’lumotlar asosida x bilan belgilangan kesmani toping (O — aylana markazi).
4. 4-rasmdan foydalanib, masalani yeching. Unda,
   a) $PC = 5$ *dm*, $OD = 7$ *dm*, $AB = 2$ *dm*, $PA = ?$
   b) $PA = 5$ *dm*, $AB = 4$ *dm*, $PC = 3$ *dm*, $OD = ?$
5. Aylananing $AB = 7$ *sm* va $CD = 5$ *sm* vatarlari P nuqtada kesishadi. Agar $CP : PD = 2 : 3$ bo‘lsa, P nuqta AB vatarni qanday nisbatda bo‘ladi?
6. Aylananing C nuqtasidan AB diametrga CD perpendikular tushirilgan. Agar $AD = 2$ *sm*, $DB = 18$ *sm* bo‘lsa, CD kesmani toping.

7*. Aylanaga ichki chizilgan $ABCD$ to‘rtburchakning diagonallari K nuqtada kesishadi. Agar $AB = 2$, $BC = 1$, $CD = 3$ va $CK : KA = 1 : 2$ bo‘lsa, AD kesmani toping.

8*. Aylanaga ichki chizilgan $ABCD$ to‘rtburchakda $AB : DC = 1 : 2$ va $BD : AC = 2 : 3$ bo‘lsa, $DA : BC$ nisbatni toping.





58 IV BOBGA DOIR QO‘SHIMCHA MASALALAR VA MA’LUMOTLAR

I. Testlar

1. **To‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligi haqida noto‘g‘ri tasdiqni ko‘rsating:**
   A. Katetlaridan kichik;
   B. Uchburchakni ikkita o‘xshash uchburchaklarga ajratadi;
   D. Katetlarining gipotenuzadagi proyeksiyalari orasida o‘rta proporsional;
   E. Gipotenuzaning yarmiga teng.
2. ***AB* va *CD* vatarlar *O* nuqtada kesishadi. Noto‘g‘ri tasdiqni toping:**
   A. $\angle DAB=\angle DCB$; B. *AOD* va *COB* uchburchaklar o‘xshash;
   D. $AO\cdot OB=CO\cdot OD$; E. $AO=CO$.
3. **To‘g‘ri tasdiqni toping:**
   A. Teng kesmalarning proyeksiyalari ham teng bo‘ladi;
   B. Katta kesmaning proyeksiyasi katta bo‘ladi;
   D. Bir to‘g‘ri chiziqdagi teng kesmalarning proyeksiyalari teng bo‘ladi;
   E. Proyeksiya uzunligi proyeksiyalanuvchi kesma uzunligiga teng bo‘ladi.
4. **To‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandlik uni ikkita uchburchakka ajratadi. Bu uchburchaklar:**
   A. Teng; B. Tengdosh; D. O‘xshash; E. Teng yonli.
5. **Uzunligi *a* va *b* bo‘lgan kesmalarning o‘rta proporsionali nimaga teng?**
   A. $a+b$; B. $\sqrt{ab}$; D. $\frac{a+b}{2}$; E. $a:b$.
6. ***ABCD* to‘rtburchak *O* markazli aylanaga ichki chizilgan. Noto‘g‘ri tasdiqni ko‘rsating:**
   A. $\Delta AOB \backsim \Delta COD$; B. $\angle A+\angle C=\angle B+\angle D$;
   D. $AO\cdot OB=CO\cdot OD$; E. $AB\cdot CD=BC\cdot AD$.

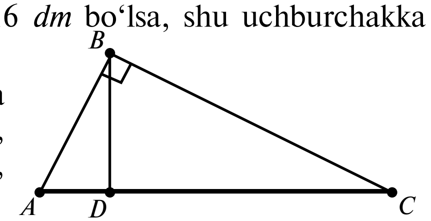
II. Masalalar.

1. To‘g‘ri burchakli uchburchak katetlarining nisbati 3:4 ga teng. Bu uchburchakning gipotenuzasi 50 *sm*. Uchburchakning to‘g‘ri burchagi uchidan tushirilgan balandligi gipotenuzadan qanday uzunlikdagi kesmalar ajratadi?
2. Aylananing *AB* va *CD* vatarlari *E* nuqtada kesishadi. Agar $AE=5$ *sm*, $BE=2$ *sm* va $EC=2{,}5$ *sm* bo‘lsa, *ED* ni toping.
3. Radiusi 6 *m* bo‘lgan aylananing markazidan 10 *m* uzoqlikda *K* nuqta olindi va *K* nuqtadan aylanaga urinma o‘tkazildi. Urinmaning urinish nuqtasi *P* bilan *K* nuqta orasidagi masofani toping.
4. *ABC* uchburchakda $\angle C=90°$ va *CD* balandlik 4,8 *dm*. Agar $AD=3{,}6$ *dm* bo‘lsa, *AB* tomonni toping.

5. Aylananing *AB* va *CD* vatarlari *O* nuqtada kesishadi. Agar $AO=6$, $OB=4$ va $CO=3$ bo‘lsa, *OD* kesmani toping.
6. Aylanada *A*, *B*, *C*, *D* nuqtalar belgilangan, *BA* va *CD* nurlar *O* nuqtada kesishadi. Agar $OA=5$, $AB=4$, $OD=6$ bo‘lsa, *DC* vatarni toping.
7. Aylanaga *B* nuqtada urinuvchi to‘g‘ri chiziq ustida *A* nuqta olindi. Agar $AB=12$ va *A* nuqtadan aylanagacha bo‘lgan eng qisqa masofa 8 bo‘lsa, aylana radiusini toping.
8. Yarim aylanadagi *C* nuqtadan *AB* diametrga tushirilgan *CD* perpendikular *AB* kesmada 4 va 9 ga teng kesmalar ajratadi. *CD* kesmani toping.
9. To‘g‘ri burchakli uchburchakning balandligi gipotenuzani 3 *dm* va 12 *dm* ga teng kesmalarga bo‘ladi. Uchburchak yuzini toping.
10. Radiusi 5 *sm* bo‘lgan *O* markazli aylananing *AB* vatarida *D* nuqta olingan. Agar $AD=2$ *sm*, $DB=4{,}5$ *sm* bo‘lsa, *OD* kesmani toping.
11. Radiusi 5 *m* bo‘lgan *O* markazli aylanani *A* va *B* nuqtalarda kesuvchi to‘g‘ri chiziqda *P* nuqta olindi. Agar $PA=5$ *m*, $AB=2{,}8$ *m* bo‘lsa, *OP* masofani toping.
12. To‘rtta parallel to‘g‘ri chiziq berilgan. Ular burchak tomonlarini *A* va A_1, *B* va B_1, *C* va C_1 hamda *D* va D_1 nuqtalarda kesadi. Bunda *A, B, C, D* nuqtalar burchakning bitta tomonida yotadi. Agar $AB=8$, $CD=12$ va $C_1D_1=9$ bo‘lsa, A_1B_1 kesmani toping.
13. Aylana burchakka ichki chizilgan. Agar burchak uchidan aylanagacha bo‘lgan masofa radiusga teng bo‘lsa, burchak kattaligini toping.
14. Aylanaga *AB* diametrning *B* uchidan *BC* urinma va *AC* kesuvchi o‘tkazilgan. *AC* aylana bilan *D* nuqtada kesishadi. Agar $AD=DC$ bo‘lsa, *CBD* burchakni toping.
15. To‘g‘ri burchakli uchburchakning katetlari nisbati 2:3 kabi. Uchburchakning gipotenuzasiga tushirilgan balandlik uni ikkita uchburchakka bo‘ladi. Ular yuzlarining nisbatini toping.

III. O‘zingizni sinab ko‘ring (namunaviy nazorat ishi)

1. Aylana tashqarisidagi nuqtadan aylanaga urinma o‘tkazilgan. Bu nuqtadan aylanagacha bo‘lgan eng qisqa masofa 2 *sm* ga, urinish nuqtasigacha bo‘lgan masofa esa 6 *sm* ga teng. Aylananing radiusini toping.
2. ΔABC to‘g‘ri burchakli, $AD=9$ *dm*, $DC=16$ *dm* bo‘lsa, shu uchburchakka ichki chizilgan aylana radiusini hisoblang.

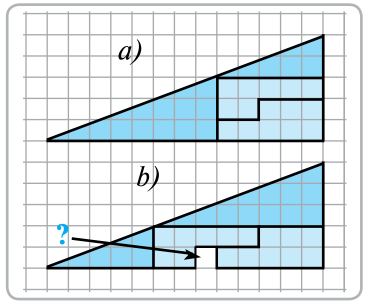


3. Nuqtadan to‘g‘ri chiziqqa ikkita og‘ma o‘tkazilgan. Agar og‘malar 1:2 nisbatda bo‘lib, ularning proyeksiyalari 1 *m* va 7 *m* bo‘lsa, og‘malarning uzunliklarini toping.

4.* (Qo‘shimcha masala) PQ va undan uzun ET kesmalar berilgan. Shunday $ABCD$ to‘rtburchak yasangki, $AB=BC=PQ$; $BD=ET$ bo‘lib, diagonallari kesishadigan O nuqta uchun $AO\cdot OC=BO\cdot OD$ tenglik o‘rinli bo‘lsin.

Qiziqarli masala

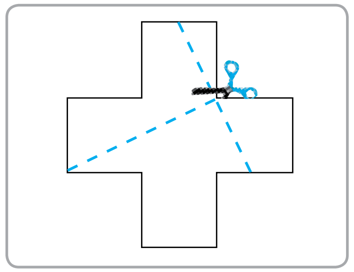
Uchburchak 4-a rasmda ko‘rsatilganidek qilib to‘rtta bo‘lakka bo‘lingan va 4-b rasmda ko‘rsatilganidek qilib qayta yig‘ilgan. Ayting-chi, ortiqcha kvadrat qayerdan paydo bo‘lib qoldi?

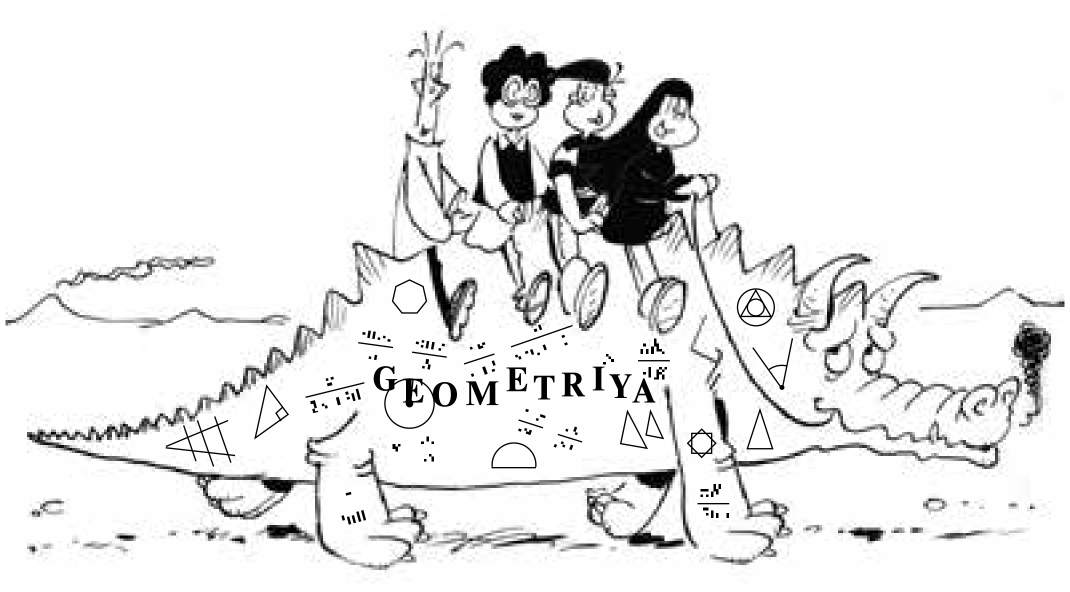


Yunon xochi

Eramizdan avvalgi 500-yillarda paydo bo‘lgan bu shaklni hayotning ramzi sifatida non ustiga chizganlar.

Bu shaklni qalin qog‘ozga chizib olib, uni rasmda ko‘rsatilgan chiziqlar bo‘ylab qirqing. Hosil bo‘lgan bo‘laklardan kvadrat yasash mumkinligiga ishonch hosil qiling.





V BOB

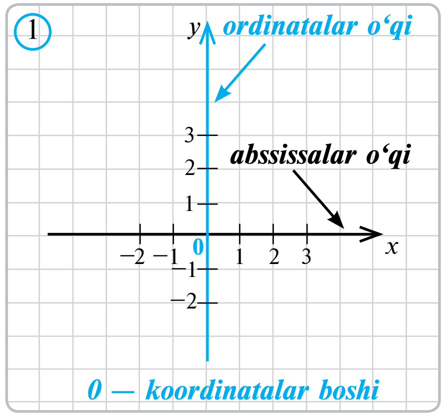
PLANIMETRIYA KURSI BO‘YICHA TAKRORLASH

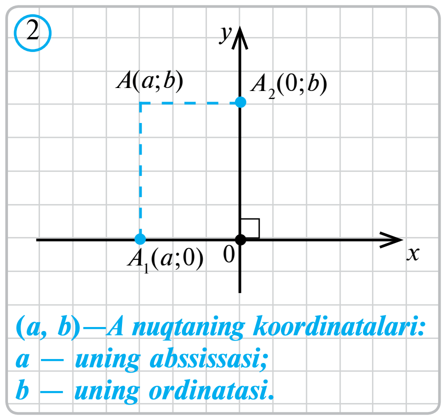
Ushbu bobni o‘rganish natijasida siz quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

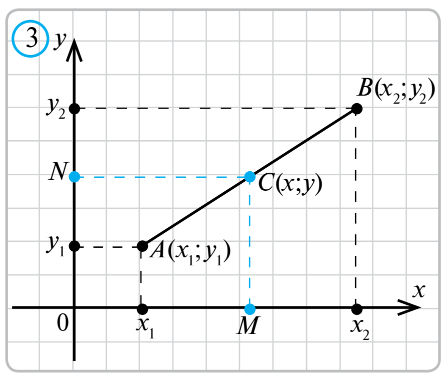
- √ *geometriyaning planimetriya qismi bo‘yicha o‘tilgan mavzularni esga olish;*
- √ *planimetriya kursi bo‘yicha o‘zlashtirilgan bilim, ko‘nikma va malakalarni mustahkamlash;*
- √ *yakunlovchi nazorat ishiga tayyorgarlik ko‘rish.*

59 KOORDINATALAR USULI

Tekislikdagi to‘g‘ri burchakli koordinatalar sistemasi bilan 7-sinf algebra kursida tanishgansiz *(1–2-rasmlar)*. Quyida shu mavzuga oid geometrik masalalarni qaraymiz.







1-masala. Uchlari koordinatalar tekisligining birinchi choragida bo‘lgan AB kesma berilgan bo‘lsin: $A(x_1;y_1)$ va $B(x_2;y_2)$, $x_1>0$, $y_1>0$, $x_2>0$, $y_2>0$ *(3-rasm)*. AB kesmaning o‘rtasi bo‘lgan $C(x;y)$ nuqtaning koordinatalarini toping.

Yechilishi. Bu holatda CN kesma asoslarining uzunliklari x_1 va x_2 bo‘lgan trapetsiyaning o‘rta chizig‘i, CM kesma esa asoslarining uzunliklari y_1 va y_2 bo‘lgan trapetsiyaning o‘rta chizig‘i bo‘ladi.

Trapetsiya o‘rta chizig‘i xossasiga ko‘ra,

$$x=\frac{x_1+x_2}{2};\quad y=\frac{y_1+y_2}{2} \qquad (1)$$

bo‘ ladi.

Bu formulalarning to‘g‘riligini AB kesmaning boshqa holatlari uchun ham shunga o‘xshash mushohada bilan ko‘rsatish mumkin.

2-masala. Uchlari $A(-1;-2)$, $B(2;-5)$, $C(1;-2)$, $D(-2;1)$ nuqtalarda bo‘lgan $ABCD$ to‘rtburchakning parallelogramm ekanligini isbotlang.

Yechilishi. (1) formuladan foydalanib, to‘rtburchakning AC va BD diagonallari o‘rtasining koordinatalarini topamiz:

$$AC:\ x=\frac{-1+1}{2}=0,\qquad y=\frac{-2+(-2)}{2}=-2;$$

$$BD:\ x=\frac{2+(-2)}{2}=0,\qquad y=\frac{-5+1}{2}=-2.$$

Demak, $ABCD$ to‘rtburchakning har ikki diagonali o‘rtasi bitta $(0;-2)$ nuqta bo‘lar ekan. Boshqacha qilib aytganda, $ABCD$ to‘rtburchak diagonallari $(0;-2)$ nuqtada kesishadi va shu nuqtada teng ikkiga bo‘linadi. Bu $ABCD$ to‘rtburchakning parallelogramm bo‘lishi alomatlaridan biridir.

Savol, masala va topshiriqlar

1. Ko‘pburchaklarning yuzlarini hisoblang *(4-rasm)*.

2. Aylananing 8 *sm* ga teng vatari aylanadan 90° ga teng yoy ajratadi. Aylana markazidan vatargacha bo‘lgan masofani toping.

3. Tomonlari a) 5, 5 va 6; b) 17, 65, 80 bo‘lgan uchburchak yuzini toping.

4. Tomonlari a) 13, 13, 12; b) 35, 29, 8 bo‘lgan uchburchakka ichki chizilgan aylana radiusini toping.

5. Uchlari quyidagicha bo‘lgan kesmalar o‘rtasi koordinatalarini toping:
a) $A(1;-2)$, $B(5;6)$;
b) $A(4;-3)$, $B(1;2)$;
d) $A(-4;5)$, $B(2;3)$;
e) $A(-0{,}7;2)$, $B(-0{,}3;4{,}2)$.

6*. Agar $A(1;0)$, $B(2;3)$, $C(3;2)$ bo‘lsa, $ABCD$ parallelogrammning D uchi koordinatalarini toping.

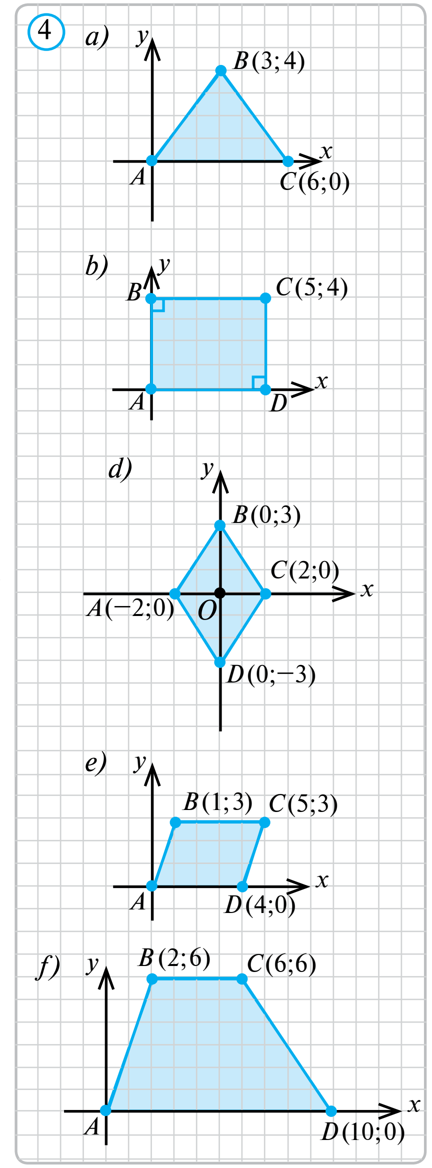
7*. Parallelogramm burchaklari bissektrisalari kesishgan nuqtalar to‘g‘ri to‘rtburchak uchlari bo‘lishini isbotlang.

8. Katetlari 40 *sm* va 30 *sm* bo‘lgan to‘g‘ri burchakli uchburchakka ichki va tashqi chizilgan aylanalarning radiusini toping.

9. Aylanaga ichki chizilgan to‘rtburchakning uchta burchagi 2:3:4 kabi nisbat hosil qilishi ma’lum. Uning burchaklarini toping.

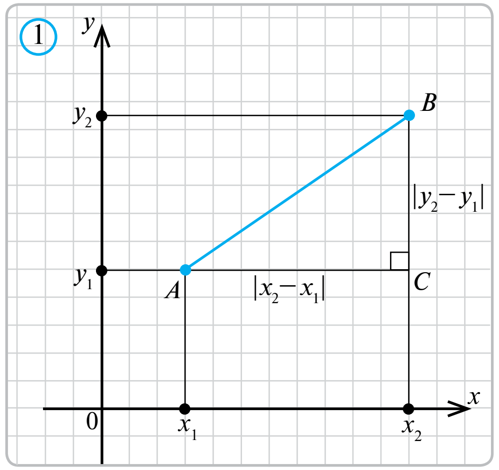
10. Radiusi 6 *sm* bo‘lgan aylananing 60° ga teng yoyini tortib turgan vatarini toping.

11. Radiuslari 6 *sm* bo‘lgan aylanalar markazlari orasidagi masofa $6\sqrt{2}$ *sm* ga teng. Aylanalarning umumiy vatari uzunligini toping.



60 KOORDINATALAR USULI VA VEKTORLAR

1-masala. Koordinatalar tekisligida berilgan $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar orasidagi masofa $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ formula bilan hisoblanishini ko‘rsating.



Yechilishi. Aytaylik, A va B nuqtalar 1-rasmdagidek joylashgan bo‘lsin ($x_1 \neq x_2$, $y_1 \neq y_2$). A va B nuqtalardan koordinata o‘qlariga parallel to‘g‘ri chiziqlar o‘tkazamiz va ularning kesishish nuqtasini C bilan belgilaymiz.

Unda, $AC=|x_2-x_1|$ hamda $BC=|y_2-y_1|$. ABC to‘g‘ri burchakli uchburchakka Pifagor teoremasini qo‘llasak,

$$AB^2=AC^2+BC^2=(x_2-x_1)^2+(y_2-y_1)^2$$

bo‘ladi. Undan,

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

formulani hosil qilamiz. Bu formulaning $x_1=x_2$ yoki $y_1=y_2$ bo‘lganda ham to‘g‘riligiga ishonch hosil qiling.

2-masala. Agar $A(-3;-1)$, $B(1;-1)$, $C(1;-3)$, $D(-3;-3)$ bo‘lsa, $ABCD$ to‘g‘ri to‘rtburchak ekanligini isbotlang.

Yechilishi. 1) AC diagonal o‘rtasining x, y koordinatalarini topamiz:

$$x=\frac{-3+1}{2}=-1; \quad y=\frac{-1-3}{2}=-2.$$

BD diagonal o‘rtasining x, y koordinatalarini topamiz:

$$x=\frac{1-3}{2}=-1; \quad y=\frac{-1-3}{2}=-2.$$

Demak, $ABCD$ to‘rtburchak diagonallari bitta $(-1;-2)$ nuqtada kesishib, shu nuqtada teng ikkiga bo‘linar ekan. Bu $ABCD$ parallelogramm ekanligini ko‘rsatadi.

2) $ABCD$ parallelogramm diagonallarining uzunligini topamiz:

$$AC=\sqrt{(1-(-3))^2+(-3-(-1))^2}=\sqrt{4^2+(-2)^2}=\sqrt{20};$$

$$BD=\sqrt{(1-(-3))^2+(-1-(-3))^2}=\sqrt{4^2+2^2}=\sqrt{20}.$$

Demak, $ABCD$ parallelogrammning diagonallari o‘zaro teng ekan. Bu (to‘g‘ri to‘rtburchak alomatiga ko‘ra) $ABCD$ — to‘g‘ri to‘rtburchak ekanligini bildiradi.

Savol, masala va topshiriqlar

1. Agar a) $A(2;7)$, $B(-2;7)$; b) $A(-5;-1)$, $B(-5;-7)$; d) $A(-3;0)$, $B(0;4)$; e) $A(0;3)$, $B(-4;0)$ bo‘lsa, AB kesmaning uzunligini hisoblang.

2. Agar $M(4;0)$, $N(12;-12)$, $P(5;-9)$ bo‘lsa, MNP uchburchak perimetrini toping.

3. Kollinear $\bar{x}$ va $\bar{y}$ vektorlar chizing va $2\bar{x}+3\bar{y}$ vektorni yasang.

4. Agar A, B, C va D nuqtalar bir to‘g‘ri chiziqda yotmasa va $\overline{AB}=0{,}7\overline{DC}$ bo‘lsa, $ABCD$ to‘rtburchak turini aniqlang.

5. Nokollinear $\bar{a}$ va $\bar{b}$ vektorlar berilgan. Agar $3\bar{a}-x\bar{b}=y\bar{a}+4\bar{b}$ bo‘lsa, x va y sonlarni toping.

6. Agar AA_1, BB_1 va CC_1 kesmalar ABC uchburchaklarning medianalari, O — ixtiyoriy nuqta bo‘lsa, $\overline{OA}+\overline{OB}+\overline{OC}=\overline{OA_1}+\overline{OB_1}+\overline{OC_1}$ tenglikni isbotlang.

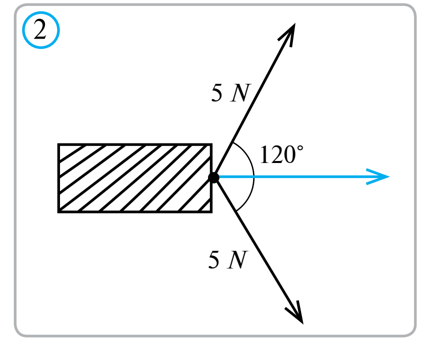
7. ABC uchburchak medianalari O nuqtada kesishadi. $\overline{AB}$, $\overline{BC}$ va $\overline{CA}$ vektorlarni $\bar{a}=\overline{OA}$ va $\bar{b}=\overline{OB}$ vektorlar orqali ifodalang.

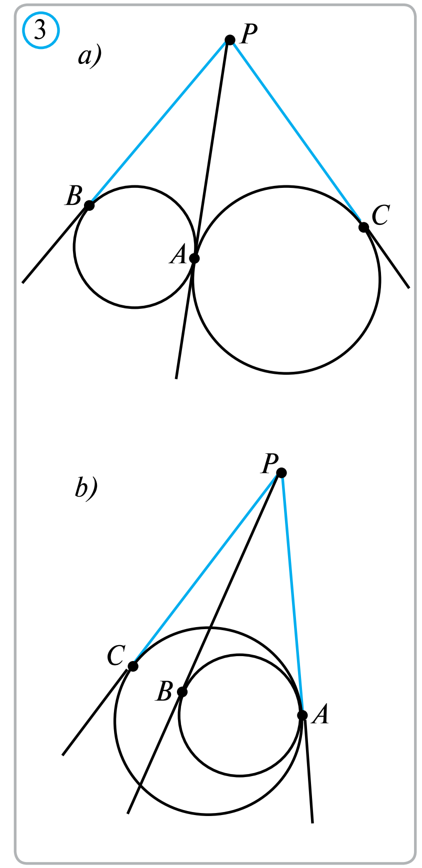
8. Jismga har biri $5N$ bo‘lgan ikkita kuch ta’sir ko‘rsatyapti *(2-rasm)*. Agar bu kuchlarning yo‘nalishlari orasidagi burchak 120° bo‘lsa, ularning teng ta’sir etuvchisi kattaligini toping.

9. Teng tomonli uchburchakka tashqi chizilgan aylananing radiusi 6 *sm*. Uchburchak perimetri va yuzini toping.

10. Aylanaga A nuqtadan o‘tkazilgan urinmada B nuqta olindi. B nuqtadan aylananing eng yaqin nuqtasigacha bo‘lgan masofa 4 *sm* ga, eng uzoq nuqtasigacha bo‘lgan masofa esa 8 *sm* ga teng. AB kesmani toping.

11*. Radiuslari turlicha bo‘lgan ikkita aylana A nuqtada PA to‘g‘ri chiziqqa urinadi. Bu aylanalarga mos ravishda PA dan farqli PB va PC urinmalar o‘tkazilgan. Agar B va C bu urinmalarning aylanaga urinish nuqtalari bo‘lsa, $PC=PB$ tenglikni isbotlang *(3-rasm)*.



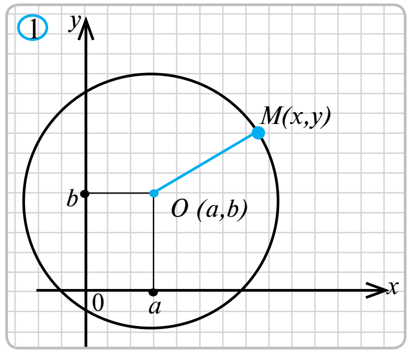


61 AYLANA VA DOIRA

1-masala. Koordinatalar tekisligida markazi $O(a;\ b)$ nuqtada va radiusi R bo'lgan aylanadagi ixtiyoriy $M(x;\ y)$ nuqtaning x va y koordinatalari

$$(x - a)^2 + (y - b)^2 = R^2 \qquad (1)$$

tenglikni qanoatlantirishini isbotlang.



Yechilishi. $O(a;\ b)$ — berilgan aylana markazi, $M(x;\ y)$ — shu aylananing ixtiyoriy nuqtasi bo'lsa, u holda $OM=R$ bo'ladi. Koordinatalar tekisligida berilgan ikki nuqta orasidagi masofani topish formulasiga (134-betdagi 1-masalaga qarang) ko'ra,

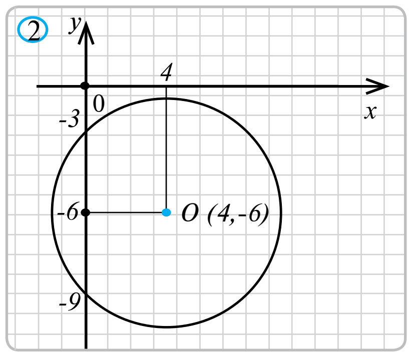
$$OM=\sqrt{(x - a)^2 + (y - b)^2}.$$

Shunday qilib,

$$\sqrt{(x - a)^2 + (y - b)^2}=R.$$

Oxirgi tenglikning har ikkala qismini kvadratga oshirib, (1) tenglikni hosil qilamiz.

Eslatma. (1) tenglama markazi $(a;\ b)$ nuqtada bo'lgan R radiusli **aylana tenglamasi** deyiladi.



2-masala. Koordinatalar tekisligida ushbu

$$(x - 4)^2 + (y + 6)^2=25$$

tenglama bilan aniqlangan aylanani ordinatalar o'qidan ajratgan kesmaning uzunligini toping.

Yechilishi. Berilgan aylana bilan ordinatalar o'qi kesishgan nuqtalarning abssissalari nolga teng bo'ladi. $x=0$ bo'lganda, berilgan tenglamadan foydalanib, bu nuqtalarning ordinatasini topamiz:

$$(0 - 4)^2 + (y + 6)^2=25,\ (y + 6)^2=9,$$

$$y = -9 \text{ yoki } y = -3.$$

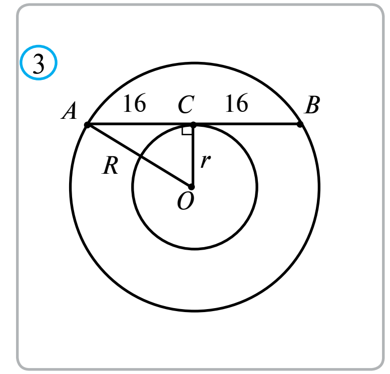
Demak, aylana va ordinatalar o'qi $(0;\ -9)$ va $(0;\ -3)$ nuqtalarda kesishadi. Bu nuqtalar orasidagi masofa 6 birlikka teng.

Javob: 6.

3-masala. Markazlari O nuqtada joylashgan ikkita doira halqa tashkil qiladi. Katta doiraning 32 *sm* ga teng AB vatari kichik doiraga C nuqtada urinadi (*3-rasm*). Agar halqaning kengligi 8 *sm* bo'lsa, u holda bu halqaning yuzini toping.

Yechilishi. Katta doiraning radiusini R bilan, kichiginikini esa r bilan belgilaymiz. Masala shartiga ko'ra, $OA=R=r+8$ (*sm*) va $OC = r$. Bundan tashqari,

C nuqta AB vatarning o'rtasi, ya'ni $AC=16$ *sm*, OCA uchburchak esa to'g'ri burchakli bo'ladi.



Pifagor teoremasiga ko'ra, $OC^2+CA^2=OA^2$ bo'lgani uchun,

$$r^2 + 16^2=(r + 8)^2$$

tenglamani hosil qilamiz. Bu tenglamani yechib, $r=12$ *sm* ekanligini topamiz. Unda $R=r+8=20$ (*sm*) bo'ladi. Katta doira yuzidan kichiginikini ayirib, berilgan halqa yuzi S ni topamiz:

$$S=\pi R^2 - \pi r^2=20^2\pi - 12^2\pi=400\pi - 144\pi=$$

$=256\pi$ (*sm*^{2}). ***Javob:*** 256π *sm*^{2}.

Savol, masala va topshiriqlar

1. Quyidagi tenglamalar bilan berilgan aylanalar markazlarining koordinatalarini va radiusini ayting. Shu aylanalarni yasang.

a) $(x - 1)^2 + (y + 2)^2 = 4$; b) $(x - 4)^2 + (y - 3)^2 = 16$;

d) $x^2 + y^2 = 25$; e) $x^2 + (y - 2)^2 = 9$.

2. Aylanaga ichki chizilgan $ABCD$ to'rtburchakning A, B va C uchlaridagi burchaklari nisbati 1:2:3 kabi. To'rtburchak ichki burchaklarini toping.

3. Aylananing 1:8 qismiga mos markaziy burchakni toping.

4. Markazi A nuqta bo'lgan aylanada B nuqta olingan. Markazi B nuqtada bo'lgan boshqa aylana A nuqtadan o'tadi. Bu ikki aylana C nuqtada kesishadi. ACB burchakni toping.

5. Aylananing AB va CD vatarlari O nuqtada kesishadi. Agar $AO=4$ *sm*, $BO=6$ *sm* va $CD=11$ *sm* bo'lsa, OC va OD kesmalarni toping.

6. Aylanaga ichki chizilgan to'g'ri to'rtburchakning diagonali bitta tomonidan ikki marta katta. Bu to'rtburchak uchlarining aylanadan ajratgan yoylarining gradus o'lchovlarini toping.

7. Aylanaga tashqi chizilgan trapetsiyaning o'rta chizig'i 7 *sm*. Trapetsiya perimetrini toping.

8*. Radiusi 15 *sm* bo'lgan doira markazidan 7 *sm* uzoqlikdagi K nuqtadan 27 *sm* uzunlikdagi AB vatar o'tkazilgan. AK va BK kesmalarni toping.

9. Muntazam sakkizburchakning bir uchidan chiqqan eng katta va eng kichik diagonallari orasidagi burchakni toping.

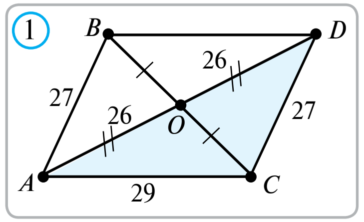
10. Uchlari koordinatalar tekisligidagi $A(-3;\ 4)$, $B(3;\ 4)$, $C(3;\ -8)$ nuqtalarda bo'lgan uchburchak berilgan.

a) $\angle ABC=90^\circ$ ekanligini ko'rsating;

b) ABC uchburchakka tashqi chizilgan doiraning markazini, radiusini va yuzini toping.

62 TAKRORLASH

Masala. ABC uchburchakda AO mediana, $AO=26$, $AB=27$ va $AC=29$. Uchburchak yuzini toping.



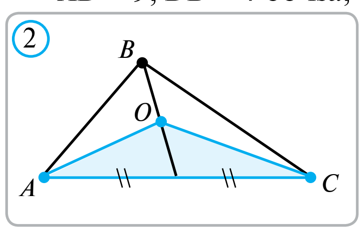
Yechilishi. AO nurda A nuqtadan $AD=2AO=52$ bo‘ladigan qilib, D nuqtani tanlaymiz (*1-rasm*). Unda $BO=OC$, $AO=OD$ bo‘lgani uchun $ABDC$ — parallelogramm bo‘ladi.

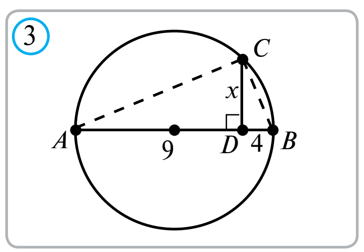
ABC va ADC uchburchakning yuzlari teng. Geron formulasidan foydalanib, ADC uchburchak yuzini hisoblaymiz:

$$P=\frac{29+52+27}{2}=54; \quad S=\sqrt{54\cdot(54-29)(54-52)(54-27)}=270.$$ ***Javob:*** 270.

Savol, masala va topshiriqlar

1. ABC va EFK uchburchaklar o‘xshash: AB va EF, BC va FK ularning mos tomonlari. Agar $AB=4$ *sm*, $BC=5$ *sm*, $CA=7$ *sm* va $EF:AB=2{,}1$ bo‘lsa, EFK uchburchakning tomonlarini toping.
2. ABC va $A_1B_1C_1$ uchburchaklar o‘xshash va ularning mos tomonlari nisbati $6:5$ ga teng. ABC uchburchak yuzi $A_1B_1C_1$ uchburchak yuzidan 77 dm^2 ga ortiq. Uchburchaklar yuzlarini toping.
3. ABC uchburchak medianalari kesishgan nuqta O bo‘lsin. Agar AOC uchburchak yuzi 4 sm^2 bo‘lsa, ABC uchburchak yuzini toping (*2-rasm*).
4. Aylananing C nuqtasidan AB diametrga CD perpendikular tushirilgan. Agar $AD=9$, $DB=4$ bo‘lsa, CD kesmani toping (*3-rasm*).

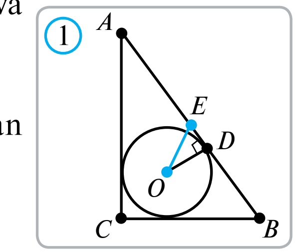




5. Tomoni 6 *m*, bu tomoniga yopishgan burchaklari 30° va 45° bo‘lgan uchburchakning yuzini toping.
6. Asoslari 28 *dm* va 16 *dm*, yon tomonlari esa 25 *dm* va 17 *dm* bo‘lgan trapetsiyaning balandligini toping.
7. Radiusi 2 *sm* bo‘lgan aylanaga yuzasi 20 sm^2 bo‘lgan teng yonli trapetsiya tashqi chizilgan. Trapetsiya tomonlarining uzunliklarini toping.
8. To‘g‘ri burchakli uchburchakka ichki chizilgan aylananing gipotenuzaga urinish nuqtasi gipotenuzani 2 *sm* va 3 *sm* bo‘lgan kesmalarga ajratadi. Uchburchakning katetlarini toping.

63 TAKRORLASH

Masala. Katetlari 3 va 4 bo‘lgan to‘g‘ri burchakli uchburchakka ichki va tashqi chizilgan aylanalarning markazlari orasidagi masofani toping (*1-rasm*).



Yechilishi. 1) ABC uchburchakda $\angle C=90°$, $AC=4$ va $BC=3$ bo‘lsin. Unda, Pifagor teoremasiga ko‘ra,

$$AB=\sqrt{3^2+4^2}=5.$$

2) To‘g‘ri burchakli uchburchakka tashqi chizilgan aylananing E markazi gipotenuzaning o‘rtasida bo‘ladi:

$$BE=\frac{AB}{2}=\frac{5}{2}.$$

3) Uchburchakka ichki chizilgan aylana radiusi OD ni topamiz (D — ichki chizilgan aylananing gipotenuzaga urinish nuqtasi):

$$OD=\frac{AC+BC-AB}{2}=\frac{4+3-5}{2}=1.$$

4) BD va DE kesmalarni topamiz:

$$BD=\frac{AB+BC-AC}{2}=\frac{5+3-4}{2}=2; \qquad ED=BE-DE=\frac{5}{2}-2=\frac{1}{2}.$$

5) To‘g‘ri burchakli ODE uchburchakdan OE kesmani topamiz:

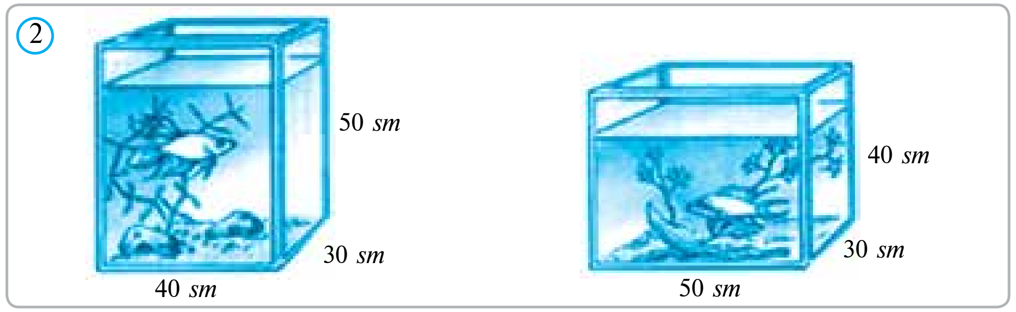
$$OE=\sqrt{OD^2+ED^2}=\sqrt{1+\frac{1}{4}}=\frac{\sqrt{5}}{2}.$$ ***Javob:*** $\frac{\sqrt{5}}{2}$.

Savol, masala va topshiriqlar

1. Teng yonli ABC uchburchakda $AB=AC=4$ *sm* va $\angle A=30°$ bo‘lsa, uning BE balandligini toping.
2. Trapetsiyaning asoslari 5 *dm* va 8 *dm*, yon tomonlari esa 3,6 *dm* va 3,9 *dm*. Trapetsiya yon tomonlarining davomi O nuqtada kesishadi. O nuqtadan trapetsiya uchlarigacha bo‘lgan masofalarni toping.
3. A burchakning bir tomoniga $AB=5$ *sm* va $AC=16$ *sm* kesmalar, ikkinchi tomoniga esa $AD=8$ *sm* va $AF=10$ *sm* kesmalar qo‘yilgan. ACD va AFB uchburchaklar o‘xshashmi? Javobingizni asoslang.
4. To‘g‘ri to‘rtburchakning yuzi 9 dm^2, diagonallari hosil qilgan burchaklardan biri esa 120° ga teng. To‘g‘ri to‘rtburchak tomonlarini toping.
5. Agar teng yonli uchburchakning asosi 24 *sm* va yon tomoni 13 *sm* bo‘lsa, u holda uchburchakka tashqi chizilgan aylana radiusini toping.
6. Rombning balandligi 12 *sm* bo‘lib, diagonallaridan biri 15 *sm*. Romb yuzini toping.

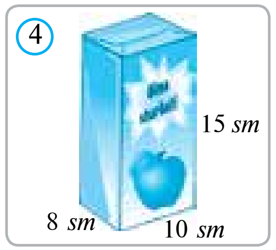
7. Agar $ABCD$ parallelogrammda $A(1;-3)$, $B(-2;4)$ va $C(-3;1)$ bo‘lsa, uning D uchi koordinatalarini toping.

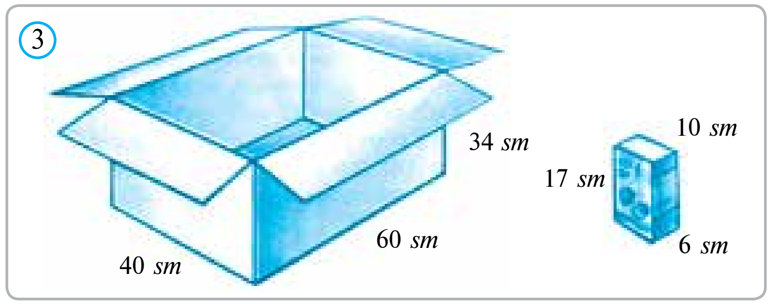
8. Ikkita akvariumga yuqori chetidan 10 *sm* past qilib suv quyildi (*2-rasm*). Qaysi akvariumda suv ko‘p?



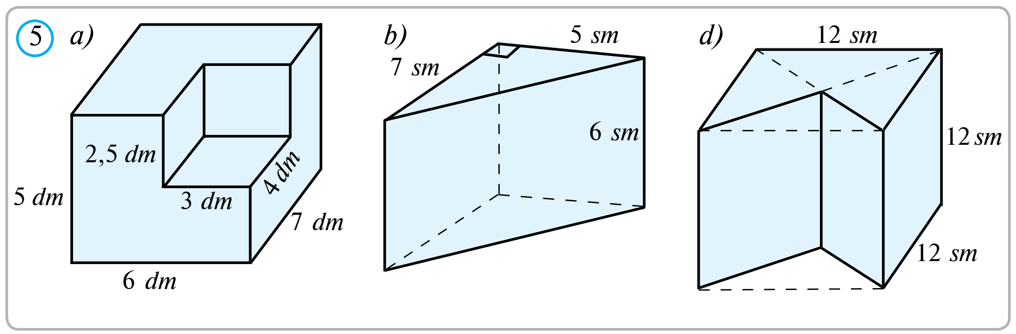
9. Qutiga necha paket meva sharbati sig‘adi (*3-rasm*)?

10. 1 litrli meva sharbati paketi to‘g‘ri to‘rtburchakli parallelepiped shaklida (*4-rasm*). Bitta qadoq uchun qancha material kerak bo‘ladi?





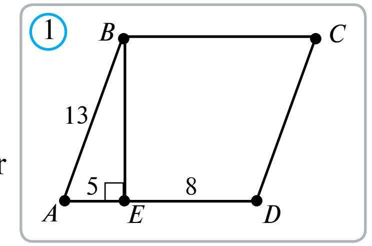
11*. 5-rasmda tasvirlangan yog‘och bo‘laklarining hajmini hisoblang.



64 TAKRORLASH

Masala. Rombning o‘tmas burchagi uchidan o‘tkazilgan balandlik romb tomonlaridan birini o‘tkir burchagi uchidan boshlab hisoblaganda 5 *sm* va 8 *sm* bo‘lgan kesmalarga ajratadi. Romb yuzini hisoblang.

Yechilishi. $ABCD$ romb, $\angle B > 90°$, BE— balandlik, $AE = 5$ *sm*, $ED = 8$ *sm* bo‘lsin (*1-rasm*).



1) Romb tomonini topamiz:

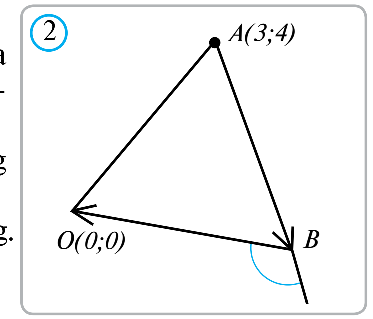
$$AD = AE + ED = 5 + 8 = 13\ (sm).$$

2) To‘g‘ri burchakli ABE uchburchakka Pifagor teoremasini qo‘llab, BE balandlikni topamiz:

$$BE = \sqrt{AB^2 - AE^2} = \sqrt{13^2 - 5^2} = 12\ (sm).$$

3) Romb yuzini topamiz: $S = AD \cdot BE = 13 \cdot 12 = 156\ (sm^2)$. ***Javob:*** 156 sm^2.

Savol, masala va topshiriqlar

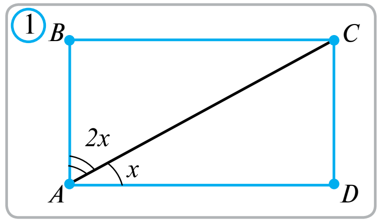


1. Agar teng tomonli AOB uchburchakda $O(0;0)$ va $A(3;4)$ ekanligi ma’lum bo‘lsa, $\overline{AB} \cdot \overline{BO}$ skalyar ko‘paytmani toping (*2-rasm*).
2. Asoslari AB va CD bo‘lgan $ABCD$ trapetsiyaning diagonallari O nuqtada kesishadi. Agar $OB = 8$ *sm*, $OD = 20$ *sm* va $OC = 50$ *sm* bo‘lsa, AO kesmani toping.
3. Agar $AB = 1{,}7$ *sm*, $BC = 3$ *sm*, $CA = 4{,}2$ *sm*, $A_1B_1 = 34$ *dm*, $B_1C_1 = 60$ *dm* va $C_1A_1 = 84$ *dm* bo‘lsa, ABC va $A_1B_1C_1$ uchburchaklar o‘xshashmi?
4. Perimetri 36 *sm* bo‘lgan parallelogrammning diagonallari kesishishidan hosil bo‘lgan ikkita uchburchakdan birining perimetri ikkinchisinikidan 8 *sm* ortiq bo‘lsa, parallelogrammning tomonlarini toping.
5. 60° ga teng burchakka bir-biriga tashqaridan urinuvchi ikkita aylana ichki chizilgan. Kichik aylananing radiusi 1 *sm* bo‘lsa, katta aylana radiusini toping.
6. Katta asosi AD bo‘lgan $ABCD$ trapetsiyaning AC diagonali CD tomoniga perpendikular va $\angle BAC = \angle CAD$. Agar trapetsiyaning perimetri 20 *sm* va $\angle D = 60°$ bo‘lsa, AD tomon uzunligini toping.
7. Agar aylana diametrining uchlari aylananing biror urinmasidan 18 *sm* va 12 *sm* uzoqlikda ekanligi ma’lum bo‘lsa, aylana uzunligini toping.
8. Asoslarining uzunliklari va yuzi mos ravishda 8 *sm*, 14 *sm* va 44 sm^2 bo‘lgan teng yonli trapetsiyaning yon tomonini toping.

65 TAKRORLASH

Masala. To'g'ri to'rtburchakning diagonallari 12 *sm* ga teng, ular to'rtburchak burchagini 2:1 nisbatda bo'ladi. To'g'ri to'rtburchak perimetrini toping.

ABCD— to'g'ri to'rtburchak, $AC=12$ *sm*, $\angle BAC : \angle CAD = 2:1$ ⇨ $P=?$



Yechilishi. 1) Agar $\angle CAD=x$ desak, $\angle BAC=2x$ va $\angle CAD+\angle BAC=x+2x=90^\circ$ bo'ladi. Bundan $x=30^\circ$.

2) To'g'ri burchakli *ADC* uchburchak katetlarini topamiz:

$$CD=AC\sin CAD=12\cdot\sin30^\circ=12\cdot\frac{1}{2}=6\ (sm),$$

$$AD=AC\cdot\cos CAD=12\cdot\cos30^\circ=12\cdot\frac{\sqrt{3}}{2}=6\sqrt{3}\ (sm).$$

3) To'rtburchak perimetrini topamiz:

$$P=2(AD+CD)=2(6+6\sqrt{3})=12(1+\sqrt{3})\ (sm).$$

Javob: $12(1+\sqrt{3})$ *sm*.

Savol, masala va topshiriqlar.

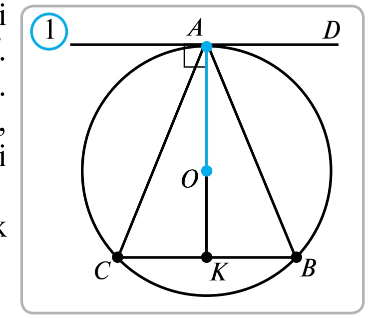
1. *ABC* va $A_1B_1C_1$ uchburchaklar o'xshash, $AB=6$ *sm*, $BC=9$ *sm* va $CA=10$ *sm*. Agar $A_1B_1C_1$ uchburchakning katta tomoni 7,5 *sm* bo'lsa, qolgan tomonlarini toping.
2. *ABC* uchburchakning *AB* tomoniga parallel to'g'ri chiziq *AC* tomonni *A* uchidan boshlab hisoblaganda 2:7 kabi nisbatda bo'ladi. Agar $AB=10$ *sm*, $BC=18$ *sm* va $CA=21{,}6$ *sm* bo'lsa, to'g'ri chiziq *ABC* uchburchakdan ajratgan uchburchak tomonlarini toping.
3. Agar teng yonli trapetsiyaning yon tomoni o'rta chizig'iga teng va perimetri 48 *sm* bo'lsa, trapetsiyaning yon tomoni uzunligini toping.
4. Asoslari 6 *sm* va 3 *sm* bo'lgan to'g'ri burchakli trapetsiyaga ichki chizilgan aylana radiusini toping.
5. Agar $A_1A_4=2{,}24$ bo'lsa, u holda $A_1A_2A_3A_4A_5A_6$ muntazam oltiburchakning perimetrini toping.
6. Agar $N(7;3)$ va $M(-3;5)$ bo'lsa, *NM* diametrli aylana uzunligini toping.
7. Uchidagi burchagi 120° bo'lib, radiusi 10 *sm* bo'lgan aylanaga ichki chizilgan teng yonli uchburchak yuzini toping.
8. Agar *ABCD* to'rtburchakda $AB=5$ *sm*, $BC=13$ *sm*, $CD=9$ *sm*, $DA=15$ *sm* va $AC=12$ *sm* bo'lsa, *ABCD* to'rtburchakning yuzini toping.

9*. 90° li markaziy burchakka mos yoyning uzunligi 15π ga teng. Aylanaga tashqi chizilgan muntazam uchburchak yuzini toping.

66 TAKRORLASH

Masala. Aylananing *AB* vatari 10 *sm*. Vatarning *A* uchidan *AD* urinma, *B* uchidan esa shu urinmaga parallel *BC* vatar o'tkazildi. Agar $BC=12$ *sm* bo'lsa, aylana radiusini toping.

Yechilishi: 1) *A* nuqta va aylana markazi — *O* nuqta orqali o'tuvchi to'g'ri chiziq *BC* vatarni *K* nuqtada kessin. *AD* urinma bo'lgani uchun $AK\perp AD$, $AD\parallel BC$ bo'lgani uchun esa $AK\perp BC$.



2) $AK\perp BC$, ya'ni $OK\perp BC$ bo'lgani uchun $CK=KB$. *AK* kesma *ABC* uchburchakning ham medianasi, ham balandligi ekan. Demak, *ABC* — teng yonli uchburchak: $AC=AB=10$ *sm*.

3) Geron formulasidan foydalanib, *ABC* uchburchak yuzini topamiz:

$$p=\frac{a+b+c}{2}=\frac{10+10+12}{2}=16\ (sm),$$

$$S=\sqrt{p\cdot(p-a)(p-b)(p-c)}=\sqrt{16\cdot(16-10)(16-10)(16-12)}=48\ (sm^2).$$

4) *ABC* uchburchakka tashqi chizilgan aylana radiusini topamiz:

$$R=\frac{abc}{4S}=\frac{12\cdot10\cdot10}{4\cdot48}=6{,}25\ (sm).$$

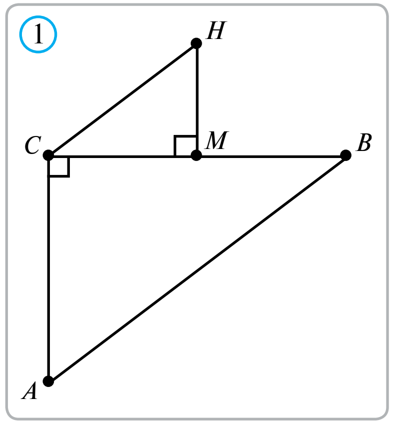
Javob: 6,25 *sm*.

Savol, masala va topshiriqlar

1. Uchburchakning tomonlari mos ravishda 13 *sm*, 14 *sm* va 15 *sm* ga teng. Uchburchakka ichki va tashqi chizilgan doiralar yuzlari nisbatini toping.
2. Agar $\angle BDC=40^\circ$ va $\angle CBD=60^\circ$ bo'lsa, aylanaga ichki chizilgan *ABCD* to'rtburchakning *A* va *C* burchaklarini toping.
3. Aylanaga tashqi chizilgan teng yonli trapetsiyaning asoslari 4 *sm* va 16 *sm* bo'lsa, aylana radiusini toping.
4. To'g'ri burchakli uchburchakning katetlariga tushirilgan medianalari $\sqrt{52}$ *sm* va $\sqrt{73}$ *sm* ga teng. Uchburchakning yuzini toping.
5. Katetlari 6 *m* va 8 *m* bo'lgan to'g'ri burchakli uchburchakning gipotenuzasiga tushirilgan balandligini toping.
6. To'g'ri burchakli uchburchakning bitta kateti 5 *sm* va gipotenuzasi 13 *sm* bo'lsa, uning yuzini toping.
7. *x* ning qanday qiymatlarida $\bar{a}(x;7)$ va $\bar{b}(5;2-x)$ vektorlari perpendikular bo'ladi?
8. Uchburchakning ikki tomoni 10 *sm* va 12 *sm*, ular orasidagi o'tkir burchakning sinusi 0,8 ga teng. Uchburchakning uchinchi tomonini toping.

9*. Agar teng yonli uchburchakning asosi 24 *sm* va yon tomoni 20 *sm* bo'lsa, bu uchburchakka ichki chizilgan doira yuzini toping.

67–68 YAKUNIY NAZORAT ISHI



I. Namunaviy nazorat ishi

1. $ABCD$ parallelogrammda $\angle A = 45°$, $AD = 4$. Parallelogramm AB tomonining davomiga $\angle PDA = 90°$ ga teng bo'ladigan BP kesma qo'yildi. BC va PD kesmalar T nuqtada kesishadi, bunda $PT : TD = 3 : 1$.

a) $\triangle BPT \sim \triangle CDT$ ekanligini isbotlang, bu uchburchaklar yuzlari nisbatini toping.

b) $ABCD$ parallelogramm yuzini toping.

d) AB va TD kesmalarning o'rtalarini tutashtiruvchi kesmaning uzunligini toping.

e) $\overline{AB}$ vektorni $\overline{CA}$ va $\overline{TB}$ vektorlar orqali ifodalang.

f) CAD burchakning sinusini toping.

2. *(Qo'shimcha)* 1-rasmda $BC \perp AC$, $MH \perp BC$, $2MC = BC$, $MH = 0{,}5AC$ bo'lsa, $AB \| CH$ ekanligini isbotlang.

II. Nazorat ishi uchun namunaviy testlar

1. Agar to'g'ri burchakli uchburchakning balandligi gipotenuzasini 6 *sm* va 54 *sm* kesmalarga ajratsa, bu uchburchakning yuzini toping:
A) 648 sm^2; B) 324 sm^2; D) 1080 sm^2; E) 540 sm^2.

2. C nuqtadan o'tkazilgan bir kesuvchi aylanani A va B, ikkinchisi esa D va E nuqtalarda kesadi. Agar $CA = 18$ *sm*, $CB = 8$ *sm*, $CD = 8$ *sm* bo'lsa, DE kesma uzunligini toping:
A) 17 *sm*; B) 1 *sm*; D) 9 *sm*; E) to'g'ri javob ko'rsatilmagan.

3. Agar $A(-5;2\sqrt{3})$, $B(-4;2)$, $C(-2;\sqrt{3})$, $D(0;2)$ bo'lsa, $ABCD$ to'rtburchakning diagonallari orasidagi burchakni toping:
A) 30°; B) 60°; D) 90°; E) to'g'ri javob ko'rsatilmagan.

4. Agar parallelogrammning diagonallari 10 *sm* va $8\sqrt{2}$ *sm* ga teng va ular orasidagi burchak 45° bo'lsa, parallelogrammning tomonlarini toping:
A) $\sqrt{17}$ *sm* va $\sqrt{97}$ *sm*; B) 5 *sm* va 6 *sm*;
D) $\sqrt{34}$ *sm* va $\sqrt{63}$ *sm*; E) to'g'ri javob ko'rsatilmagan.

5. Radiusi 8 *sm* bo'lgan aylanaga ichki chizilgan muntazam oltiburchakning yuzini toping:
A) $48\sqrt{3}$ sm^2; B) $192\sqrt{3}$ sm^2; D) $96\sqrt{2}$; E) to'g'ri javob ko'rsatilmagan.

6. Markaziy burchagi 140°, yuzi $31{,}5\pi$ sm^2 bo'lgan doiraviy sektorning radiusini aniqlang:
A) 9 *sm*; B) 18 *sm*; D) 9π *sm*; E) to'g'ri javob ko'rsatilmagan.

7. Asosining uzunligi 15 *sm* bo'lgan uchburchak asosiga parallel kesma o'tkazilgan. Agar hosil bo'lgan trapetsiyaning yuzi uchburchak yuzining $\frac{3}{4}$ qismini tashkil qilishi ma'lum bo'lsa, kesmaning uzunligini toping:
A) 6,5; B) 7; D) 7,5; E) 5.

8. Yon tomoni $2\sqrt{39}$ *sm* bo'lgan teng yonli uchburchak balandligining asosiga nisbati 3:4 ga teng bo'lsa, uchburchakning yuzini toping:
A) 260; B) 245; D) 310; E) 72.

9. $a(4;4\sqrt{3})$ va $b(8\sqrt{3};8)$ vektorlar orasidagi burchakni toping:
A) 45°; B) 90°; D) 30°; E) 60°.

10. Teng yonli trapetsiyaning asoslari 10 *sm* va 16 *sm*, yon tomoni esa 5 *sm*. Trapetsiyaning yuzini toping:
A) 45; B) 50; D) 48; E) 52.

11. To'g'ri burchakli uchburchakning gipotenuzasi 13 *sm* bo'lib, katetlaridan biri ikkinchisidan 7 sm katta. Uchburchakning yuzini toping:
A) 30 sm^2; B) 25 sm^2; D) 45 sm^2; E) 40 sm^2.

12. Tomoni 5 *sm* bo'lgan rombning bitta diagonali 6 *sm* ga teng. Rombning yuzini toping:
A) 24 sm^2; B) 30 sm^2; D) 29 sm^2; E) 40 sm^2.

13. Diagonali $6\sqrt{2}$ bo'lgan kvadratga ichki chizilgan aylana uzunligini toping:
A) 10π; B) 8π; D) 9π; E) 6π.

14. Tomoni $6\sqrt{2}$ *sm* bo'lgan kvadratga tashqi chizilgan doira yuzini toping:
A) 9π; B) 12π; D) 15π; E) 18π.

15. Balandliklari 4 *sm* va 6 *sm* bo'lgan parallelogramm yuzi 36 sm^2 ga teng. Uning perimetrini toping:
A) 26 *sm*; B) 30 *sm*; D) 29 *sm*; E) 36 *sm*.

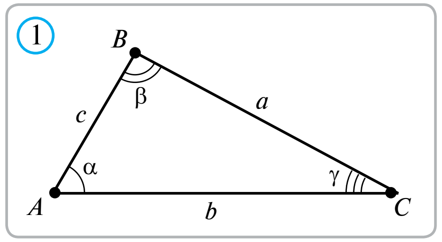
16. Perimetri 30 *sm* bo'lgan parallelogrammning tomonlari 2:3 nisbatda. Agar parallelogrammning o'tkir burchagi 30° bo'lsa, uning yuzini toping:
A) 26 sm^2; B) 27 sm^2; D) 29 sm^2; E) 30 sm^2.

17. Agar ABC uchburchakda $AB = 6\sqrt{3}$ *sm*, $BC = 12$ *sm* va $\angle C = 60°$ bo'lsa, uchburchakning A burchagini toping:
A) 45°; B) 90°; D) 30°; E) 60°.

PLANEMETRIYAGA OID ASOSIY TUSHUNCHA VA MA'LUMOTLAR

UCHBURCHAKLAR

1°. Asosiy tushunchalar



Tekislikda bir to'g'ri chiziqda yotmagan uchta nuqta berilgan bo'lsin. Shu nuqtalarning har ikkitasini kesmalar bilan tutashtiramiz. Hosil bo'lgan shakl *uchburchak* deyiladi. Nuqtalar uchburchakning *uchlari*, kesmalar esa *tomonlari* deyiladi. Belgilanishi: A, B, C — uchlar, a, b, c — tomonlar (*1-rasm*).

Uchburchak uchta ichki burchakka ega: $\angle BAC$, $\angle CBA$, $\angle ACB$. Belgilanishi: α, β, γ.

Mediana — uchburchak uchini uning qarshisidagi tomon o'rtasi bilan tutashtiruvchi kesma. Uchburchakda 3 ta mediana bo'lib, ular m_a, m_b, m_c kabi belgilanadi.

Bissektrisa — uchburchak uchini uning qarshisidagi tomon bilan tutashtiruvchi va shu uchdagi burchak bissektrisasida yotuvchi kesma. Uchburchakda uchta bissektrisa bo'lib, ular l_a, l_b, l_c kabi belgilanadi.

Balandlik — uchburchak uchidan uning qarshisidagi tomon yotgan to'g'ri chiziqqa tushirilgan perpendikular.

Uchburchakda uchta balandlik bo'lib, ular h_a, h_b, h_c kabi belgilanadi.

O'rta chiziq — ikki tomon o'rtalarini tutashtiruvchi kesma.

O'rta chiziqlar soni ham 3 ta.

Perimetr — uchala tomon uzunliklari yig'indisi. Belgilanishi: P.

Uchburchaklar tomonlariga qarab uch turga ajratiladi:

a) teng tomonli ($a=b=c$); b) teng yonli (a, b, c larning qandaydir ikkisi teng);

d) turli tomonli (a, b, c larning hech qaysi ikkisi teng emas).

Uchburchakning uchala tomoniga urinib o'tuvchi aylana unga ichki chizilgan *aylana* deyiladi (bunday aylana mavjud va yagona). Ichki chizilgan aylana radiusi r orqali belgilanadi.

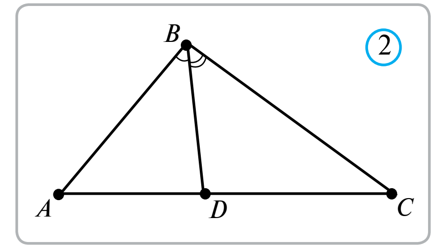
Uchburchakning uchala uchidan o'tuvchi aylana unga *tashqi chizilgan aylana* deyiladi va uning radiusi R orqali belgilanadi (bunday aylana mavjud va yagona).

2°. Asosiy munosabatlar

1) $\alpha+\beta+\gamma=180°$. Uchburchak ichki burchaklari yig'indisi 180° ga teng.

2) Uchala mediana bir nuqtada kesishadi. Bu nuqta medianani 2:1 nisbatda bo'ladi. Mediana uchburchakni ikkita yuzlari teng uchburchaklarga ajratadi. Medianalar uzunliklari $m_a=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}$; $m_b=\frac{1}{2}\sqrt{2a^2+2c^2-b^2}$; $m_c=\frac{1}{2}\sqrt{2a^2+2b^2-c^2}$ formulalardan topiladi.

3) Uchala bissektrisa bir nuqtada kesishadi. Bu nuqta ichki chizilgan aylana markazi bo'ladi. Bissektrisa o'zi tushirilgan tomonni qolgan tomonlarga proporsional bo'laklarga ajratadi (*2-rasm*).



BD bissektrisa bo'lsa, $\frac{AB}{AD}=\frac{BC}{DC}$.

Bissektrisa uzunliklarini

$$l_a=\frac{2\sqrt{bc}}{b+c}\sqrt{p(p-a)};\quad l_b=\frac{2\sqrt{ac}}{a+c}\sqrt{p(p-b)};$$

$l_c=\frac{2\sqrt{ab}}{a+b}\sqrt{p(p-c)}$, $p=\frac{1}{2}(a+b+c)$ formulalardan topish mumkin.

4) Uchburchak balandliklari yoki ularning davomlari bir nuqtada kesishadi. Balandlik uzunliklarini

$$h_a=\frac{2S}{a};\qquad h_b=\frac{2S}{b};\qquad h_c=\frac{2S}{c}$$

formulalardan topish mumkin. Bu yerda S — uchburchak yuzi.

5) Uchburchak tomonlarining o'rta perpendikulari bir nuqtada kesishadi. Bu nuqta uchburchakka *tashqi chizilgan aylana markazi* bo'ladi.

6) Uchburchakning *o'rta chizig'i* uchinchi tomonga parallel va uning yarmiga teng.

7) Sinuslar teoremasi:

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}=2R.$$

8) Kosinuslar teoremasi:

$$a^2=b^2+c^2-2bc\cos\alpha,\quad b^2=a^2+c^2-2ac\cos\beta,\quad c^2=a^2+b^2-2ab\cos\gamma$$

9. Uchburchak yuzini hisoblash formulalari:

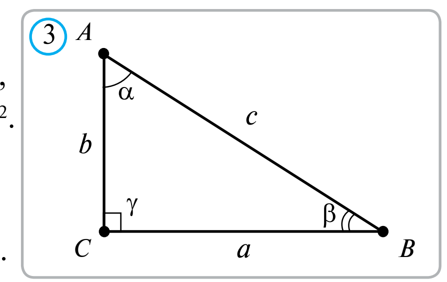
$$S=\frac{1}{2}ah_a=\frac{1}{2}bh_b=\frac{1}{2}ch_c;\qquad S=\frac{1}{2}ab\sin\gamma=\frac{1}{2}bc\sin\alpha=\frac{1}{2}ac\sin\beta;$$

10. Geron formulasi:

$$S=\sqrt{p(p-a)(p-b)(p-c)},\qquad p=\frac{a+b+c}{2};\qquad S=\frac{abc}{4R},\qquad S=pr.$$

3°. Muhim xususiy hollar

a) *To'g'ri burchakli uchburchak* (*3-rasm*).



$\angle\gamma=90°$, $\alpha+\beta=90°$, AC va BC — katetlar, AB — gipotenuza. Pifagor teoremasi: $a^2+b^2=c^2$.

$$S=\frac{1}{2}ab;\qquad R=\frac{c}{2};\qquad r=\frac{a+b-c}{2};$$

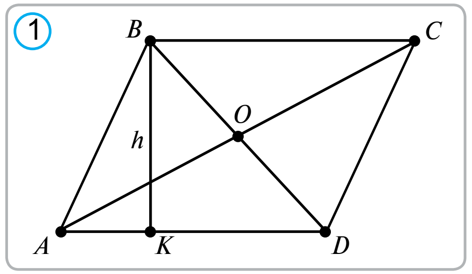
$$\frac{a}{c}=\sin\alpha;\quad \frac{a}{c}=\cos\beta;\quad \frac{b}{c}=\sin\beta;\quad \frac{b}{c}=\cos\alpha.$$

$$\frac{a}{b}=\text{tg}\alpha; \quad \frac{a}{b}=\text{ctg}\beta; \quad \frac{b}{a}=\text{ctg}\alpha; \quad \frac{b}{a}=\text{tg}\beta.$$

b) *Teng tomonli uchburchak*

$$\alpha=\beta=\gamma=60^\circ, \quad S=\frac{a^2\sqrt{3}}{4}, \quad r=\frac{a\sqrt{3}}{6}, \quad R=\frac{a\sqrt{3}}{3}.$$

TO‘RTBURCHAKLAR



1°. Parallelogramm

Qarama-qarshi tomonlari parallel bo‘lgan to‘rtburchak *parallelogramm* deyiladi (*1-rasm*).

Qo‘shni bo‘lmagan uchlarni tutashtiruvchi kesma *diagonal* deyiladi.

AB va *CD; AD* va *BC* parallel tomonlar; *BD* va *AC* diagonallar.

Asosiy xossalar va munosabatlar

1) Diagonallar kesishish nuqtasi parallelogrammning simmetriya markazi bo‘ladi.

2) Qarama-qarshi tomonlarning uzunliklari o‘zaro teng:

$$AB=CD \quad \text{va} \quad AD=BC.$$

3) Parallelogrammning qarama-qarshi burchaklari o‘zaro teng:

$$\angle BAD=\angle BCD \quad \text{va} \quad \angle ABC=\angle ADC.$$

4) Qo‘shni burchaklar yig‘indisi 180° ga teng.

5) Diagonallar kesishish nuqtasida teng ikkiga bo‘linadi: $BO=OD$ va $AO=OC$

6) Tomonlari kvadratlarining yig‘indisi diagonallari kvadratlarining yig‘indisiga teng:

$$AB^2+BC^2+CD^2+DA^2=AC^2+BD^2 \quad \text{yoki} \quad 2(AB^2+BC^2)=AC^2+BD^2.$$

7) Parallelogramm yuzi: a) $S=ah_a$, bu yerda: $a=AD$ tomon, $h_a=BK$ — balandlik; b) $S=ab\sin\alpha$, bu yerda: $b=AB$ — tomon, $\alpha=\angle BAD$ — *AB* va *AD* tomonlar orasidagi burchak.

2°. Romb

Barcha tomonlari o‘zaro teng bo‘lgan parallelogramm *romb* deyiladi.

Parallelogramm uchun o‘rinli bo‘lgan barcha xossalar romb uchun ham o‘rinli.

Rombning qo‘shimcha xossalari.

1) Romb diagonallari o‘zaro perpendikular.

2) Romb diagonallari ichki burchaklarning bissektrisalari bo‘ladi.

3) Romb yuzi $S=\frac{1}{2}d_1d_2$, bu yerda: d_1, d_2 — romb diagonallari.

3°. To‘g‘ri to‘rtburchak

Barcha burchaklari 90° ga teng bo‘lgan parallelogramm *to‘g‘ri to‘rtburchak* deyiladi.

1) To‘g‘ri to‘rtburchak diagonallari o‘zaro teng.

2) To‘g‘ri to‘rtburchak yuzi $S=ab$, bu yerda: a va b — to‘g‘ri to‘rtburchakning qo‘shni tomonlari.

4°. Kvadrat

Barcha tomonlari o‘zaro teng bo‘lgan to‘g‘ri to‘rtburchak *kvadrat* deyiladi.

Romb va to‘g‘ri to‘rtburchaklar uchun o‘rinli bo‘lgan barcha xossalar kvadrat uchun ham o‘rinli.

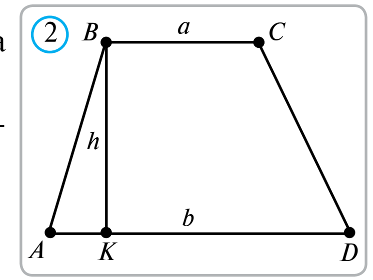
Agar a — kvadrat tomoni, d esa diagonali bo‘lsa: $S=a^2$; $S=\frac{d^2}{2}$; $d=a\sqrt{2}$.

5°. Trapetsiya

Asoslar deb ataluvchi ikki tomoni o‘zaro parallel va yon tomonlar deb ataluvchi, qolgan ikki tomoni esa parallel bo‘lmagan to‘rtburchak *trapetsiya* deyiladi.

Yon tomonlar o‘rtalarini tutashtiruvchi kesma trapetsiyaning *o‘rta chizig‘i* deyiladi.

Asosiy xossalar



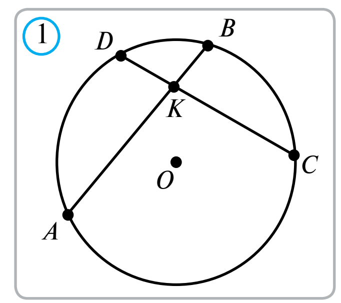
1) Trapetsiya o‘rta chizig‘i asoslarga parallel va asoslar yig‘indisining yarmiga teng bo‘ladi.

2) Trapetsiya yuzi $S=\frac{a+b}{2}h$, bu yerda a va b — asoslar, h esa balandlik (*2-rasm*).

AYLANA, DOIRA

1°. Musbat son R va tekislikda O nuqta berilgan bo‘lsin. O nuqtadan R masofada joylashgan nuqtalardan tashkil topgan shakl *aylana* deyiladi. O nuqta *aylana markazi*, markaz bilan aylanadagi nuqtani tutashtiruvchi kesma *radius*, R son esa *radius uzunligi* deyiladi. Aylanadagi ikki nuqtani tutashtiruvchi kesma *vatar*, markazdan o‘tuvchi vatar esa *diametr* deyiladi.

Tekislikning aylana bilan chegaralangan chekli qismi *doira* deb ataladi.



Asosiy munosabatlar

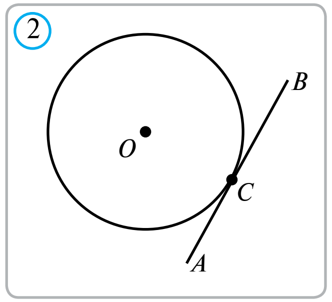
1) $D=2R$, bu yerda: D — diametr uzunligi.

2) $L=2\pi R$ — aylana uzunligi.

3) $S=\pi R^2$ — doira yuzi.

4) *AB* va *CD* vatarlar K nuqtada kesishsa (*1-rasm*), $AK\cdot KB=CK\cdot KD$ munosabat bajariladi.

5) Vatarni teng ikkiga bo‘luvchi diametr shu vatarga perpendikulardir.



6) Teng vatarlar markazdan teng masofalarda joylashgan va, aksincha, markazdan teng masofada joylashgan vatarlar o‘zaro teng.

2°. Urinma

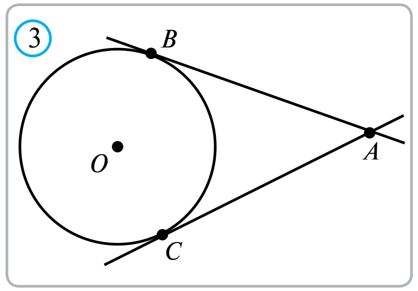
Aylana (yoki doira) bilan yagona umumiy nuqtaga ega bo‘lgan to‘g‘ri chiziq *urinma* deyiladi. Nuqta esa *urinish nuqtasi* deyiladi (*2-rasm*).

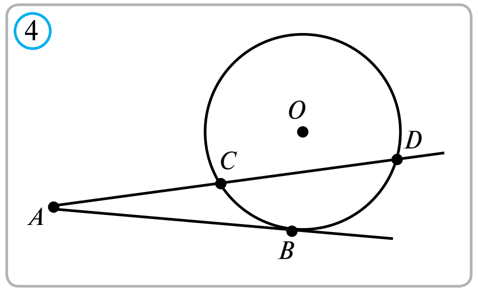
Aylana bilan 2 ta umumiy nuqtaga ega bo‘lgan to‘g‘ri chiziq *kesuvchi* deb ataladi.

Urinmaning xossalari

1) Urinish nuqtasiga o‘tkazilgan radius urinmaga perpendikulardir.

2) Doira tashqarisidagi nuqtadan shu doiraga ikkita urinma o‘tkazish mumkin. Bu urinmalarning kesmalari o‘zaro teng (*3-rasm*): $AB=AC$.



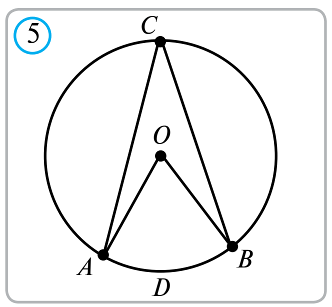


3) Agar AC kesuvchi bo‘lib, aylanani C va D nuqtalarda kesib o‘tsa, AB esa urinma bo‘lsa, $AB^2=AD\cdot AC$ tenglik o‘rinli (*4-rasm*).

3°. Markaziy va ichki chizilgan burchaklar

Aylanadagi ikki nuqta yordamida aylana ikki bo‘lakka ajraladi. Bu bo‘laklar *yoylar* deb ataladi. Belgilanishi: $\overset{\frown}{ADB}$; $\overset{\frown}{ACB}$.

AOB burchak ADB yoyga tiralgan *markaziy burchak* (*5-rasm*), ACB burchak esa ADB yoyga tiralgan va aylanaga *ichki chizilgan burchak* deyiladi. Bu burchaklar orasida



$$\angle ACB=\frac{1}{2}\angle AOB$$

munosabat o‘rinli.

Xususan, yarim aylanaga tiralgan ichki burchak to‘g‘ri burchak bo‘ladi (*6-rasm*). Bitta yoyga tiralgan aylanaga ichki chizilgan burchaklar o‘zaro teng bo‘ladi.

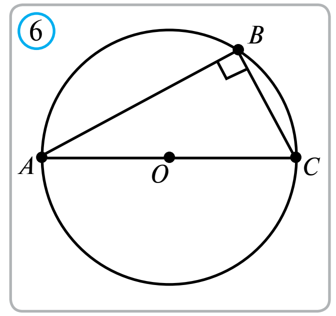
4°. Sektor va segment

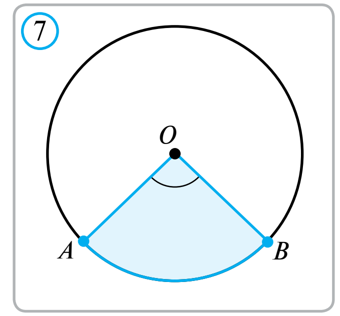
Doiraning ikki radius bilan chegaralangan bo‘lagi

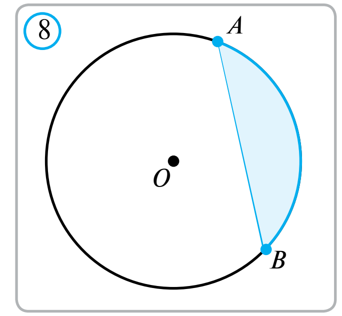
sektor deyiladi (*7-rasm*). Sektor yoyining uzunligi: $l=\frac{\pi R\alpha}{180}$, bu yerda α — markaziy burchakning gradus o‘lchovi.

Sektor yuzi: $S=\frac{\pi R^2\alpha}{360}$; $S=\frac{1}{2}Rl$.

Segment — doiraning vatar va shu vatar tiralgan yoy bilan chegaralangan bo‘lagi (*8-rasm*).







Segment yuzi: $S=S_{sektor}\pm S_{\Delta}=\frac{\pi R^2}{360}\cdot\alpha\pm\frac{1}{2}R^2\sin\alpha$

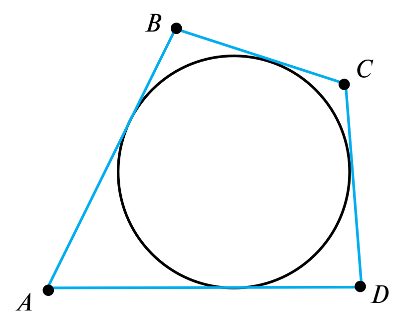
MUNTAZAM KO‘PBURCHAKLAR

Muntazam n burchakning tomoni a_n, perimetri P_n, yuzi S_n, ichki chizilgan aylana radiusi r_n, tashqi chizilgan aylana radiusi R_n, ichki burchagi α_n bo‘lsa,

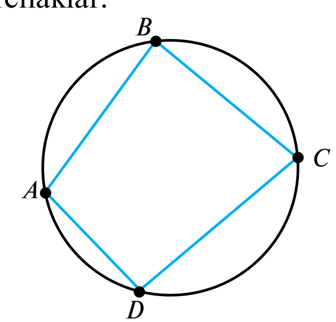
$$P_n=na_n,\qquad S_n=\frac{1}{2}P_nr_n=\frac{1}{2}na_nr_n,\qquad \alpha_n=\frac{(n-2)\cdot 180^\circ}{n}$$

$$R_n=\frac{a_n}{2\sin\frac{180^\circ}{n}},\qquad r_n=\frac{a_n}{2\operatorname{tg}\frac{180^\circ}{n}}$$

Aylanaga tashqi va ichki chizilgan to‘rtburchaklar.



$BC+AD=AB+CD$



$\angle A+\angle C=180^\circ$

$\angle B+\angle D=180^\circ$

10 dan 99 gacha bo‘lgan natural sonlar kvadratlarining jadvali

o‘nlik / *birlik*	1	2	3	4	5	6	7	8	9
0	100	400	900	1600	2500	3600	4900	6400	8100
1	121	441	961	1681	2601	3721	5041	6561	8281
2	144	484	1024	1764	2704	3844	5184	6724	8464
3	169	529	1089	1849	2809	3969	5329	6889	8649
4	196	576	1156	1936	2916	4036	5476	7056	8836
5	225	625	1225	2025	3025	4225	5625	7225	9025
6	256	676	1296	2116	3136	4356	5776	7396	9216
7	289	729	1369	2209	3249	4489	5929	7569	9409
8	324	784	1444	2304	3364	4624	6084	7744	9604
9	361	841	1521	2401	3481	4761	6241	7921	9801

Ayrim kattaliklar jadvali

$\pi \cong 3{,}1416$	$\sqrt{8} \cong 2{,}8284$
$\sqrt{2} \cong 1{,}4142$	$\sqrt{10} \cong 3{,}1623$
$\sqrt{3} \cong 1{,}7320$	$\frac{1}{\sqrt{2}} \cong 0{,}7071$
$\sqrt{5} \cong 2{,}2360$	$\frac{1}{\sqrt{3}} \cong 0{,}5774$
$\sqrt{6} \cong 2{,}4495$	$\frac{1}{\sqrt{\pi}} \cong 0{,}3183$
$\sqrt{7} \cong 2{,}6457$	

Trigonometrik funksiyalar qiymatlarining jadvali

α°	sinα	cosα	tgα	ctgα	α°	sinα	cosα	tgα	ctgα
1	0,0175	1,000	0,0175	57,3	**46**	0,719	0,695	1,036	0,966
2	0,0349	0,999	0,0349	28,6	**47**	0,731	0,682	1,072	0,933
3	0,0523	0,999	0,0524	19,1	**48**	0,743	0,669	1,111	0,900
4	0,0698	0,998	0,0699	14,3	**49**	0,755	0,656	1,150	0,869
5	0,0872	0,996	0,0875	11,4	**50**	0,766	0,643	1,192	0,839
6	0,1045	0,995	0,1051	9,51	**51**	0,777	0,629	1,235	0,810
7	0,1219	0,993	0,1228	8,14	**52**	0,788	0,616	1,280	0,781
8	0,139	0,990	0,141	7,11	**53**	0,799	0,602	1,327	0,754
9	0,156	0,988	0,158	6,31	**54**	0,809	0,588	1,376	0,727
10	0,174	0,985	0,176	5,67	**55**	0,819	0,574	1,428	0,700
11	0,191	0,982	0,194	5,145	**56**	0,829	0,559	1,483	0,675
12	0,208	0,978	0,213	4,507	**57**	0,839	0,545	1,540	0,649
13	0,225	0,974	0,231	4,331	**58**	0,848	0,530	1,600	0,625
14	0,242	0,970	0,249	4,011	**59**	0,857	0,515	1,664	0,601
15	0,259	0,966	0,268	3,732	**60**	0,866	0,500	1,732	0,577
16	0,276	0,961	0,287	3,487	**61**	0,875	0,485	1,804	0,554
17	0,292	0,956	0,306	3,271	**62**	0,883	0,469	1,881	0,532
18	0,309	0,951	0,325	3,078	**63**	0,891	0,454	1,963	0,510
19	0326	0,946	0,344	2,904	**64**	0,899	0,438	2,050	0,488
20	0,342	0,940	0,364	2,747	**65**	0,906	0,423	2,145	0,466
21	0,358	0,934	0,384	2,605	**66**	0,914	0,405	2,246	0,445
22	0,375	0,927	0,404	2,475	**67**	0,921	0,391	2,356	0,424
23	0,391	0,921	0,424	2,356	**68**	0,927	0,375	2,475	0,404
24	0,405	0,914	0,445	2,246	**69**	0,934	0,358	2,605	0,384
25	0,423	0,906	0,466	2,145	**70**	0,940	0,342	2,747	0,364
26	0,438	0,899	0,488	2,050	**71**	0,946	0,326	2,904	0,344
27	0,454	0,891	0,510	1,963	**72**	0,951	0,309	3,078	0,325
28	0,469	0,883	0,532	1,881	**73**	0,956	0,292	3,271	0,306
29	0,485	0,875	0,554	1,804	**74**	0,961	0,276	3,487	0,287
30	0,500	0,866	0,577	1,732	**75**	0,966	0,259	3,732	0,268
31	0,515	0,857	0,601	1,664	**76**	0,970	0,242	4,011	0,249
32	0,530	0,848	0,625	1,600	**77**	0,974	0,225	4,331	0,231
33	0,545	0,839	0,649	1,540	**78**	0,978	0,208	4,507	0,213
34	0,559	0,829	0,675	1,483	**79**	0,982	0,191	5,145	0,194
35	0,574	0,819	0,700	1,428	**80**	0,985	0,174	5,67	0,176
36	0,588	0,809	0,727	1,376	**81**	0,988	0,156	6,31	0,158
37	0,602	0,799	0,754	1,327	**82**	0,990	0,139	7,11	0,141
38	0,616	0,788	0,781	1,280	**83**	0,993	0,1219	8,14	0,1228
39	0,629	0,777	0,810	1,235	**84**	0,995	0,1045	9,51	0,1051
40	0,643	0,766	0,839	1,192	**85**	0,996	0,0872	11,4	0,0875
41	0,656	0,755	0,869	1,150	**86**	0,998	0,0698	14,3	0,0699
42	0,669	0,743	0,900	1,111	**87**	0,999	0,0523	19,1	0,0524
43	0,682	0,731	0,933	1,072	**88**	0,999	0,0349	28,6	0,0349
44	0,695	0,719	0,966	1,036	**89**	1,000	0,0175	57,3	0,0175
45	0,707	0,707	1,000	1,000	**90**	1,000	0,0000	-	0,0000

JAVOBLAR VA KO'RSATMALAR

1-dars. 1. 50°; 130°; 133°; 97°. 2. 12 *sm*. 3. 65°; 70°; 45°. 4. 105°; 130°; 125°. 5. 35°; 35°; 110°. 6. 94°; 56°; 30°. 7. 110°; 130°; 120°. 8. *Ko'rsatma:* to'rtta uchburchakning har birining tomonlari dastlabki uchburchakning mos tomonlarining yarmiga teng. 9. *Ko'rsatma: DF* kesma *ABH* uchburchakning ham, *CEB* uchburchakning ham o'rta chizig'i bo'ladi. 10. *Ko'rsatma: ANC* va *CKA* uchburchaklarning hamda ichki almashinuchi burchaklarning tengligidan foydalaning.

2-dars. 1. $6\sqrt{6}$. 2. 36. 3. 30°. 4. a) 80°; 80°; 20°; b) 70°; 70°; 40°. 5. 6,72 *sm*. 6. 54. 7. 5 *sm*; $25\sqrt{3}$ sm^2; 120°; 30°; 30°. 8. 55°; 60°; 65°. 9. 90°. 10. 140°. 11. 50°.

3-dars. 2. 78°; 102°; 78°; 102°. 3. 53°; 37°. 4. 110°; 70°; 110°; 70°. 5. 45°; 135°; 45°; 135°. 6. 20 *sm* yoki 28 *sm*. 7. *Ko'rsatma:* Oldin tomonlari $AB=2$ *sm*, $BC=6$ *sm* bo'lgan *ABCD* to'g'ri to'rtburchak yasang. Keyin markazlari *B* va *C* nuqtalarda radiusi 3 *sm* bo'lgan aylanalar yasang.

4-dars. 1. 30 *sm*. 2. 13 *sm*. 3. *Ko'rsatma:* 3-darsdan 7-masalaga qarang. 4. $880\sqrt{41}$ sm^2. 5. a) 4 *sm*, 8 *sm*; b) 45°, 90°; d) $16+8\sqrt{2}$ *sm*, 32 sm^2. 6. $18\sqrt{3}$ sm^2. 7. 30 sm^2. 8. 28 *sm*; $28\sqrt{2}$ *sm*.

5-dars. 3. Uchburchaklar o'xshash. 5. 5; 8; $\frac{1}{2}$. 6. 72; 162; 90.

6-dars. 3. 12 *m*. 4. 7,5 *sm*; 12,5 *sm*; 15 *sm*. 5. 73,5 m^2; 37,5 m^2. 6. Uchburchaklar o'xshash.

7-dars. 3. a) 4,5; b) 10,5; d) 4,5. 4. a) 10; b) 6; d) 4,5. 5. a) 5 *sm*, 3,5 *sm*; b) $5\frac{5}{7}$ *sm*, $2\frac{2}{7}$ *sm*. 6. a) 8; b) 3,5; d) 12,5. 8. 12 *sm*.

8-dars. 4. a) ha; b) ha; d) yo'q. 5. $2\frac{2}{3}$ *sm*, 9. 6. a) 15 *sm*; 20 *sm*; b) 24 *sm*; 18 *sm*; d) 144 sm^2; 256 sm^2. 8. 19,2 *m*.

9-dars. 2. ha. 3. a) va d); e) va f). 4. 108 sm^2. 5. 4 *sm*; 6 *sm*. 7. 4,8 *sm*. 9. 12.

10-dars. 2. a) va d); b) va e); g) va f). 3. 36 *m* yoki 20,25 *m*. 4. 12 *sm*; 14 *sm*. 6. $5\frac{5}{11}$ *sm*.

11-dars. 3. a) 15; b) $3\frac{1}{12}$; d) $3\frac{5}{17}$. 4. 18 *sm*; 6 *sm*. 5. 29 dm^2. 6. 6 *dm*. 7. *m*:*n*.

12-dars. 1. $3\frac{3}{17}$ *m*; 13,6 *sm*. 7. *n*:*m*. 8. a) *S*:4; b) *S*:2; d) *S*:4.

13-dars. II. 1. 12 sm^2. 2. 8,4. 3. 2,4. 4. 24. 5. 8. 6. 1,6. III. 1. 6 *sm*. 2. 65 *dm*; 52 *dm*. 3. $AB \| EF$.

14-dars. 5. 1 *km* 750 *m*. 8. 7,2 *sm*. 9. $k=\frac{1}{2}$ yoki $k=2$.

15-dars. 4. $k=2$. 5. 6 sm^2; 24 sm^2. 6. 104 sm^2. 7. Har ikki holda $k=1$. 8. 1,2 m^2. 9. 16 *sm*, 32 *sm*.

16-dars. 4. $\frac{2}{3}$; $\frac{4}{9}$. 5. $X*X$ va $Y*Y$ nurlarning kesishish nuqtasi gomotetiya markazidir. 6. $OX_1=2\cdot OX$. 7. *Ko'rsatmalar:* Mavzudagi masalaning yechimidan foydalaning.

8. a) $OA_1=\frac{2}{3}OA$; b) $OA_1=20A$; d) $OA_1=30A$; e) $OA_1=OA$. 9. *Ko'rsatma:* Mavzudagi 3-rasmdan foydalaning.

17-dars. 4. a) $P_2=42$; $k=\frac{1}{2}$; b) $S_1=12$, $k=2$; d) $P_1=150\sqrt{2}$, $k=\sqrt{2}$; e) $P_1=10$, $S_2=216$.

18-dars. 1. ≈6,97 *m*. 2. 300 *m*. 3. ≈72 *m*. 4. 6,6 *m*.

19-dars. 1. 9. 2. 12 *dm*. 3. 8 *m*. 4. 24 dm^2. 6. *Ko'rsatma: ABC* uchburchak chizing, ko'pburchaklar yasash mavzusidagi 1-masaladan foydalanib, chizilgan uchburchak tomonlaridan uch marta kichik uchburchak yasang.

20-dars. 1. 72°; 72°; 36°. 3. 12 sm^2. 4. 15 000 000 *km*. 5. a) Ha; b) Ha. 7. 6 *sm*, 12 *sm*, 18 *sm*. 8. 63 *m*.

21-dars. II. 1. 8 *sm*. 2. $4\frac{4}{9}$ *sm*. 3. 48 *m*. 4. 4 *sm*; 0,5 sm^2. 5. $5\frac{1}{3}$ *m*. 6. 867 *km*. III. 1. 7,5 *m*. 2. 6 *sm*. 3. a) 7,5 *sm*; b) 6 *sm*; d) 16,2 *sm*. *Qiziqarli masalalar:* 1. O'zgarmaydi. 2. a) Ha; b) Yo'q. 3. *Ko'rsatma:* Chizg'ich bilan har bir qo'g'irchoqning bo'yini o'lchang va ularning nisbatini toping.

22-dars. 4. $\sin A=\frac{5}{13}$; $\cos A=\frac{12}{13}$; $\text{tg}A=\frac{5}{12}$; $\text{ctg}A=\frac{12}{5}$. 5. a) $\sin A=\frac{7}{25}$; $\cos A=\frac{24}{25}$; $\text{tg}A=\frac{7}{24}$; $\text{ctg}A=\frac{24}{7}$. $\sin B=\frac{24}{25}$; $\cos B=\frac{7}{25}$; $\text{tg}B=\frac{24}{7}$; $\text{ctg}B=\frac{7}{25}$. 6. $BC=$ [illegible]; $AB=$ [illegible]. 7. $AB=34$; $AC=30$.

23-dars. 4. a) 15 *sm*; b) 8 *sm*; d) 36,125 *sm*; e) 31,875 *sm*. 5. [illegible] *sm*. 7. 42 sm^2. 8. 21 sm^2. 9. 32 sm^2. 10. 180 sm^2.

24-dars. 3. $2\sqrt{3}$ *dm*, $4\sqrt{3}$ *dm*. 4. a) $12+4\sqrt{3}$; b) $6+6\sqrt{3}$; d) $16+8\sqrt{2}$. 5. a) $\angle A=45°$, $\angle B=45°$; b) $\angle A=60°$; $\angle B=30°$; d) $\angle A=30°$, $\angle B=60°$. 6. [illegible] sm^2. 7. 7 *sm*; 24 *sm*, $\cos A=\frac{24}{25}$, $\text{tg}A=\frac{7}{24}$; $\text{ctg}A=\frac{24}{7}$. 8. 120°; 120°; 60°; 60°.

25-dars. 1. 36 sm^2. 2. 24 *sm*. 3. a) $6\sqrt{3}$; b) 30; d) $\frac{105\sqrt{3}}{4}$. 4. $(24+4\sqrt{3})$ *sm*; $(24+8\sqrt{3})$ sm^2. 5. $10\sqrt{3}$ *sm*. 6. a) $\frac{\sqrt{3}}{6}$; b) $\frac{1}{2}$; d) $\frac{\sqrt{3}}{2}$. 7. ≈807 m^2. 8. ≈88 *m*.

26-dars. 2. tangens 90° da, kotangens 0° va 180° da. 3. $\sin\alpha>0$, $\cos\alpha<0$, $\text{tg}\alpha<0$, $\text{ctg}\alpha<0$. 6. $\sin 45°=\sin 135°$; $\cos 45°>\cos 135°$. 7. 24 *sm*; 18 sm^2.

27-dars. 2. 1) $\sin^2\alpha$; 2) $\cos^2\alpha$; 3) 1; 4) $\cos^2\alpha$; 5) $\cos^2\alpha$; 6) $\sin^2\alpha$. 3. a) [illegible]; b) $\frac{\sqrt{3}}{4}$; d) 0. 4. $6\sqrt{3}$ sm^2. 5. $0,8\sqrt{3}$ *sm*, $1,6\sqrt{3}$ *sm*. 6. a) [illegible]; b) $\frac{\sqrt{3}}{4}$; e) 0. 9. a) *A* [illegible]; e) $A(-2;0)$; f) *A* [illegible].

28-dars. 4. a) 150°; b) 135°; d) 135°; e) 150°. 5. a) 0; b) 1; d) 0; e) −3,5; 6. a) 1; b) 1; d) 1. 7. 3,5 *sm*. 8. $36\sqrt{3}$ sm^2. 9. a) $\frac{1}{2}$; $-\frac{1}{2}$; b) ± [illegible]; d) 0. 10*. a) 30°; b) 135°; d) 150°.

29-dars. III. 2. 1000, 37°. 3. 2°. 4. 34°. 5. $2\sqrt{3}$; $4\sqrt{3}$. 6. $3\sqrt{3}$ *sm*. 7. 5 *sm*. 8. 12, $24\sqrt{3}$. 9. 20 *sm*, 100 sm^2. 10. 4, $16\sqrt{3}$. 11. 30°; 60°. 13. 12 *sm*; $4\sqrt{3}$ *sm*; $8\sqrt{3}$ *sm*. 14. 32 sm^2. 15. $\frac{15}{17}$; [illegible]; [illegible]. 16. [illegible]. 17. $12(\sqrt{3}+1)$, $72(\sqrt{3}+1)$. IV. 1. $\frac{15}{17}$; $\frac{8}{15}$; $\frac{15}{8}$.

2. $2\sqrt{77}$; 13°; 77°. **4.** *Ko'rsatma:* Uchburchak tengsizligi haqidagi teoremadan foydalaning.

30-dars. **2.** a) 6 sm^2; b) 73,5 sm^2; d) 6 sm^2. **3.** 36 sm^2. **4.** $49\sqrt{2}$ sm^2. **5.** $54\sqrt{3}$ sm^2. **6.** $2\frac{2}{3}$ *sm*; $4,5\sqrt{2}$ *sm*. **7.** $\frac{h_a h_b}{2\sin\alpha}$. **8.** $4,8\sqrt{3}$ *sm*.

31-dars. **2.** a) $BC=6$; b) $AB=8\sqrt{2}$; d) $AC=7\sqrt{2}$. **3.** a) $\sin C=$ [illegible]; b) $\sin A=$ [illegible]; d) $\sin B=$ [illegible]. **4.** 4,8 *dm*. **5.** 30° yoki 150°. **6.** Mumkin. **7.** $AB\approx21,1$ *m*; $\angle B\approx37°$, $\angle C\approx76°$. **8.** 76°; 26,1 *sm*; 23,8 *sm*.

32-dars. **2.** a) $\sqrt{13}$ *sm*; b) 4 *m*; d) $\sqrt{283}$ *dm*. **3.** $\frac{1}{5}$; $\frac{19}{35}$; [illegible]. **4.** $2\sqrt{13}$ *sm* yoki $2\sqrt{109}$ *sm*. **5.** $\sqrt{31}$ *sm*, $\sqrt{91}$ *sm*. **6.** $\sqrt{109}$ *sm*, $\sqrt{39}$ *sm*.
7. *Ko'rsatma: ADC* va *BDC* uchburchaklarga kosinuslar teoremasini qo'llab, a^2 va c^2 ni toping, so'ngra bu tengliklarni hadma-had qo'shing. **8.** [illegible] *sm*; [illegible] *sm*; [illegible] *sm*.

33-dars. **1.** $\angle B$ va $\angle C$. **2.** *AB* va *BC*. **3.** a) o'tkir burchakli; b) to'g'ri burchakli; d) o'tmas burchakli. **4.** a) 8[illegible]; b) 8[illegible]; d) 24[illegible]; e) [illegible]. **6.** *Ko'rsatma:* Sinuslar teoremasidan foydalaning. **7.** *Ko'rsatma:* 6-masalaga o'xshash yechiladi. **8.** *Ko'rsatma:* Sinuslar teoremasidan foydalaning.

34-dars. **1.** a) $10\sqrt{3}$; b) $28\sqrt{2}$; d) 12; e) ≈0,1532. **2.** a) —2,5; b) 0; d) 2. **3.** a) 8; b) 24; d) 8; e) 0. **5.** a) —7,5; d) 0. **6.** $a\perp b$, $c\perp d$.

35-dars. **1.** a) $\alpha=90°$, $a=b=5$, $c=5\sqrt{2}$. b) $\gamma\approx45°$; $b\approx17,9$, $c\approx14,6$; d) $\alpha=20°$; $b\approx65.8$; $c\approx88,6$; e) $\gamma=119°$; $a\approx16,7$; $b\approx11,2$. **2.** a) $c\approx5,29$; $\alpha\approx79°6'$; $\beta\approx138°21'$; b) $c\approx53,09$; $\alpha\approx11°39'$; $\beta\approx38°21'$; d) $a\approx19,9$; $\beta\approx58°19'$; $\gamma\approx936°41'$; e) $a\approx22,9$; $\beta\approx21°$; $\gamma\approx15°$. **3.** a) $\alpha\approx29°$; $\beta\approx47°$; $\gamma\approx104°$; b) $\alpha\approx54°$; $\beta\approx13°$; $\gamma\approx113°$; d) $\alpha\approx34°$; $\beta\approx44°$; $\gamma\approx102°$; e) $\alpha\approx39°$; $\beta\approx93°$; $\gamma\approx48°$.

36-dars. **1.** a) $2\sqrt{3}$ *sm*; b) 16 *sm*; d) [illegible]. **2.** $4\sqrt{2}$ *m*; 8 *m* va $4+4\sqrt{3}$ *m*. **3.** $50\sqrt{3}$ *kg*. **4.** 14 *sm*. **5.** $2\sqrt{14}$ *sm*. **6.** $6\sqrt{3}$ *sm*. **7.** 50 *sm*.

37-dars. **1.** ≈10,8 *m*. **2.** ≈15 *m*. **3.** ≈43,4 *m*. **4.** ≈35°. **5.** ≈ 73,2 *m*. **6.** ≈ 49 *m*. **7.** Asfalt yo'ldan.

38-39-dars. **II. 1.** $3\sqrt{6}$, $3\sqrt{2}$. **2.** [illegible]; 0,89; —0,65. **3.** $2\sqrt{7}$ *sm*; [illegible] *sm*. **4.** 30[illegible] *sm*. **5.** 28 *sm*. **6.** 8 sm^2; $(4+4\sqrt{5})$ *sm*; $h_a=4$ *sm*, $h_b=0,8\sqrt{5}$ *sm*. **7.** $2\sqrt{13}$. **8.** a) o'tkir burchakli; b) to'g'ri burchakli, d) o'tmas burchakli. **9.** 63 sm^2. **10.** ≈3,7 *sm*. **11.** 7 *sm*. **12.** 6. **13.** 0. **14.** —9. **15.** 135°. **16.** OC≈9,6. **17.** $(24+24\sqrt{3})$ *sm*. **18.** 5. **III. 1.** ≈109°. **2.** $\gamma=100°$, $a\approx3,25$; $c\approx6,43$. **3.** 6,25; 14,76.

40-dars. **2.** a) Har qanday uchburchak aylanaga ichki chizilishi mumkin; b) Qarama-qarshi burchaklari yig'indisi 180° bo'lgan to'rtburchaklar. **3.** Bitta yoyga tiralgan burchaklar teng. **4.** 10 *sm*. **5.** 672 sm^2. **6.** a) $10\sqrt{3}$ *sm*; b) $10\sqrt{2}$ *sm*; d) $10\sqrt{2}$ *sm*; $10\sqrt{2}$ *sm*; 20 *sm*. **7.** 8[illegible] *sm*. **8.** ΔABF da, $\angle BAF+\angle AFB=90°$, $\angle ABF=90°$. Demak, AF – diametr. **9.** Qarama-qarshi burchaklari yig'indisi 180°, ya'ni aylanaga ichki chizish mumkin. **10.** *Ko'rsatma:* Bitta asos va bir yon tomonning o'rta perpendikulari kesishgan nuqta aylana markazi bo'ladi.

41-dars. **2.** 7,2 *sm*. **3.** a) 16,6; b) 22; d) 22,6. **4.** a) 2,5; b) 3,5; d) 2. **8.** 6 *sm*.

42-dars. **3.** a) 60°; b) 108°; d) 120°; e) 144°; f) 160°. **4.** a) 120°; b) 72°; d) 120°; e) 36°; f) 30°. **5.** a) 3; b) 4; d) 8; e) 12.

43-dars. **1.** 3 *sm* va $3\sqrt{2}$ *sm*. **2.** $\sqrt{3}$ va $2\sqrt{3}$. **7.** a) 6; b) 12; d) 10; e) 20; f) 5.

44-dars. **3.** 8 *sm*; $8\sqrt{2}$ *sm*; $8\sqrt{3}$ *sm*; $8\sqrt{2}+3$ *sm*; 16 *sm*. **4.** [illegible] *sm*; **5.** a) $20\sqrt{2}$ *sm*; b) 40 *sm*. **6.** [illegible] *sm*.

45-dars. **I. 1.** E; **2.** D; **3.** D; **4.** B; **5.** B; **6.** E; **7.** E. **III. 1.** $\sqrt{3}$:4: $6\sqrt{3}$. **2.** 3:4. **3.** a) ≈5,780 *sm*; b) ≈4,142 *sm*; d) ≈2,679 *sm*. **4.** $S=\sqrt{2}R^2$. **5.** 24 sm^2. **IV. 1.** 4 sm; 13 sm. **2.** a) 80 *sm*; b) $20\sqrt{2-\sqrt{3}}$ *sm*; $40\sqrt{2-\sqrt{3}}$ *sm*; d) 200 sm^2. **3.** $4\sqrt{3}$ *sm*; 8 *sm*. 4. $\frac{27\sqrt{3}}{4}$ sm^2.

46-dars. **2.** a) 3 marta ortadi; b) 6π *sm* ga ortadi; d) 3 marta kamayadi; e) 6π *sm* ga kamayadi. **3.** 6369 *km*. **4.** a) [illegible]; b) $\pi\sqrt{a^2+b^2}$; d) [illegible]. **5.** a) πa; b) $\pi c(\sqrt{2}-1)$; d) $\pi c(\sin\alpha+\cos\alpha-1)$. **6.** 1,5 *m*. **7.** 66348 marta.

47-dars. **1.** a) π *sm*; b) $1,5\pi$ *sm*; d) 3π *sm*; e) 4π *sm*. **2.** a) [illegible]; b) [illegible]; d) [illegible]. **3.** a) ≈69°; b) 120°; d) 150°. **4.** a) [illegible] *sm*; b) 2π *sm*; d) [illegible] *sm*; **5.** a) 4π; b) 16π. **7.** 2.

48-dars. **3.** k^2 marta ortdi; b) k^2 marta kamayadi. **4.** $6,25\pi$ sm^2; $12,5\pi$ sm^2. **5.** $2,25\pi$ sm^2; 9π sm^2. **6.** $(\pi-2)R^2$. **7.** 21,25 π sm^2. **8.** 7,5 sm^2.

49-dars. **3.** a) [illegible]π sm^2; [illegible] sm^2; b) $6,125\pi$ sm^2; [illegible] sm^2; d) [illegible] sm^2; [illegible] sm^2; e) [illegible] sm^2; [illegible] sm^2. **4.** a) a^2[illegible]; b) a^2[illegible]; d) [illegible]a^2; **5.** π sm^2; 3π sm^2; 5π sm^2; 7π sm^2. **6.** $\frac{25(2\pi-3\sqrt{3})}{3}$ sm^2; [illegible] sm^2; **7.** [illegible] sm^2. **8.** $S_1<S<S_2$; 300 sm^2 < 314sm^2 < 321,48 sm^2.

50-dars. **1.** Doiraniki katta. **2.** $\frac{160\pi}{3}$ sm^2. **3.** $5,76\pi$ sm^2. **4.** $8.(\pi-2)$ sm^2. **6.** 6π sm^2; 10π *sm*.

51-dars. **II. 1.** $6\sqrt{2+\sqrt{2}}$. **2.** [illegible] *dm*. **3.** 30 *sm*. **4.** 90°. **5.** 3. **6.** π va $6,25\pi$. **7.** $\frac{10\pi+3\sqrt{3}}{2\pi-3\sqrt{3}}$. **8.** [illegible]. **9.** [illegible]$.a^2$. **10.** $1,5\pi$. **11.** 7. **12.** ≈9π—26,04. **13.** π. **14.** $54\sqrt{3}$—24π. **15.** $\frac{3\pi}{8}$. **III. 2.** $8\sqrt{3}$ *sm*. **3.** a) [illegible] *sm*; b) [illegible] sm^2; d) $\frac{216\pi+81\sqrt{3}}{\pi^2}$ sm^2.

52-dars. **3.** $5\sqrt{2}$ *sm*. **4.** 12 *sm*. **5.** 44 *m*, 60 *m*. **7.** 1:7. **8.** $AB\cos\alpha$.

53-dars. **1.** a) 30 *sm*, 12 *sm*; b) 9 *sm*, 12 *sm*, 21 *sm*; d) 3 *sm*, 15 *sm*, 3 *sm*, 21 *sm*.

3. 6 *sm*; 10,5 *sm*. 4. 9 *sm*, 12 *sm*, 15 *sm*, 18 *sm*. 5. 60°. 6. 21 *sm*. 7. 20 *sm*.

54-dars. 1. *Ko'rsatma:* $\Delta ACD \sim \Delta CBD \sim \Delta ABC$. 2. 25 *sm*, 15 *sm*, 20 *sm*. 3. [illegible] *sm*. 4. a) 5, 4; b) 24, 25; d) 8,10. 5. 16:25. 6. 56,16 sm^2. 7. 60 sm^2. 8. [illegible]; $\frac{4}{9}$; [illegible].

55-dars. 2. *Ko'rsatma:* a) katetlari *a* va *b* bo'lgan to'g'ri burchakli uchburchak yasang; b) gipotenuzasi *a*, bir kateti *b* bo'lgan to'g'ri burchakli uchburchak yasang. 3. *Ko'rsatma:* Katetlari $AB=BC=1$ bo'lgan ΔABC yasang. So'ng kateti $CC_1=1$ va $\angle C_1=90°$ bo'lgan ΔBCC_1 yasang va hokazo. 4. a) 20; b) 45; d) 37,5. 5. 225 sm^2. 6. 180 sm^2. 7. 25:9. 9. $OC \geq OD$ bo'lgani uchun tengsizlik har doim to'g'ri.

56-dars. 1. a) 6,25; b) 12; d) 0,25. 2. a) 8 *sm*; b) 2,5 *sm*; d) 0,9 *sm*. 3. a) 4 *dm*; b) 4 *dm*. 4. 4 *sm*. 6. 9 *dm*; 16 *dm*.

57-dars. 1. 10 *sm*. 2. 2 *sm*. 3. a) 2,5; b) 4; d) 2. 4. a) $4\sqrt{6}-1$ *sm*; b) 6 *sm*. 5. 1:6. 6. 6 *sm*. 7. 3. 8. 1:4.

58-dars. II. **1.** 18 *sm*; 32 *sm*. **2.** 4 *sm*; **3.** 8 *sm*; **4.** 6,4 *dm*. **5.** 8 *sm*. **6.** 1,5. **7.** 5. **8.** 6. **9.** 45 dm^2. **10.** 4 *sm*. **11.** 8 *sm*. **12.** 6. **13.** 60°. **14.** 45°. **15.** 4:9. III. **1.** 8 *sm*. **2.** 5 *dm*. **3.** 4 *sm*; 8 *sm*.

59-dars. 1. a) 12 (kv.b.); b) 20 (kv.b.); d) 12 (kv.b.); e) 12 (kv.b.); f) 42 (kv.b.). 2. 4 *sm*. 3. a) 12; b) 288. 4. a) [illegible]; b) 2[illegible]. 5. a) (3;2); b) (2,5;—0,5); d) (—1;4); e) (—0,5;3,1). 6. D(2;—1). 8. 10 *sm*; 25 *sm*. 9. 60°; 90°; 120°; 90°. 10. 6 *sm*. 11. $6\sqrt{2}$ *sm*.

60-dars. 1. a) 4; b) 6; d) 5; e) 5. 2. $4\sqrt{13}+\sqrt{82}+\sqrt{58}$. 4. Trapetsiya. 5. $x=4$, $y=3$. 7. $b-a$; $-a-2b$; $2a+b$. 8. 5*N*. 9. $18\sqrt{3}$; $27\sqrt{3}$. 10. $4\sqrt{2}$ *sm*. 11. $PA=PB$ va $PA=PC$ bo'lgani uchun $PB=PC$.

61-dars. 2. 45°; 90°; 135°; 90°. 3. 45°. 4. 60°. 5. 3 sm; 8 sm. 7. 28 sm. 9. 45°.

62-dars. 1. 8,4 *sm*, 10,5 *sm*, 14,7 *sm*. 2. 175 dm^2; 252 dm^2. 3. 12 sm^2. 4. 6. 5. $9(3-\sqrt{3})$ sm^2. 6. 8 *sm*. 7. 5 *sm*; 2 *sm*; 5 *sm*; 8 *sm*. 8. 3 *sm*, 4 *sm*.

63-dars. 1. 2 *sm*. 2. 6 *dm*; 9,6 *dm*; 6,5 *dm*; 10,4 *dm*. 3. Ha. 4. $\sqrt[4]{27}$; $3\sqrt[4]{3}$. **5.** 16,9 *sm*. 6. 150 sm^2. 7. (0;—6). 8. Birinchisida. 9. 80 ta. 10. 7 dm^2. 11. a) 180 dm^3; b) 105 sm^3; d) 1296 sm^3.

64-dars. 1. —12,5. 2. 20 *sm*. 3. Ha. 4. 5 *sm*, 13 *sm*. 5. 3 *sm*. 6. 8 *sm*. 7. 30π *sm*. 8. 5 *sm*. 9. 25 *sm* yoki 20 $\sqrt{2}$ *sm*.

65-dars. 1. 4,5 *sm*; 6,75 *sm*. 2. [illegible] *sm*, 4 *sm*; 4,8 *sm*. 3. 12 *sm*. 4. 2 *sm*. 5. 6,72. 6. $2\sqrt{26}\pi$ 7. $25\sqrt{3}$ sm^2. 8. 84 sm^2. 9. $675\sqrt{3}$ sm^2.

66-dars. 1. $4\frac{129}{1024}$. 2. 100°; 80°. 3. 4 *sm*. 4. 24 sm^2. 5. 4,8 *m*. 6. 30 sm^2. **7.** 7. 8. 10 *sm* yoki $2\sqrt{97}$ *sm*.

67–68-dars. 1. a) 9; b) 4 sm^2; d) 3,5 *sm*; e) [illegible] $TB-CA$; f) 0,2. 2. $\Delta CMH \sim \Delta BCA$.

X 18 Haydarov Bahodir Qayumovich
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GEOMETRIYA

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Ijaraga berilgan darslik holatini ko‘rsatuvchi jadval

T/r	O‘quvchining ismi va familiyasi	O‘quv yili	Darslikning olingandagi holati	Sinf rahbari-ning imzosi	Darslikning topshiril-gandagi holati	Sinf rahbari-ning imzosi
1						
2						
3						
4						
5						
6						

Darslik ijaraga berilib, o‘quv yili yakunida qaytarib olinganda yuqoridagi jadval sinf rahbari tomonidan quyidagi baholash mezonlariga asosan to‘ldiriladi:

Yangi	Darslikning birinchi marotaba foydalanishga berilgandagi holati.
Yaxshi	Muqova butun, darslikning asosiy qismidan ajralmagan. Barcha varaqlari mavjud, yirtilmagan, ko‘chmagan, betlarida yozuv va chiziqlar yo‘q.
Qoniqarli	Muqova ezilgan, birmuncha chizilib, chetlari yedirilgan, darslikning asosiy qismidan ajralish holati bor, foydalanuvchi tomonidan qoniqarli ta’mirlangan. Ko‘chgan varaqlari qayta ta’mirlangan, ayrim betlariga chizilgan.
Qoniqarsiz	Muqovaga chizilgan, yirtilgan, asosiy qismdan ajralgan yoki butunlay yo‘q, qoniqarsiz ta’mirlangan. Betlari yirtilgan, varaqlari yetishmaydi, chizib, bo‘yab tashlangan. Darslikni tiklab bo‘lmaydi.